2/11/11

- warm up (last problem on 7.2 worksheet)
- Lecture 7.3
  ↳ Shell method for volume of a solid of revolution

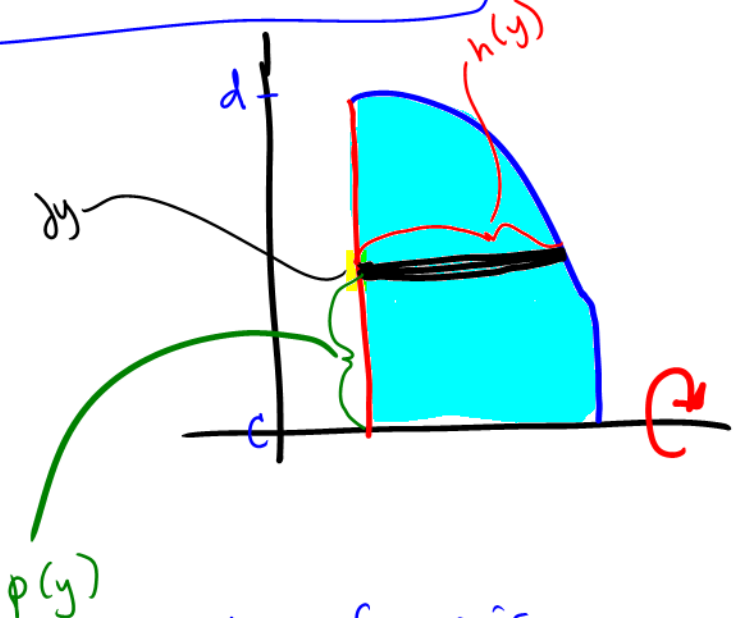
Monday

- Lecture 7.4
  ↳ Arc length

wednesday

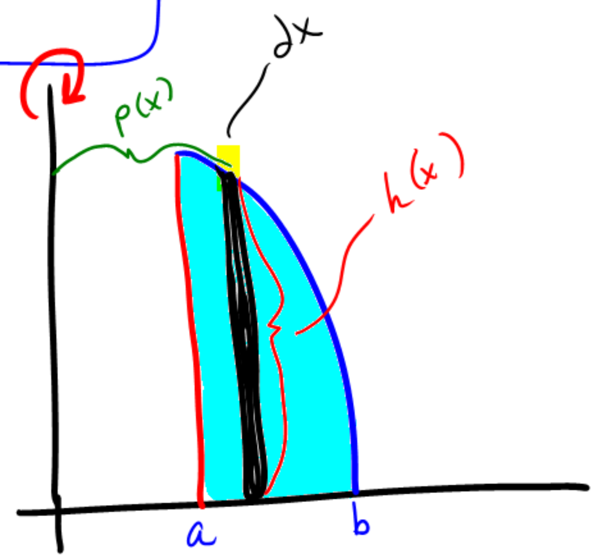
Review

Exam is on 2/23



Axis of rev. is horizontal

$$V = 2\pi \int_c^d p(y) \cdot h(y)\, dy$$



Axis of rev is vertical

$$V = 2\pi \int_a^b p(x) h(x)\, dx$$

MATH 251/GRACEY NAME____________________________________

WS/7.3

Volume using the Shell Method

- The shell method uses cylindrical shells.
  - $V = lwh$
    - The length is the circumference of the cylinder, or $2\pi r$
      - $r = p(x)$ or $r = p(y)$
      - The radius is the distance between any given rectangle you draw and the axis of revolution
    - The width is the change in x or the change in y
    - The height is the height of any rectangle you draw
      - $h(x)$ or $h(y)$

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1. (15 POINTS) Find the volume of the solid bounded by the graph of $y = \cos x$, $x = 0$, $y = 0$, and $x = \frac{\pi}{2}$, which is then rotated about the line ~~x = 2~~.

