Section 1.8: EXPONENTS AND ORDER OF OPERATIONS

When you are done with your homework you should be able to…

- π Evaluate exponential expressions
- π Simplify algebraic expressions with exponents
- π Use the order of operations agreement
- π Evaluate mathematical models

WARM-UP:

1. Determine whether the given number is a solution of the equation.

$\frac{5m-1}{6}=\frac{3m-2}{4};\ -4$

$\frac{5(-4)-1}{6} \stackrel{?}{=} \frac{3(-4)-2}{4}$ → $\frac{-20-1}{6} \stackrel{?}{=} \frac{-12-2}{4}$ → $\frac{\cancel{-21}^{-7}}{\cancel{6}_{2}} \stackrel{?}{=} \frac{\cancel{-14}^{-7}}{\cancel{4}_{2}}$ → $-\frac{7}{2} = -\frac{7}{2}$ ✓

$\boxed{\text{yes}}$

2. Write a numerical expression for each phrase. Then simplify the numerical expression.

   a. 14 added to the product of 4 and -10

$4(-10)+14$

$= -40+14$

$= \boxed{-26}$

   b. The quotient of $\underbrace{-18}$ and $\underbrace{\text{the sum of -15 and 12}}$

$\frac{-18}{-15+12} = \frac{-18}{-3}$

$= \boxed{6}$

or $-18 \div (-15+12) = -18 \div (-3)$

$= \boxed{6}$

DEFINITION OF A NATURAL NUMBER EXPONENT

If b is a real number and n is a natural number,

$$b^n = \underbrace{b \cdot b \cdot b \cdots b}_{n \text{ times}}$$

b^n is read "the nth power of b" or "b to the nth power. The expression b^n is called an exponential expression.

Example 1: Evaluate.

1. $(-5)^3 = (-5)(-5)(-5) = \boxed{-125}$

2. $(-12)^2 = (-12)(-12) = \boxed{144}$

3. $-12^2 = -(12 \cdot 12) = \boxed{-144}$

ORDER OF OPERATIONS

1. Perform all operations within grouping symbols
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in the order in which they occur, working from left to right.
4. Finally, do all additions and subtractions using one of the following procedures:
   - Work from left to right and do additions and subtractions in the order in which they occur.

   or
   - Rewrite subtractions as additions of opposites. Combine positive and negative numbers separately, and then add these results.

Example 2: Simplify.

1. $40 \div 4 \cdot 2$ $= 10 \cdot 2$

$= \boxed{20}$

2. $\dfrac{-5(7-2)-3(4-7)}{-13-(-5)}$

$= \dfrac{-5(5)-3(-3)}{-13+5}$

$= \dfrac{-25+9}{-8}$

$= \dfrac{-16}{-8}$ $= \boxed{2}$

3. $(3 \cdot 5)^2 - 3 \cdot 5^2$ $= 15^2 - 3 \cdot 25$

$= 225 - 75$

$= \boxed{150}$

4. $\left[-\dfrac{4}{7}-\left(-\dfrac{2}{5}\right)\right]\left[-\dfrac{3}{8}+\left(-\dfrac{1}{9}\right)\right]$

$= \left[-\dfrac{4}{7}\cdot\dfrac{5}{5}+\dfrac{2}{5}\cdot\dfrac{7}{7}\right]\left[-\dfrac{3}{8}\cdot\dfrac{9}{9}-\dfrac{1}{9}\cdot\dfrac{8}{8}\right]$

$= \left[-\dfrac{20}{35}+\dfrac{14}{35}\right]\left[-\dfrac{27}{72}-\dfrac{8}{72}\right]$

$= \left[-\dfrac{\cancel{6}^{1}}{\cancel{35}_{1}}\right]\left[-\dfrac{\cancel{35}^{1}}{\cancel{72}_{12}}\right]$ $= \boxed{\dfrac{1}{12}}$

Example 3: Simplify each algebraic expression.

1. $-6x^2 + 18x^2$ $= \boxed{12x^2}$

2. $4(7x^3-5)-\left[2(8x^3-1)+1\right]$

$= 4(7x^3)-4(5)-\left[2(8x^3)-2(1)+1\right]$

$= 28x^3-20-\left[16x^3-1\right]$

$= 28x^3-20-16x^3-(-1)$

$= 28x^3-20-16x^3+1$

$= \boxed{12x^3-19}$

3. $6-5\left[8-(2y-4)\right]$

APPLICATIONS

In Palo Alto, CA, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers). The mathematical model $C=\frac{200x}{100-x}$ describes the cost, *C*, in tens of thousands of dollars, for removing *x* percent of the contaminants.

