1/31/11

- Warm up: Ex 4 on 5.8 worksheet
- Finish 5.8

Wednesday

Have 5.6-5.8 homework done so you know what questions to ask me!

Friday

- Exam 1/Ch. 5.6 - 5.8
- Homework due:
  - ↳ review
  - ↳ 5.6 - 5.8

Example 3: Differentiate with respect to *x*.

a. $y=\dfrac{e^x-e^{-x}}{2}$

b. $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

Did last time

Example 4: Find the integral.

a. $\int \dfrac{e^x+e^{-x}}{2}dx = \frac{1}{2}\int (e^x+e^{-x})dx$

$= \frac{1}{2}(e^x-e^{-x})+C$

$= \dfrac{e^x-e^{-x}}{2}+C$

$= \sinh x + C$

cosh x

b. $\int \dfrac{e^x+e^{-x}}{e^x-e^{-x}}dx$

THEOREM: DERIVATIVES AND INTEGRALS OF HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}[\sinh u] = (\cosh u)u' \qquad \int \cosh u\,du = \sinh u + C$$

$$\frac{d}{dx}[\cosh u] = (\sinh u)u' \qquad \int \sinh u\,du = \cosh u + C$$

$$\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u' \qquad \int \operatorname{sech}^2 u\,du = \tanh u + C$$

$$\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u' \qquad \int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u' \qquad \int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$$

$$\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u' \qquad \int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$

Example 5: Differentiate with respect to *x*.

a. $f(x) = \tanh(3x^2 - 2)$

$$f'(x) = \operatorname{sech}^2(3x^2-2)(6x)$$

$$f'(x) = \boxed{6x\operatorname{sech}^2(3x-2)}$$

b. $y = \ln(\cosh x)$

$$\frac{dy}{dx} = \frac{\sinh x}{\cosh x}$$

$$\frac{dy}{dx} = \boxed{\tanh x}$$

Example 6: Find the integral.

a. $\int \frac{\cosh\sqrt{x}}{\sqrt{x}}dx = \int \frac{\cosh u}{\cancel{\sqrt{x}}}\left(2\cancel{\sqrt{x}}\,du\right)$

$u=\sqrt{x}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$= 2\sinh u + C$

$= \boxed{2\sinh\sqrt{x}+C}$

b. $\int \frac{\cosh x}{1+\sinh^2 x}dx = \int \frac{\cancel{\cosh x}}{1^2+u^2}\left(\frac{du}{\cancel{\cosh x}}\right)$

$u=\sinh x$

$\frac{du}{dx}=\cosh x$

$dx=\frac{du}{\cosh x}$

$= \int \frac{du}{1^2+u^2}$

$= \frac{1}{1}\arctan\left(\frac{u}{1}\right)+C$

$= \boxed{\arctan(\sinh x)+C}$

INVERSE HYPERBOLIC FUNCTIONS Do NOT Need to memorize

FUNCTION	DOMAIN
1. $\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$	$(-\infty,\infty)$
2. $\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$	$[1,\infty)$
3. $\tanh^{-1}x=\frac{1}{2}\ln\frac{1+x}{1-x}$	$(-1,1)$
4. $\coth^{-1}x=\frac{1}{2}\ln\frac{x+1}{x-1}$	$(-\infty,-1)\cup(1,\infty)$
5. $\operatorname{sech}^{-1}x=\ln\frac{1+\sqrt{1-x^2}}{x}$	$(0,1]$
6. $\operatorname{csch}^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	$(-\infty,0)\cup(0,\infty)$

* $|2|=2$

* $1+\left(\frac{2}{3}\right)^2$ $\frac{9+4}{9}$

Example 7: Evaluate each function.

a. $\sinh^{-1}0 = \ln\left(0+\sqrt{(0)^2+1}\right)$

$= \ln 1$

$= \boxed{0}$

b. $\operatorname{csch}^{-1}\frac{2}{3} = \ln\left(\frac{1}{(2/3)}+\frac{\sqrt{1+(2/3)^2}}{|2/3|}\right)$

$= \ln\left(\frac{3}{2}+\frac{\sqrt{13}}{\cancel{3}\left|\frac{2}{3}\right|}\right) = \boxed{\ln\left(\frac{3+\sqrt{13}}{2}\right)}$

