4/13/11

· Review

Friday

Exam 4 / Ch. 14.1-14.3, 14.5-14.7

Hw due

$\ln a + \ln b = \ln(ab)$

14.1

(26) $\displaystyle\int_1^3 \int_0^y \frac{4}{x^2+y^2}\,dx\,dy = \int_1^3 \frac{4}{y}\arctan\frac{x}{y}\Big|_0^y\,dy$

$\displaystyle = \int_1^3 \frac{4}{y}(\arctan 1 - \arctan 0)\,dx$

$\displaystyle = \int_1^3 \frac{4}{y}\left(\frac{\pi}{4} - 0\right)dy$

$\displaystyle = \pi\int_1^3 \frac{dy}{y}$

$\displaystyle = \pi \ln|y|\Big|_1^3$

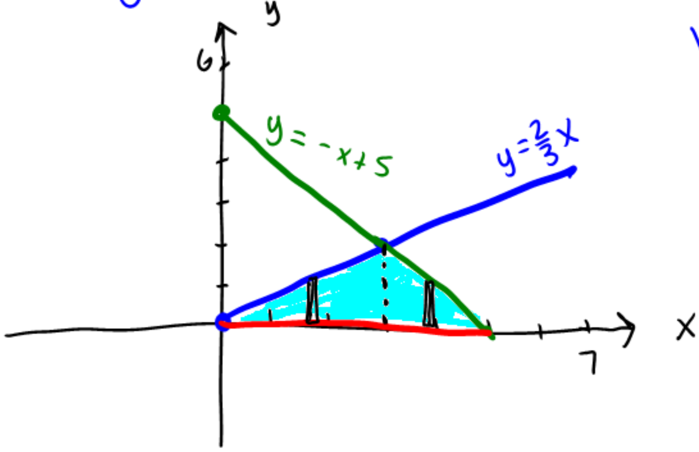
$= \pi(\ln 3 - \ln 1)$

$= \pi \ln 3$

$= \boxed{\ln 3^{\pi}}$

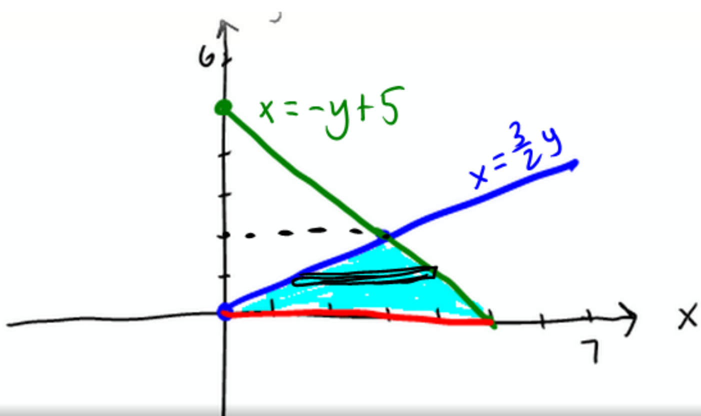
(41) $2x - 3y = 0$, $x + y = 5$, $y = 0$

$y = \frac{2}{3}x$ $y = -x+5$



Vertically Simple

$\displaystyle\int_0^3\int_0^{\frac{2}{3}x} dy\,dx + \int_3^5\int_0^{-x+5} dy\,dx$



Horizontally simple

$\displaystyle\int_0^2\int_{\frac{3}{2}y}^{-y+5} dx\,dy$

14.2: 43 41 8th ed.

$z = x^2 + y^2, \quad x^2 + y^2 = 4, \quad z = 0$

$$2\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2)\, dy\, dx = 2\int_0^2 \left(x^2 y + \frac{y^3}{3}\right)\Bigg|_0^{\sqrt{4-x^2}} dx$$

$$= 2\int_0^2 \left(x^2(4-x^2)^{1/2} + \frac{(4-x^2)^{3/2}}{3}\right) dx$$

$$= 2\int_0^{\pi/2} \left[(4\sin^2\theta)(2\cos\theta) + \frac{8\cos^3\theta}{3}\right] 2\cos\theta\, d\theta$$

$$= 2\int_0^{\pi/2}$$

trig sub:

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = \sqrt{4 - 4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta)}$

$= 2\cos\theta$

$(\sqrt{4-x^2})^3 = (2\cos\theta)^3$

$= 8\cos^3\theta$

$x^2 = (2\sin\theta)^2$

$= 4\sin^2\theta$

limits

$0 = 2\sin\theta$

$0 = \sin\theta$

$\theta = 0$

$2 = 2\sin\theta$

$1 = \sin\theta \rightarrow \theta = \frac{\pi}{2}$

$$\int x^{-2}\, dx = \frac{x^{-2+1}}{-2+1}$$

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29 $z = \frac{1}{(x+1)^2 (y+1)^2}, \quad 0 \le x < \infty, \quad 0 \le y < \infty$

$$\int_0^\infty \int_0^\infty \frac{1}{(x+1)^2 (y+1)^2}\, dy\, dx = \int_0^\infty \frac{1}{(x+1)^2}\left[\int_0^\infty \frac{dy}{(y+1)^2} dy\right] dx$$