1. Find the cost, in tens of thousands of dollars, for removing 50% of the contaminants.

2. Find the cost, in tens of thousands of dollars, for removing 60% of the contaminants.

3. Describe what is happening to the cost of the cleanup as the percentage of contaminant removed increases.

Section 2.1: THE ADDITION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- π Identify linear equations in one variable
- π Use the addition property of equality to solve equations
- π Solve applied problems using formulas

WARM-UP:

Simplify:

1. $\frac{1}{2}-\frac{2}{3}\div\frac{5}{9}+\frac{3}{10}$

$=\frac{1}{2}-\frac{2}{\cancel{3}_1}\cdot\frac{\cancel{9}^3}{5}+\frac{3}{10}$

$=\frac{1\cdot5}{2\cdot5}-\frac{6\cdot2}{5\cdot2}+\frac{3}{10}$

$=\frac{5}{10}-\frac{12}{10}+\frac{3}{10}$

$=\frac{5-12+3}{10}$

$=-\frac{4}{10}$

$=\boxed{-\frac{2}{5}}$

2. $-40\div5\cdot2=-8\cdot2$

$=\boxed{-16}$

LINEAR EQUATIONS IN ONE VARIABLE

In Chapter 1, we learned that an ___equation___ is a statement that two ___algebraic___ expressions are ___equal___. We determined whether a given number is an equation's ___solution___ by substituting that number for each occurrence of the ___variable___. When the ___substitution___ resulted in a true statement, that ___number___ was a ___solution___. When the substituted number resulted in a ___false___ statement, that number was ___not___ a ___solution___.

VOCABULARY

Solving an equation: The _process_ of finding the _number_ (or _numbers_) that make the equation a _true_ statement. These numbers are called the _solutions_ or _roots_ of the equation, and we say that they _satisfy_ the equation.

DEFINITION OF A LINEAR EQUATION IN ONE VARIABLE

A _linear_ _equation_ in _one_ _variable_ _x_ is an equation that can be written in the form

$$ax + b = c$$

where _a_, _b_, and _c_ are real numbers, and _$a \neq 0$_.

Example 1: Give three examples of a linear equation in one variable.

1. $2x+3=7$ $(a=2, b=3, c=7)$
2. $5x = 12$ $(a=5, b=0, c=12)$
3. $\frac{x}{2}-8=x$ since it is equivalent to $-\frac{x}{2}-8=0$ $\left(a=-\frac{1}{2}, b=-8, c=0\right)$

Example 2: Give two examples of a nonlinear equation in one variable.

1. $\frac{5}{x}=50$
2. $|x|=12$
3. $x^2-4x+3=12$

VOCABULARY

Equivalent equations: Equations that have the ___same___ solution are ___equalent___ ___equations___.

$x+2=0$ | $x+2-2=0-2$

$x=-2$ | $x+0=-2$

$x=-2$

THE ADDITION PROPERTY OF EQUALITY

The ___Same___ real number or ___algebraic___ expression may be ___added___ to ___both___ sides of an ___equation___ without changing the equation's ___Solution___. That is,

If $a=b$ then $a+c=b+c$

Example 3: Solve the following equations. Check your solutions.

1. $y-5=-18$

$+5 \quad +5$

$y+0=-13$

$y=-13$

$y-5+5=-18+5$

$y-0=-13$

$y=-13$

$\boxed{\{-13\}}$

2. $18+z=14$

$-18 \quad -18$

$z=-4$

$\boxed{\{-4\}}$

3. $x+10.6=-9.0$

$-10.6 \quad -10.6$

$x=-19.6$

$\boxed{\{-19.6\}}$

4. $-\frac{1}{8}+x=-\frac{1}{4}$

$+\frac{1}{8} \quad +\frac{1}{8}$

$x=-\frac{1}{4}\cdot\frac{2}{2}+\frac{1}{8}$

$x=-\frac{2}{8}+\frac{1}{8}$

$x=-\frac{1}{8}$

$\boxed{\{-\frac{1}{8}\}}$

5. $-3x-5+4x=9$

$x-5=9$

$+5 \quad +5$

$x=14$

$\boxed{\{14\}}$

6. $7x+3=6(x-1)+9$

$7x+3=6\cdot x-6\cdot 1+9$

$7x+3=6x+3$

$-3 \quad -3$

$7x=6x$

$-6x \quad -6x$

$1x=0x$

$x=0$

$\boxed{\{0\}}$

ADDING AND SUBTRACTING VARIABLE TERMS ON BOTH SIDES OF AN EQUATION

Our goal is to __isolate__ all the __variable__ terms on one side of the equation. We can use the __addition__ __property__ of __equality__ to do this.

APPLICATIONS

1. The cost, *C,* of an item (the price paid by a retailer) plus the markup, *M*, on that item (the retailer's profit) equals the selling price, *S*, of the item. The formula is $C + M = S$.

   The selling price of a television is \$650. If the cost to the retailer for the television is \$520, find the markup.

   $C + M = S$

   $520 + M = 650$

   $-520 \qquad -520$

   $M = 130$

   $\boxed{\text{The markup is \$130.00.}}$

2. What is the difference between solving an equation such as $5y + 3 - 4y - 8 = 6 + 9$ and simplifying an algebraic expression such as $5y + 3 - 4y - 8$?

   In the equation, we could solve for y.