DERIVATIVES AND INTEGRALS OF INVERSE HYPERBOLIC FUNCTIONS

$$\frac{d}{dx}[\sinh u] = (\cosh u)u' \qquad \frac{d}{dx}[\cosh u] = (\sinh u)u'$$

$$\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u' \qquad \frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$$

$$\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u' \qquad \frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$$

$$\frac{d}{dx}\left[\sinh^{-1} u\right] = \frac{u'}{\sqrt{u^2+1}} \qquad \frac{d}{dx}\left[\cosh^{-1} u\right] = \frac{u'}{\sqrt{u^2-1}}$$

$$\frac{d}{dx}\left[\tanh^{-1} u\right] = \frac{u'}{1-u^2} \qquad \frac{d}{dx}\left[\operatorname{sech}^{-1} u\right] = \frac{-u'}{u\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left[\coth^{-1} u\right] = \frac{u'}{1-u^2} \qquad \frac{d}{dx}\left[\operatorname{csch}^{-1} u\right] = \frac{-u'}{|u|\sqrt{1+u^2}}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\ln\frac{a+\sqrt{a^2 \pm u^2}}{|u|} + C$$

Do not need to memorize

Example 8: Find the derivative of the function. Simplify your result to a single rational expression with positive exponents.

a. $f(x) = \coth^{-1} x^2$

$u = x^2$
$u' = 2x$

$f'(x) = \frac{u'}{1-u^2}$

$f'(x) = \frac{2x}{1-(x^2)^2}$

$\boxed{f'(x) = \frac{2x}{1-x^4}}$

b. $g(x) = \underbrace{x\tanh^{-1} x}_{\text{product}} + \underbrace{\ln\sqrt{1-x^2}}_{\frac{1}{2}\ln(1-x^2)}$

zero out

$g'(x) = 1\tanh^{-1}x + \cancel{x\left(\frac{1}{1-x^2}\right)} + \cancel{\frac{1}{2}\left(\frac{-2x}{1-x^2}\right)}$

$\boxed{g'(x) = \tanh^{-1} x}$

c. $y = \operatorname{sech}^{-1}(\cos 2x)$, $0 < x < \frac{\pi}{4}$

$u = \cos 2x$
$u' = -2\sin 2x$

$\frac{dy}{dx} = \frac{-u'}{u\sqrt{1-u^2}}$

$\frac{dy}{dx} = \frac{-(-2\sin 2x)}{\cos 2x\sqrt{1-\cos^2 2x}}$

$\frac{dy}{dx} = \frac{2\sin 2x}{\cos 2x\sqrt{\sin^2 2x}}$

$\frac{dy}{dx} = \frac{2\cancel{\sin 2x}^{\,1}}{\cos 2x\,\cancel{\sin 2x}_{\,1}}$

$\boxed{\frac{dy}{dx} = 2\sec 2x}$

Example 9: Find the limit.

a. $\lim\limits_{x\to-\infty} \sinh x = \boxed{-\infty}$

b. $\lim\limits_{x\to 0^-} \coth x = \boxed{-\infty}$

Example 10: Find the integral.

a. $\displaystyle\int \frac{1}{2x\sqrt{1-4x^2}}\,dx = \int \frac{1}{2x\sqrt{1-u^2}}\left(\frac{du}{2}\right)$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$a^2 = 1$

$a = 1$

$$= \frac{1}{2}\int \frac{du}{u\sqrt{1^2-u^2}}$$

$$= \frac{1}{2}\left(-\frac{1}{1}\ln \frac{1+\sqrt{1^2-(u)^2}}{|u|}\right) + C$$

$$= \boxed{-\frac{1}{2}\ln\frac{1+\sqrt{1-4x^2}}{|2x|} + C}$$

c. $\displaystyle\int \frac{3}{\sqrt{x}\sqrt{9+x}}\,dx$

b. $\displaystyle\int \frac{dx}{(x+2)\sqrt{x^2+4x+8}}$

$$x^2+4x+8 = \left(x^2+4x+(2)^2\right)+8-4$$

$$= (x+2)^2+4$$

$$= \int \frac{dx}{(x+2)\sqrt{(2)^2+(x+2)^2}}$$

$$= -\frac{1}{2}\ln\frac{2+\sqrt{(2)^2+(x+2)^2}}{|x+2|} + C$$

$$= \boxed{-\frac{1}{2}\ln\frac{2+\sqrt{x^2+4x+8}}{|x+2|}}$$

d. $\displaystyle\int \frac{1}{1-4x-2x^2}\,dx = \frac{1}{2}\int \frac{dx}{\left(\sqrt{\frac{3}{2}}\right)^2-(x+1)^2}$

$a = \sqrt{\frac{3}{2}}$

$2+1-2\left(x^2+2x+(1)^2\right)$

$3-2(x+1)^2$

$2\left(\frac{3}{2}-(x+1)^2\right)$

$$= \frac{1}{2}\left(\frac{1}{2\sqrt{\frac{3}{2}}}\ln\left|\frac{\sqrt{\frac{3}{2}}+(x+1)}{\sqrt{\frac{3}{2}}-(x+1)}\right|\right) + C$$

*simplication left to student