$$= \int_0^\infty \frac{1}{(x+1)^2}\left[-\frac{1}{y+1}\Bigg|_0^\infty\right] dx$$

$$= \int_0^\infty \frac{1}{(x+1)^2}\left[-0 - (1)\right] dx$$

$$= \int_0^\infty \frac{1}{(x+1)^2}\, dx$$

$$= -\frac{1}{x+1}\Bigg|_0^\infty$$

$$= -(0 - 1) \rightarrow \boxed{1}$$

43 cont:

$$2\int_0^{\pi/2}\left[(4\sin^2\theta)(2\cos\theta)+\frac{8\cos^3\theta}{3}\right]2\cos\theta\,d\theta$$

$$=2\left[\int_0^{\pi/2}16\sin^2\theta\cos^2\theta\,d\theta+\int_0^{\pi/2}\frac{16}{3}\cos^4\theta\,d\theta\right]$$

$$=32\int_0^{\pi/2}(1-\cos^2\theta)\cos^2\theta\,d\theta+\frac{32}{3}\int_0^{\pi/2}\left[\cos^2\theta\right]^2d\theta$$

$$=32\int_0^{\pi/2}\left[\cos^2\theta-\left[\cos^2\theta\right]^2\right]d\theta+\frac{32}{3}\int_0^{\pi/2}\left(\frac{1+\cos2\theta}{2}\right)^2d\theta$$

$$=32\int_0^{\pi/2}\left[\left(\frac{1+\cos2\theta}{2}\right)-\left(\frac{1+\cos2\theta}{2}\right)^2\right]d\theta+\frac{8}{3}\int_0^{\pi/2}\left(1+2\cos2\theta+\cos^2 2\theta\right)d\theta$$

$$=16\int_0^{\pi/2}(1+\cos2\theta)\,d\theta-4\int_0^{\pi/2}\left(1+2\cos2\theta+\overbrace{\cos^2 2\theta}^{\frac{1+\cos4\theta}{2}}\right)d\theta$$

$$+\frac{8}{3}\int_0^{\pi/2}\left(1+2\cos2\theta+\frac{1}{2}+\frac{\cos2\theta}{2}\right)d\theta$$

$$=16\left(\theta+\frac{\sin2\theta}{2}\right)\Big|_0^{\pi/2}-4\left(\frac{3}{2}\theta+\sin2\theta+\frac{\sin4\theta}{8}\right)\Big|_0^{\pi/2}$$

$$+\frac{8}{3}\left(\frac{3}{2}\theta+\sin2\theta+\frac{\sin4\theta}{8}\right)\Big|_0^{\pi/2}$$

$$=16\left[(\pi/2+0)-(0)\right]-4\left[\left(\frac{3}{2}(\pi/2)+0+0\right)-(0)\right]+\frac{8}{3}\left[\left(\frac{3}{2}\left(\frac{\pi}{2}\right)+0+0\right)-(0)\right]$$

$$=8\pi-3\pi+2\pi$$

$$=\boxed{7\pi}$$

(41) $\displaystyle\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \left[(4-x^2-y^2)-(4-2x)\right] dy\,dx$

found from graph

$z = 4-x^2-y^2$

$z = 4-2x$

$\cancel{4}-x^2-y^2 = \cancel{4}-2x$

$-y^2 = x^2-2x$

$-1+y^2 = -(x^2-2x+(-1)^2)$

$y^2-1 = -(x-1)^2$

$y^2 = 1-(x-1)^2$

$y = \pm\sqrt{1-(x-1)^2}$

14.3

(23) $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} xy\,dy\,dx$

$y = \sqrt{2x-x^2} \to y^2 = 2x-x^2$

and $y = 0$

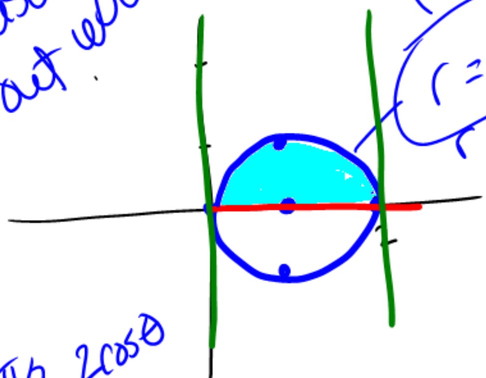
$y^2+x^2-2x = 0$

$y^2+(x-1)^2 = 1$

or Reason it out using

$r = a\cos\theta$

$r = 2\cos\theta$



$\displaystyle\int_0^{\pi/2} \int_0^{2\cos\theta} r(r\cos\theta)(r\sin\theta)\,dr\,d\theta$

One way

$y = \sqrt{2x-x^2}$

$y^2 = 2x-x^2$

$(r\sin\theta)^2 = 2r\cos\theta-(r\cos\theta)^2$

$r^2\sin^2\theta = 2r\cos\theta - r^2\cos^2\theta$

$r^2\sin^2\theta + r^2\cos^2\theta = 2r\cos\theta$

$r^2(\sin^2\theta+\cos^2\theta) = 2r\cos\theta$

$r^2-2r\cos\theta = 0$

$r(r-2\cos\theta) = 0$

$r = 0$ or $r = 2\cos\theta$