   In the algebraic expression we could write a simplified expression. $5y + 3 - 4y - 8 = y - 5$

Section 2.2: THE MULTIPLICATION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- π Use the multiplication property of equality to solve equations
- π Solve equations in the form of $-x = c$
- π Use the addition and multiplication properties to solve equations
- π Solve applied problems using formulas

WARM-UP:

Solve:

1. $5z - 12 = z + 8$

$+12 \quad +12$

$5z = z + 20$

$-z \quad -z$

$\frac{1}{4} \cdot 4z = 20 \cdot \frac{1}{4}$

$1z = 5$

$z = 5$

$\boxed{\{5\}}$

2. $x = -7(2 - x) + 18$

$x = -7 \cdot 2 - (-7)(x) + 18$

$x = -14 + 7x + 18$

$x = 4 + 7x$

$-7x \quad -7x$

$\frac{-6x}{-6} = \frac{4}{-6}$

$1x = -\frac{2}{3}$

$x = -\frac{2}{3}$

$\boxed{\{-\frac{2}{3}\}}$

THE MULTIPLICATION PROPERTY OF EQUALITY

The __same__ __nonzero__ real number or __algebraic__ expression may __multiply__ __both__ sides of an __equation__ without changing the __solution__. That is,

If $a = b$ and $c \neq 0$ then $a \cdot c = b \cdot c$

Example 1: Solve the following equations. Check your solutions.

1. $-5z = -20$

$\frac{-5z}{-5} = \frac{-20}{-5}$

$z = 4$

$\boxed{\{4\}}$

4. $(-8)\left(-\frac{1}{8}x\right) = 6(-8)$

$1x = -48$

$x = -48$

$\boxed{\{-48\}}$

2. $-51=-y$

$\frac{-51}{-1}=\frac{-y}{-1}$

$51=y$

$y=51$

$\boxed{\{51\}}$

3. $8x-3x=-45$

$\frac{5x}{5}=\frac{-45}{5}$

$x=-9$

$\boxed{\{-9\}}$

5. $6z-3=z+2$

$-z \quad -z$

$5z-3=2$

$+3 \quad +3$

$\frac{5z}{5}=\frac{5}{5}$

$z=1$

$\boxed{\{1\}}$

6. $5y+6=3y-6$

$-3y \quad -3y$

$2y+6=-6$

$-6 \quad -6$

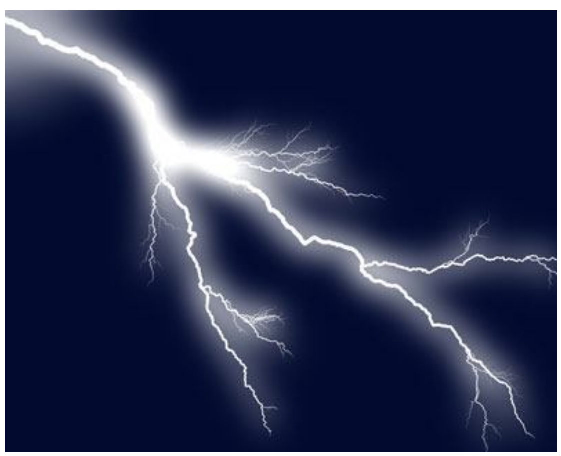
$\frac{2y}{2}=\frac{-12}{2}$

$y=-6$

$\boxed{\{-6\}}$

APPLICATIONS

The formula $M=\frac{n}{5}$ models your distance, *M,* from a lightning strike in a thunderstorm if it takes *n* seconds to hear thunder after seeing the lightning.



If you are three miles away from the lightning flash, how long will it take the sound of thunder to reach you?

$M=\frac{n}{5}$

$5\cdot 3=\frac{n}{5}\cdot 5$

$15=1n$

$15=n$

$n=15$

We have $\frac{n}{5} \to \frac{1}{5}n$

we want $1n$

It takes 15 seconds to hear the sound of thunder when the lightning is 3 miles away.

Section 2.3: SOLVING LINEAR EQUATIONS

When you are done with your homework you should be able to…

- Solve linear equations
- Solve linear equations containing fractions
- Identify equations with no solution or infinitely many solutions
- Solve applied problems using formulas

WARM-UP:

Solve:

1. $-12z = 144$

$\frac{-12z}{-12} = \frac{144}{-12}$

$z = -12$

$\boxed{\{-12\}}$

2. $-x = -7x + 24$

$+7x \quad +7x$

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$

$\boxed{\{4\}}$

Check: $x = 4$

$-(4) \stackrel{?}{=} -7(4) + 24$

$-4 = -28 + 24$

$-4 = -4$ ✓

A STEP-BY-STEP PROCEDURE FOR SOLVING LINEAR EQUATIONS

1. Simplify the algebraic expression on each side.
2. Collect all the variable terms on one side and all the constant terms on the other side.
3. Isolate the variable and Solve.
4. Check the proposed solution in the original equation.

Example 1: Solve the following equations. Check your solutions.