$x = \pi$, $x = x$

$x = \pi$

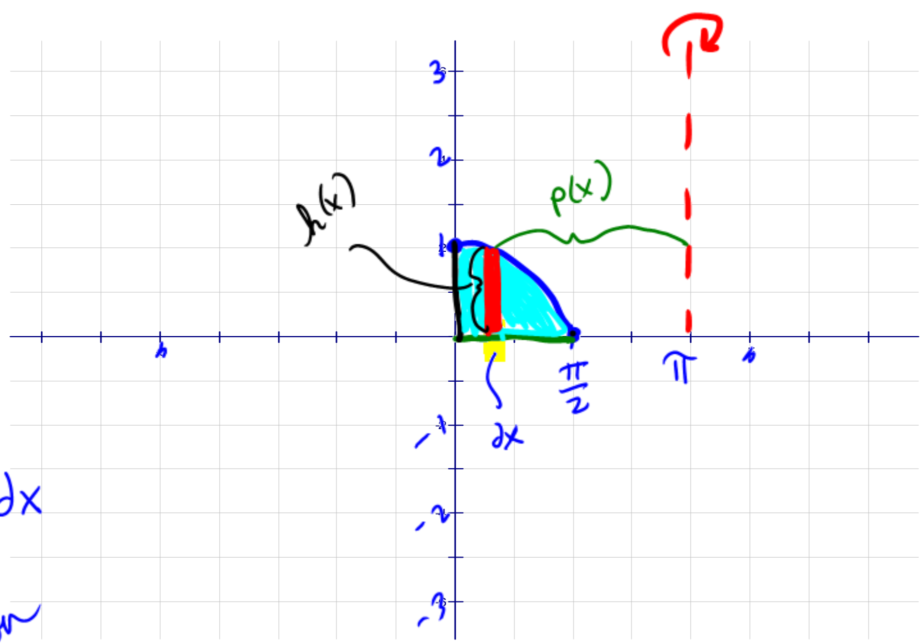
$p(x) = \pi - x$

$h(x) = \cos x - 0 = \cos x$

$$V = 2\pi \int_0^{\pi/2} (\pi - x)(\cos x)\, dx$$

$$V = 2\pi \int_0^{\pi/2} (\pi \cos x - x\cos x)\, dx$$

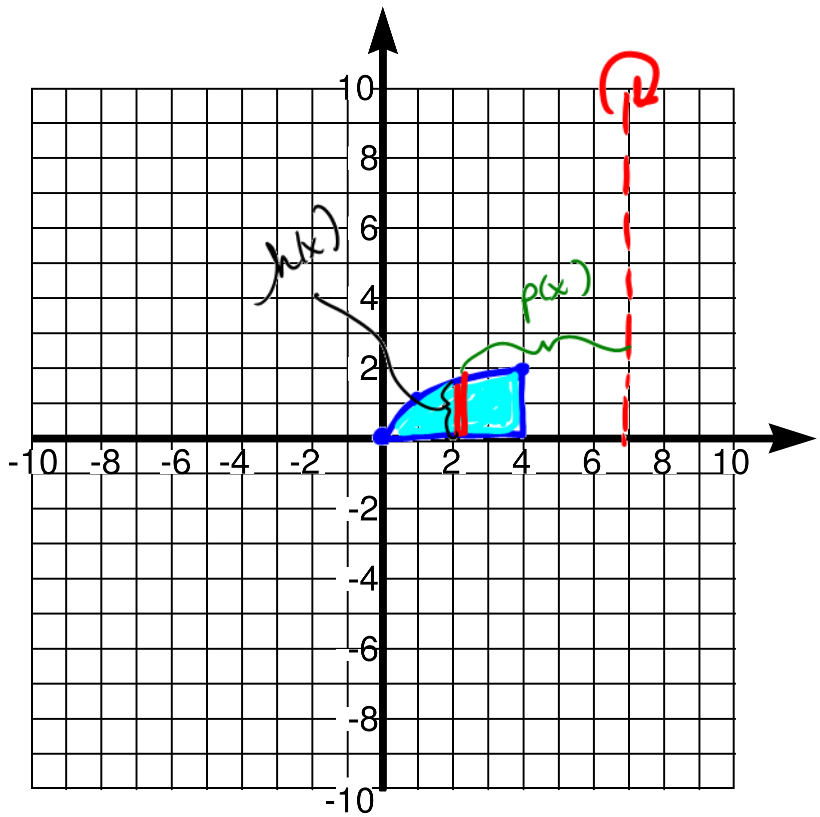
need Integration by Parts



2. (15 POINTS) Find the volume of the solid bounded by the graph of $y=\sqrt{x}$, $y=0$ and $x=4$ which is then rotated about the line $x=7$.

