2/23/11

· Review Ch.12

Friday

· Exam 2

↳ both sides of a 3x5 card for formulas and general steps - be sure to include the position function for a projectile

· HW 12.1 - 12.5 is due

12.1: 8, 26, 55, 58

8 $\vec{r}(t) = \vec{F}(t) \times \vec{G}(t)$

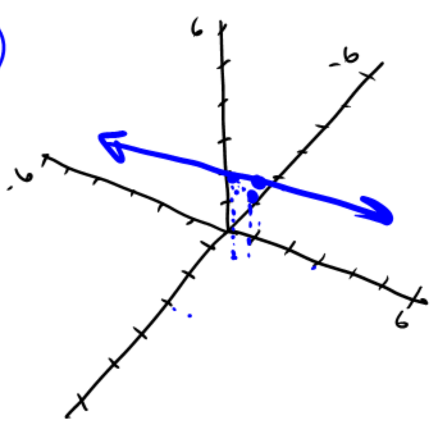
$\vec{F}(t) = t^3\hat{i} - t\hat{j} + t\hat{k}$, $\vec{G}(t) = \sqrt[3]{t}\,\hat{i} + \frac{1}{t+1}\hat{j} + (t+2)\hat{k}$

$$\vec{r}(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^3 & -t & t \\ t^{1/3} & \frac{1}{t+1} & (t+2) \end{vmatrix}$$

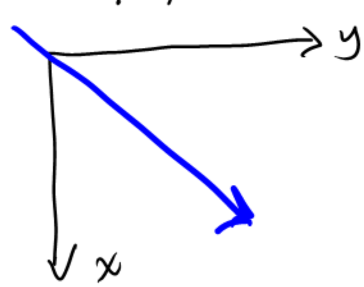
$$= \left[-t(t+2) - \frac{t}{t+1}\right]\hat{i} - \left[t^3(t+2) - t^{4/3}\right]\hat{j} + \left[\frac{t^3}{t+1} + t^{4/3}\right]\hat{k}$$

$\boxed{\text{Domain: } (-\infty, -1) \cup (-1, \infty)}$

26 $\vec{r}(t) = t\hat{i} + t\hat{j} + 2\hat{k}$

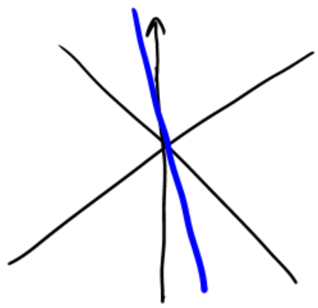


a) from (0, 0, 20)



b) from (10, 0, 0)

c) from $(5,5,5)$



55. $\frac{x^2}{16} - \frac{y^2}{4} = 1$

$\frac{y^2}{4} = \frac{x^2}{16} - 1$

$y^2 = \frac{x^2}{4} - 4$

$y = \pm\sqrt{\frac{x^2}{4} - 4}$

not very pretty

let $x = \sqrt{t}$

$y = \pm$

$\tan^2 t + 1 = \sec^2$

$1 = \sec^2 t - \tan^2$

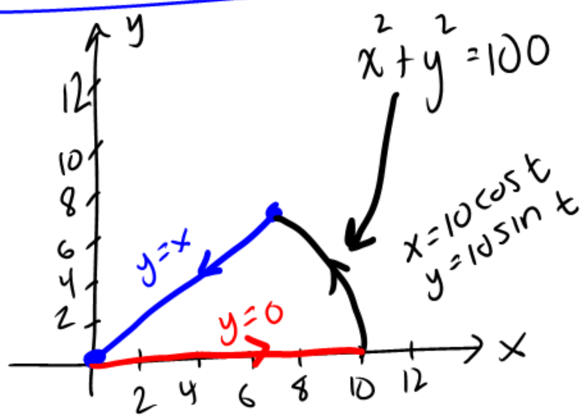
$\frac{x^2}{16} = \left(\frac{x}{4}\right)^2, \quad \frac{y^2}{4} = \left(\frac{y}{2}\right)^2$

$x = 4\sec t$

$y = 2\tan t$

$$\boxed{\vec{r}(t) = 4\sec t\,\hat{i} - 2\tan t\,\hat{j}}$$

58.