1. $-z - 34 + 10z = 2 + 10z - 54$

$9z - 34 = 10z - 52$

$-9z \quad -9z$

$-34 = z - 52$

$+52 \quad +52$

$18 = z$

$\boxed{\{18\}}$

4. $3(x+2) = x + 30$

$3 \cdot x + 3 \cdot 2 = x + 30$

$3x + 6 = x + 30$

$-x \quad -x$

$2x + 6 = 30$

$-6 \quad -6$

$\frac{2x}{2} = \frac{24}{2}$

$x = 12$

$\boxed{\{12\}}$

2. $20=44-8(2-x)$

$20=44-16+8x$

$20=28+8x$

$-20 \quad -20$

$0=8+8x$

$-8x \quad -8x$

$-8x=8$

$\frac{-8x}{-8}=\frac{8}{-8}$

$x=-1$

$\boxed{\{-1\}}$

5. $2(x-15)+3x=(6+4x)-(9x-2)$

$2x-30+3x=6+4x-9x+2$

$5x-30=-5x+8$

$+5x \quad +5x$

$10x-30=8$

$+30 \quad +30$

$10x=38$

$\frac{10x}{10}=\frac{38}{10}$

$x=\frac{19}{5}$

$\boxed{\left\{\frac{19}{5}\right\}}$

3. $5x-4(x+9)=2x+3$

$5x-4x-36=2x+3$

$x-36=2x+3$

$-2x \quad -2x$

$-x-36=3$

$+36 \quad +36$

$-x=39$

$\frac{-x}{-1}=\frac{39}{-1}$

$x=-39$

$\boxed{\{-39\}}$

6. $100=-(x-1)+4(x-6)$

$100=-x+1+4x-24$

$100=3x-23$

$+23 \quad +23$

$123=3x$

$\frac{123}{3}=\frac{3x}{3}$

$41=x$

$\boxed{\{41\}}$

LINEAR EQUATIONS WITH FRACTIONS

Equations are __easier__ to solve when they do not contain __fractions__. To remove fractions, we can __multiply__ __both__ sides of the equation by the __least__ __common__ __denominator__ of any fractions in the equation. Remember…the __LCD__ is the __smallest__ number that all __denominators__ will __divide__ into. This is often called "__clearing__ an equation of __fractions__".

Example 2: Solve the following equations. Clear the fractions first. Check your solutions.

1. $\frac{x}{2}+13=-22$

$-13 \quad -13$

$2\cdot\frac{x}{2} = -35\cdot 2$

$x=-70$

$\boxed{\{-70\}}$

3. $12\cdot\left(\frac{3y}{4}-\frac{2}{3}\right)=\left(\frac{7}{\cancel{12}}\right)\cdot\cancel{12}$

$\frac{\cancel{12}}{1}\cdot\frac{3y}{\cancel{4}}-\frac{\cancel{12}}{1}\cdot\frac{2}{\cancel{3}}=7$

$3(3y)-4(2)=7$

$9y-8=7$

$+8 \quad +8$

$\frac{9y}{9}=\frac{\cancel{15}\,5}{\cancel{9}\,3}$

$y=\frac{5}{3}$

$\boxed{\left\{\frac{5}{3}\right\}}$

2. $30\cdot\left(\frac{z}{5}-\frac{1}{2}\right)=\left(\frac{z}{\cancel{6}}\right)\cdot\cancel{30}\,5$

$\frac{\cancel{30}}{1}\cdot\frac{z}{\cancel{5}}-\frac{\cancel{30}}{1}\cdot\frac{1}{\cancel{2}}=5z$

$6z-15=5z$

$-6z \quad -6z$

$\frac{-15}{-1}=\frac{-z}{1}$

$15=z$

$\boxed{\{15\}}$

4. $12\cdot\left(\frac{x-2}{3}-\frac{4}{1}\right)=\left(\frac{x+1}{\cancel{4}}\right)\cdot\cancel{12}\,3$

$\frac{\cancel{12}}{1}\cdot\frac{x-2}{\cancel{3}}-12\cdot 4=3(x+1)$

$4(x-2)-48=3x+3$

$4x-8-48=3x+3$

$4x-56=3x+3$

$-3x \quad -3x$

$x-56=3$

$+56 \quad +56$

$x=59$

$\boxed{\{59\}}$

RECOGNIZING INCONSISTENT EQUATIONS AND IDENTITIES

If you attempt to solve an equation with no solution or one that is true for every real number, you will eliminate the variable.

- π An inconsistent equation with no solution results in a false statement, such as $0=1$.
- π An identity that is true for every real number results in a true statement, such as $0=0$.

Example 3: Solve the following equations. Use words or set notation to identify equations that have no solution, or equations that are true for all real numbers. Check your solutions.

1. $2(x-5)=2x+10$

$$2x-10=2x+10$$
$$-2x \qquad -2x$$
$$0x-10=0x+10$$
$$0-10=0+10$$
$$-10=10$$

False → no solution!

$$\boxed{\{\ \} \text{ or } \emptyset}$$

2. $5x-5=3x-7+2(x+1)$

$$5x-5=3x-7+2x+2$$
$$5x-5=5x-5$$
$$-5x \qquad -5x$$
$$0x-5=0x-5$$
$$0-5=0-5$$
$$-5=-5$$

true → infinitely many solutions

$$\boxed{\{x \mid x \text{ is a real number}\}}$$

such that

3. $\dfrac{x}{2}+\dfrac{2x}{3}+3=x+3$

$$-3 \qquad -3$$
$$6\cdot\left(\frac{x}{2}+\frac{2x}{3}\right)=(x)\cdot 6$$
$$\frac{6}{1}\cdot\frac{x}{2}+\frac{6}{1}\cdot\frac{2x}{3}=6x$$
$$3x+4x=6x$$
$$7x=6x$$
$$-6x \qquad -6x$$
$$x=0x$$
$$x=0$$
$$\boxed{\{0\}}$$

4. $\dfrac{x}{4}+3=\dfrac{x}{4}$

$$-\frac{x}{4} \qquad -\frac{x}{4}$$
$$0x+3=0x$$
$$0+3=0$$
$$3=0$$

False → no solution

$$\boxed{\emptyset \text{ or } \{\ \}}$$

APPLICATIONS

The formula $p = 15 + \frac{5d}{11}$ describes the pressure of sea water, *p,* in pounds per square foot, at a depth of *d* feet below the surface.