$p(x)=7-x$

$h(x)=\sqrt{x}-0=\sqrt{x}$



$$V=2\pi\int_0^4(7-x)x^{1/2}\,dx$$

$$V=2\pi\int_0^4\left(7x^{1/2}-x^{3/2}\right)dx$$

$$V=2\pi\left(\frac{14}{3}x^{3/2}-\frac{2}{5}x^{5/2}\right|_0^4$$

$$V=2\pi\left[\left(\frac{14}{3}\cdot 8-\frac{2}{5}\cdot 32\right)-(0-0)\right]$$

$$V=2\pi\left(\frac{560-192}{15}\right)$$

$$V=\frac{2\pi}{15}(368)$$

$$V=\frac{736\pi}{15}\text{ units}^3$$

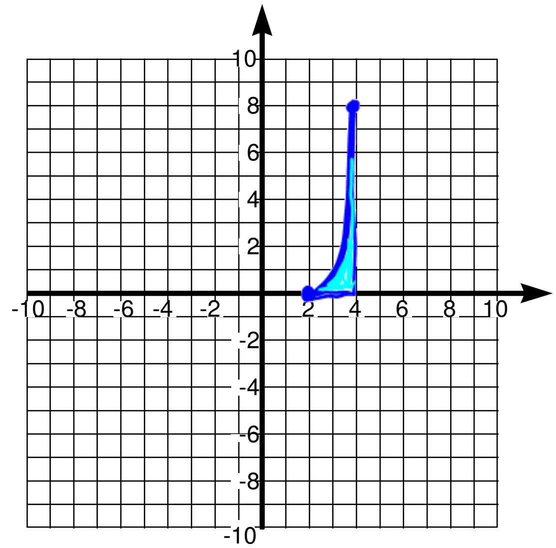
$y = (x-2)^3$

$\sqrt[3]{y} = x-2$

$x = y^{1/3}+2$

3. Sketch the region bounded by the graphs of $y=(x-2)^3$, $y=0$ and $x=4$, then **SET UP** the integrals which will find the volume of the solid created by rotated the region about

a. the line $y=0$



i. using the disc/washer method

$R(x) = (x-2)^3 - 0 = (x-2)^3$

$$\boxed{V = \pi \int_2^4 \left[(x-2)^3\right]^2 dx}$$

ii. using the shell method.

$p(y) = y - 0 = y$, $h(y) = 4 - (y^{1/3}+2)$

$$\boxed{V = 2\pi \int_0^8 (y)\left[4-(y^{1/3}+2)\right] dy}$$

b. the line $x=0$

i. using the disc/washer method

$R(y) = 4 - 0 = 4$, $r(y) = (y^{1/3}+2) - 0 = y^{1/3}+2$

$$\boxed{V = \pi \int_0^8 \left[(4)^2 - (y^{1/3}+2)^2\right] dy}$$

ii. using the shell method.

$p(x) = x - 0 = x$, $h(x) = (x-2)^3 - 0 = (x-2)^3$

$$\boxed{V = 2\pi \int_2^4 (x)(x-2)^3 dx}$$

c. the line $x=8$

i. using the disc/washer method

$R(y) = 8 - (y^{1/3}+2)$, $r(y) = 8 - 4 = 4$

$$\boxed{V = \pi \int_0^8 \left[[8-(y^{1/3}+2)]^2 - (4)^2\right] dy}$$

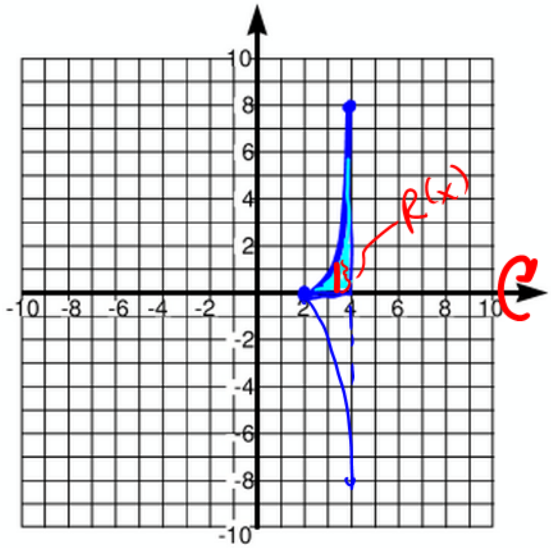
ii. using the shell method.