$\vec{r}_1(t) = t\,\hat{i}, \quad 0 \le t \le 10$

$\vec{r}_2(t) = 10\cos t\,\hat{i} + 10\sin t\,\hat{j}, \quad 0 \le t \le \frac{\pi}{4}$

$\vec{r}_3(t) = 5\sqrt{2}(1-t)\,\hat{i} + 5\sqrt{2}(1-t)\,\hat{j}$

$0 \le t \le 1$

Side = side $\sqrt{2}$

$10 = s\sqrt{2}$

$\frac{10}{\sqrt{2}} = s$

$s^2 + s^2 = d^2$

$s^2 = \frac{d^2}{2}$

$\frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

12.2: 35, 40, 50, 68

(35) $\vec{r}(\theta) = 2\cos^3\theta\,\hat{i} + 3\sin^3\theta\,\hat{j}$

$\vec{r}\,'(\theta) = -6\cos^2\theta\sin\theta\,\hat{i} + 9\sin^2\theta\cos\theta\,\hat{j}$ cont. on $(-\infty, \infty)$

$0 = 3\cos\theta\sin\theta\left(-2\cos\theta\,\hat{i} + 3\sin\theta\,\hat{j}\right)$

$3\cos\theta\sin\theta = 0$ or $3\sin\theta = 0$ and $-2\cos\theta = 0$

$\cos\theta = 0$ or $\sin\theta = 0$ $\qquad \theta = n\pi$ and $\theta = \frac{\pi}{2} + n\pi$

$\theta = \frac{\pi}{2} + n\pi$, $\theta = n\pi$

$\boxed{\text{smooth when } \theta \neq \frac{\pi}{2} + n\pi,\ n\pi,\ n \in \mathbb{Z}}$

(40) $\vec{r}(t) = e^{t}\,\hat{i} - e^{-t}\,\hat{j} + 3t\,\hat{k}$

$\vec{r}\,'(t) = e^{t}\,\hat{i} + e^{-t}\,\hat{j} + 3\,\hat{k}$ cont on $(-\infty, \infty)$

$e^{t} = 0$, $e^{-t} = 0$, $3 = 0$

can't happen

$\boxed{\text{So } \vec{r}(t) \text{ is smooth on } (-\infty, \infty)}$

(50) $\vec{r}(t) = \sqrt{t}\,\hat{i} + \frac{3}{t}\,\hat{j} - 2t\,\hat{k}$

$$\vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\left(\sqrt{t+\Delta t} - \sqrt{t}\right)\hat{i} + \left(\frac{3}{t+\Delta t} - \frac{3}{t}\right)\hat{j} - \left[2(t+\Delta t) - 2t\right]\hat{k}}{\Delta t}$$

$$\frac{\left(\sqrt{t+\Delta t}-\sqrt{t}\right)}{(\Delta t)}\frac{\left(\sqrt{t+\Delta t}+\sqrt{t}\right)}{\left(\sqrt{t+\Delta t}+\sqrt{t}\right)}=\frac{t+\Delta t-t}{\Delta t\left(\sqrt{t+\Delta t}+\sqrt{t}\right)}$$

$$=\frac{\cancel{\Delta t}\,1}{\cancel{\Delta t}\,1\left(\sqrt{t+\Delta t}+\sqrt{t}\right)}$$

So... $\lim_{\Delta t\to 0}=\frac{1}{2\sqrt{t}}$

$$\frac{1}{\Delta t}\left(\frac{3}{t+\Delta t}-\frac{3}{t}\right)=\frac{3t-3(t+\Delta t)}{t(t+\Delta t)}$$

$$=\frac{\cancel{3t}-\cancel{3t}-3\Delta t}{(\cancel{\Delta t})t(t+\Delta t)}$$

$$\lim_{\Delta t\to 0}\left(\frac{-3\Delta t}{t(t+\Delta t)}\right)=\frac{-3}{t(t+0)}$$

$$=-\frac{3}{t^2}$$

$$\frac{-2(t+\Delta t)+2t}{\Delta t}=\frac{-\cancel{2t}-2\cancel{\Delta t}\,1+\cancel{2t}}{\cancel{\Delta t}\,1}$$

$$\lim_{\Delta t\to 0}(-2)=-2$$

$$\boxed{\vec{r}\,'(t)=\frac{1}{2\sqrt{t}}\hat{\imath}-\frac{3}{t^2}\hat{\jmath}-2\hat{k}}$$

68 $\vec{r}\,'(t) = 3t^2\,\hat{j} + 6\sqrt{t}\,\hat{k}$, $\vec{r}(0) = \hat{i} + 2\hat{j}$

$$\int \left(0\hat{i} + 3t^2\,\hat{j} + 6\sqrt{t}\,\hat{k}\right) dt = t^3\,\hat{j} + \cancel{6}^{\,2} \cdot \frac{2}{\cancel{3}}\, t^{3/2}\,\hat{k} + \vec{C}$$

general solution $\vec{r}(t) = C_1\,\hat{i} + \left(t^3 + C_2\right)\hat{j} + \left(4t^{3/2} + C_3\right)\hat{k}$

$\vec{r}(0) = \hat{i} + 2\hat{j}$

So $C_1 = 1$, $C_2 = 2$, $C_3 = 0$

particular solution and $\boxed{\vec{r}(t) = \hat{i} + \left(t^3 + 2\right)\hat{j} + 4t^{3/2}\,\hat{k}}$