1. The record depth for breath-held diving, by Francisco Ferreras (Cuba) off Grand Bahama Island, on November 14, 1993, involved pressure of 201 pounds per square foot. To what depth did Francisco descend on this venture? (He was underwater for 2 minutes and 9 seconds!)

$p = 15 + \frac{5d}{11}$

$201 = 15 + \frac{5d}{11}$

$-15 \quad -15$

$11 \cdot (186) = \left(\frac{5d}{\cancel{11}}\right) \cdot \cancel{11}$

$\frac{2046}{5} = \frac{5d}{5}$

$409\frac{1}{5} = d$

He ventured to a depth of $409\frac{1}{5}$ ft.

2. At what depth is the pressure 20 pounds per square foot?

Section 2.4: FORMULAS AND PERCENTS

When you are done with your homework you should be able to...

- π Solve a formula for a variable
- π Express a percent as a decimal
- π Express a decimal as a percent
- π Use the percent formula
- π Solve applied problems involving percent change

WARM-UP:

Solve:

1. $4 = 0.25B$

$\frac{4}{0.25} = \frac{0.25B}{0.25}$

$16 = B$

$\boxed{\{16\}}$

2. $1.3 = P \cdot 26$

$\frac{1.3}{26} = \frac{P \cdot 26}{26}$

$0.05 = P$

$\boxed{\{0.05\}}$

SOLVING A FORMULA FOR ONE OF ITS VARIABLES

Solving a formula for a variable means rewriting the formula so that the variable is isolated on one side of the equation. To solve a formula for one of its variables, treat that variable as if it were the only variable in the equation.

PERIMETER

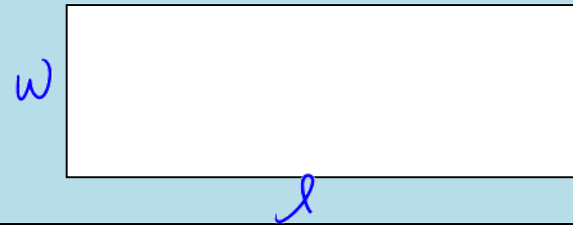
The perimeter of a 2 dimensional figure is the sum of the lengths of its sides. Perimeter is measured in linear units, such as feet, inches, miles, or Kilometers.

PERIMETER OF A RECTANGLE

The perimeter, __P__, of a rectangle with length __l__ and width __w__ is given by the formula

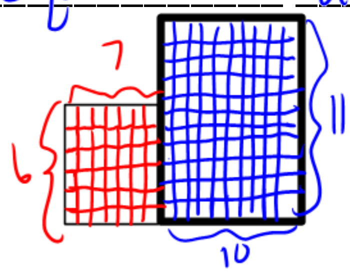
$P = 2w + 2l$ or

$P = 2(w + l)$



SQUARE UNITS

A __square__ unit is a __square__, each of whose sides is __1__ unit in length. The __area__ of a __2__ __dimensional__ figure is the number of __square__ __units__ it takes to fill the interior of the figure.



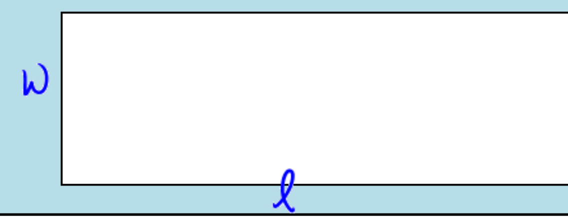
$A = 42 + 110$

$A = 152$ sq. units

AREA OF A RECTANGLE

The area, __A__, of a rectangle with length __l__ and width __w__ is given by the formula

$A = l \cdot w$



Example 1: Solve the following formulas for the specified variable.

$a(b+c) = ab + ac$

1. $d = rt;\ t$

$\frac{d}{r} = \frac{\cancel{r} \cdot t}{\cancel{r}}$

$\frac{d}{r} = t \rightarrow \boxed{t = \frac{d}{r}}$

2. $P = C + MC;\ C$

$P = C \cdot 1 + C \cdot M$

$\frac{P}{1+M} = \frac{C(1+M)}{1+M}$

$\frac{P}{1+M} = C \rightarrow \boxed{C = \frac{P}{1+M}}$

Example 2: Consider a rectangle which has an area of 15 square feet and a width of 3 feet.

$A=15,\ l=?,\ w=3$

1. Find the length.

$A = l \cdot w$

$\frac{15}{3} = \frac{l \cdot 3}{3}$

$5 = l$

$\boxed{l = 5\text{ft.}}$

2. Find the perimeter.

$P = 2l + 2w$

$P = 2(5) + 2(3)$

$P = 10 + 6$

$\boxed{P = 16\text{ft.}}$

BASICS OF PERCENTS

__Percents__ are the result of __expressing__ numbers as __part__ of __100__. The word __percent__ means __per__ __hundred__.