$p(x) = 8 - x$, $h(x) = (x-2)^3 - 0 = (x-2)^3$

$$\boxed{V = 2\pi \int_2^4 (8-x)(x-2)^3 dx}$$

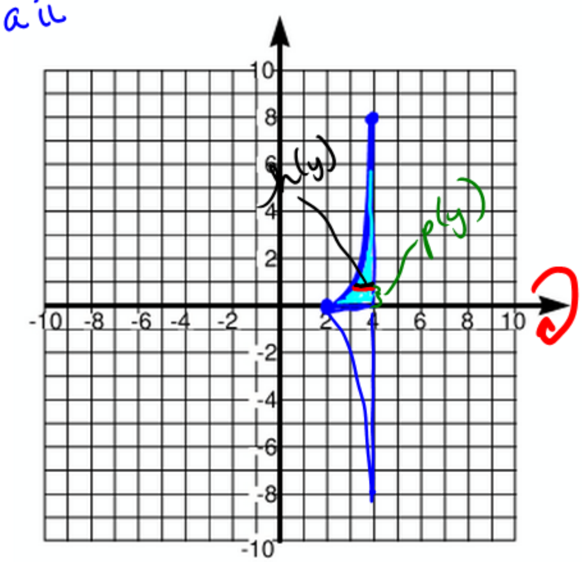
See individual pictures on the next page

3ai



disk - no hole

3aii



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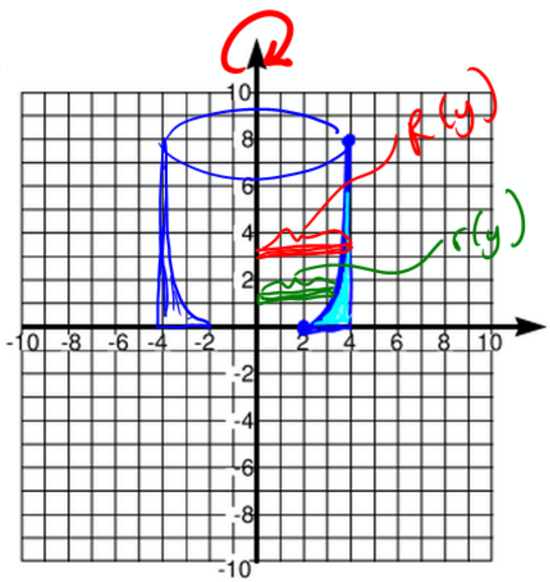
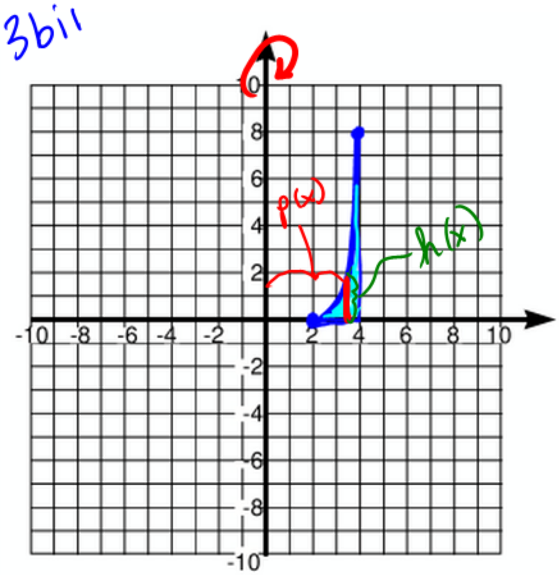


3bii



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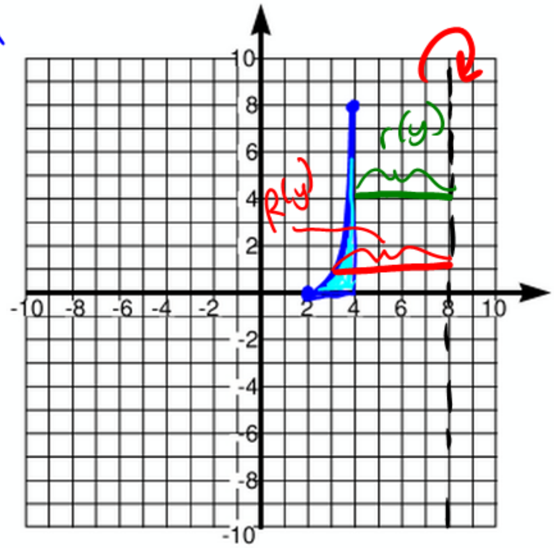

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