PERCENT NOTATION

__$n\%$__ means __$\frac{n}{100}$__. OR $n\%$ means $n\left(\frac{1}{100}\right)$

$100\% = 1$

STEPS FOR EXPRESSING A PERCENT AS A DECIMAL NUMBER

1. Move the __decimal__ point __2__ places to the __left__.
2. Remove the __percent__ sign.

Example 3: Express each percent as a decimal.

1. 9.5% $= \frac{9.5}{100} = \boxed{0.095}$
2. 235% $= \frac{235}{100} = \boxed{2.35}$

$1 = 100\%$

STEPS FOR EXPRESSING A DECIMAL NUMBER AS A PERCENT

1. Move the ___decimal___ point ___2___ places to the ___right___.
2. Attach a ___percent___ sign.

Example 4: Express each decimal as a percent.

1. $1.75 (100\%) = \boxed{175\%}$
2. $0.01 (100\%) = \boxed{1\%}$

A FORMULA INVOLVING PERCENT

___Percents___ are useful in comparing two ___numbers___. To ___compare___ the number ___A___ to the number ___B___ using a percent ___P___, the following formula is used:

P percent

$$A = P \cdot B$$

is

Example 5: Solve.

1. What is 12% of 50?

$A = 12\% \cdot 50$

$A = 0.12(50)$

$\boxed{A = 6}$

$\frac{A}{50} = \frac{12}{100} \rightarrow \frac{A(100)}{100} = \frac{600}{100}$

$\boxed{A = 6}$

2. 6 is 30% of what?

$6 = 30\% \cdot B$

$\frac{6}{0.30} = \frac{0.30 \cdot B}{0.30}$

$\boxed{20 = B}$

3. 200 is what percent of 20?

$200 = 1P\% \cdot 20$

$\frac{200}{20} = \frac{0.01P(20)}{20}$

$\frac{10}{0.01} = \frac{0.01P}{0.01}$

$\boxed{1000 = P}$

1000%

PERCENT INCREASE **PERCENT DECREASE**

watch video on mathchick.net

APPLICATIONS

1. The average, or mean, *A*, of four exam grades, *x, y, z,* and *w,* is given by the formula $A = \frac{x+y+z+w}{4}$.

   a. Solve the formula for *w*.

   b. Use the formula in part (a) to solve this problem: On your first three exams, your grades are 76%, 78%, and 79%: $x = 76,\ y = 78,\ \text{and } z = 79$. What must you get on the fourth exam to have an average of 80%?

2. A charity has raised $225,000, with a goal of raising $500,000. What percent of the goal has been raised?

3. Suppose that the local sales tax rate is 7% and you buy a graphing calculator for $96.
   a. How much tax is due?

   b. What is the calculator's total cost?

Section 2.5: AN INTRODUCTION TO PROBLEM SOLVING

When you are done with your homework you should be able to…

- π Translate English phrases into algebraic expressions
- π Solve algebraic word problems using linear equations

WARM-UP:

Solve:

A fax machine regularly sells for \$380. The sale price is \$266. Find the percent decrease in the machine's price.

$A = 380 - 266 = 114$

$B = 380$

$A = P\% \cdot B$

$A = .01P \cdot B$

$\frac{114}{380} = \frac{.01P(380)}{380}$

$\frac{0.3}{.01} = \frac{.01P}{.01}$

$30 = P$

$\boxed{\text{The machine's price decreased by } 30\%.}$

STEPS FOR SOLVING WORD PROBLEMS

1. Analysis: READ the problem. Then, __read__ the problem again!!!

   Draw a __picture__ and/or make a __chart__. Identify and name all known and unknown __quantities__.

2. Translate to Mathese: Write an equation that translates, or __models__, the conditions of the problem.

3. Solve: __Solve__ the equation. Then __check__ your solution.

4. Conclusion: Write your result, in __words__.

Example 1: Solve the following word problems.

1. The sum of a number and 28 is 245. Find the number.

① Analysis

Let x be the number

② Translate

$x + 28 = 245$

③ Solve

$$x + 28 = 245$$
$$-28 \quad -28$$
$$x = 217$$

④ Conclusion

The number 217.

2. Three times the sum of five and a number is 48. Find the number.

① Analysis

Let x be the number

② Translate

$3(5 + x) = 48$

③ Solve

$$3(5+x) = 48$$
$$15 + 3x = 48$$
$$-15 \quad -15$$
$$\frac{3x}{3} = \frac{33}{3}$$
$$x = 11$$

④ Conclusion

The number is 11.

3. Eight subtracted from six times a number is 298. Find the number.

① Analysis

Let x be the number

② Translate

$6x - 8 = 298$

③ Solve

$$6x - 8 = 298$$
$$+8 \quad +8$$
$$\frac{6x}{6} = \frac{306}{6}$$
$$x = 51$$

④ Conclusion

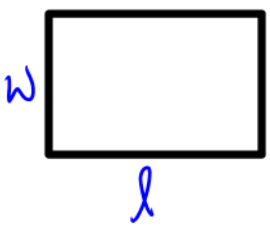
The number is 51.

4. If the quotient of three times a number and four is decreased by three, the result is nine. Find the number.

5. A car rental agency charges $180 per week plus $0.25 per mile to rent a car. How many miles can you travel in one week for $395?

6. A basketball court is a rectangle with a perimeter of 86 meters. The length is 13 meters more than the width. Find the width and length of the basketball court.

① Analysis



$P = 86$ and $P = 2l + 2w$

$l = w + 13$

② Translate

$86 = 2l + 2w$

$86 = 2(w+13) + 2w$

③ Solve

$86 = 2(w+13) + 2w$

$86 = 2w + 26 + 2w$

$86 = 4w + 26$

$\quad -26 \quad\; -26$

$60 = 4w$

$\frac{60}{4} = \frac{4w}{4}$

$15 = w$

$l = w + 13$

$l = 15 + 13 = 28$

④ Conclusion

The width is 15m and the length is 28m

7. This year's salary, \$42,074, is a 9% increase over last year's salary. What was last year's salary?

① Analysis

Let x be last year's salary

② Translate

$42074 = 100\%x + 9\%x$

$42074 = 109\%x$

③ Solve

$42074 = 109\%x$

$42074 = 1.09x$

$\frac{42074}{1.09} = \frac{1.09x}{1.09}$

$38600 = x$

④ Conclusion

$\boxed{\text{Last year's Salary was \$38600.}}$

8. A repair bill on a sailboat came to $1603, including $532 for parts and the remainder for labor. If the cost of labor is $35 per hour, how many hours of labor did it take to repair the sailboat?

① Analysis

Let x be the number of hours of labor

② Translate

$532 + 35x = 1603$

③ Solve

$$532 + 35x = 1603$$
$$-532 \qquad -532$$
$$\frac{35x}{35} = \frac{1071}{35}$$
$$x = 30\frac{21}{35}$$
$$x = 30\frac{3}{5}$$

$$\begin{array}{r} 30 \\ 35\,\overline{)1071} \\ \underline{105} \\ 21 \end{array}$$

④ Conclusion

It took $30\frac{3}{5}$ hrs of labor to repair the sailboat.

Section 2.6: PROBLEM SOLVING IN GEOMETRY

When you are done with your homework you should be able to…

- π Solve problems using formulas for perimeter and area
- π Solve problems using formulas for a circle's area and circumference
- π Solve problems using formulas for volume
- π Solve problems involving the angles of a triangle
- π Solve problems involving complementary and supplementary angles

WARM-UP:

Solve:

After a 30% reduction, you purchase a DVD player for $98. What was the selling price before the reduction?

$A = P \cdot B$

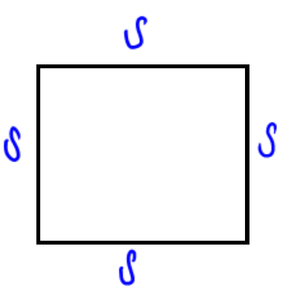
$98 = 70\% \cdot B$

$\dfrac{98}{0.70} = \dfrac{0.70B}{0.70}$

$140 = B$

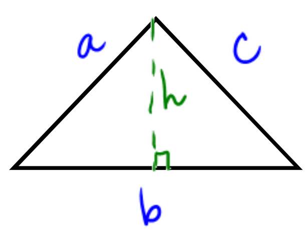
$\boxed{\text{The original price was \$140.}}$

COMMON FORMULAS FOR PERIMETER AND AREA



$P = 4s$ units

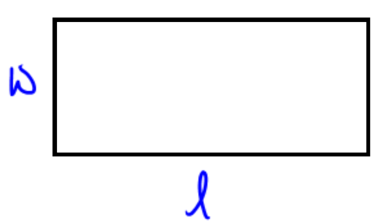
$A = s^2$ units squared



$P = (a+b+c)$ units

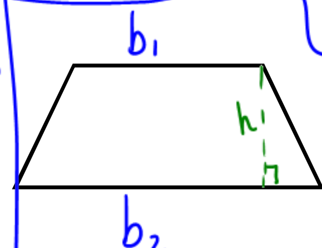
$A = \frac{1}{2}(\text{base})(\text{height})$

$A = \frac{1}{2}bh$ units squared



$P = (2l + 2w)$ units

$A = l \cdot w$ units squared



$\dfrac{b_1 + b_2}{2} \rightarrow$ average of the bases

A = (average of bases) times the height

$A = \frac{1}{2}(b_1 + b_2)h$

P = sum of the lengths of sides

Example 1: Solve.

1. A triangle has a base of 6 feet and an area of 30 square feet. Find the triangle's height.

base = 6 ft

area = 30 ft^2

$A_\Delta = \frac{1}{2}bh$

$30 = \frac{1}{2} \cdot 6 \cdot h$

$\frac{30}{3} = \frac{3h}{3}$

$10 = h$

$\boxed{\text{The height is 10 ft.}}$

2. A rectangle has a width of 46 cm and a perimeter of 208 cm. What is the rectangle's length?

$w = 46$

$P = 208$

$P = 2w + 2\ell$

$208 = 2(46) + 2 \cdot \ell$

$208 = 92 + 2\ell$

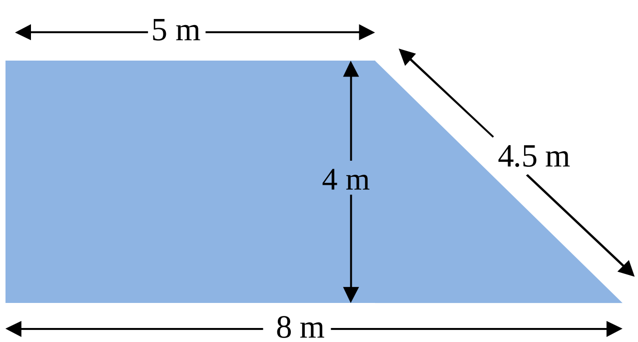
$-92 \quad -92$

$\frac{116}{2} = \frac{2\ell}{2}$

$58 = \ell$

$\boxed{\text{The length is 58 cm.}}$

3. Find the area of the trapezoid.



$b_1 = 5$

$b_2 = 8$

$h = 4$

$A = \frac{1}{2}(b_1 + b_2) \cdot h$

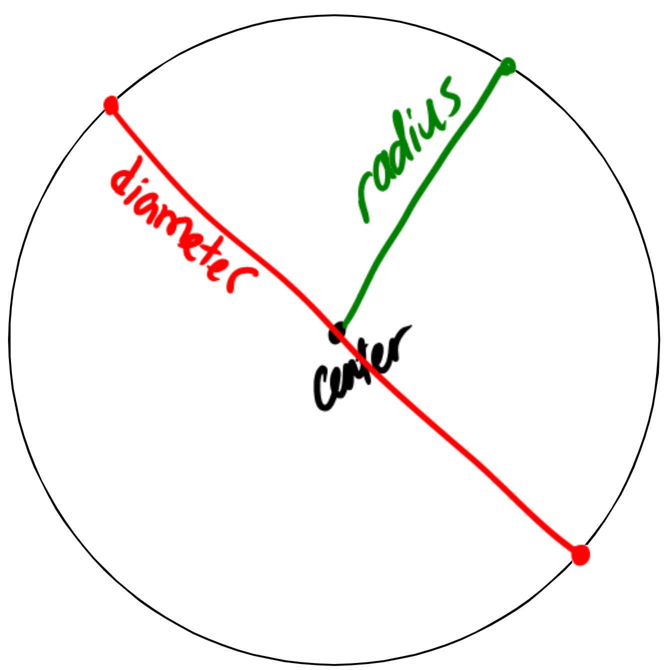
$A = \frac{1}{2}(5 + 8) \cdot 4$

$A = \frac{1}{2} \cdot 13 \cdot 4$

$\boxed{A = 26\ cm^2}$

GEOMETRIC FORMULAS FOR CIRCUMFERENCE AND AREA OF A CIRCLE

A circle is the set of all points in the plane equally distant from a given point, its center. A radius (plural radii), r, is a line segment from the center to any point on the circle. For a given circle, all radii have the same length. A diameter, d, is a line segment through the center whose endpoints both lie on the circle. For a given circle, all diameters have the same length. In any circle, the length of a diameter is twice the length of a radius and the length of a radius is half the length of a diameter.



Area

$A = \pi r^2$ units squared

Circumference

$C = 2\pi r$ units

or

$C = \pi d$ units

Example 2: Solve.

1. Find the area and circumference of a circle which has a diameter of 40 feet.

$A = \pi r^2 \rightarrow d = 40$ and $d = 2r$

$C = \pi \cdot d$

So

$\frac{2r}{2} = \frac{40}{2}$

$r = 20$

$A = \pi (20)^2$

$\boxed{A = 400\pi \text{ ft}^2}$ — exact

$A \approx 1256.6 \text{ ft}^2$ — approximate

$C = \pi \cdot 40$

$\boxed{C = 40\pi \text{ ft}}$ — exact

$C \approx 125.7 \text{ ft}$ — approximate

2. Which one of the following is a better buy: a large pizza with a 16-inch diameter for \$12 or two small pizzas, each with a 10-inch diameter, for \$12?

Large Pizza

$d = 16 \rightarrow r = \frac{1}{2}(16) = 8$

$A = \pi (8)^2$

$A = 64\pi \text{ in}^2$

$A \approx 201.1 \text{ in}^2$

Small pizza

$d = 10 \rightarrow r = \frac{1}{2}(10) = 5$

$A = \pi (5)^2$

$A = 25\pi \text{ in}^2$

$A \approx 78.5 \text{ in}^2$

2 smalls

$2(78.5 \text{ in}^2)$

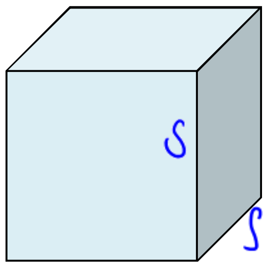
$= 157.1 \text{ in}^2$

$\boxed{\text{The large pizza is the better buy.}}$

GEOMETRIC FORMULAS FOR VOLUME

__Volume________ refers to the amount of __space________ occupied by a ____3_____-__dimensional________ figure. To measure this space, we use __cubic______ units.

cube

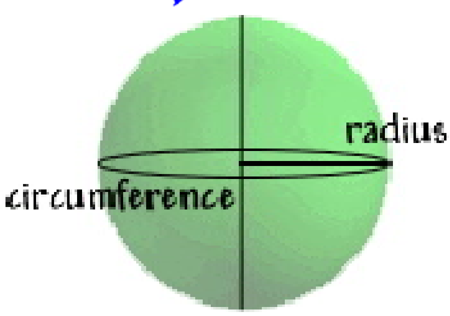


$V = s^3$ units cubed

rectangular solid

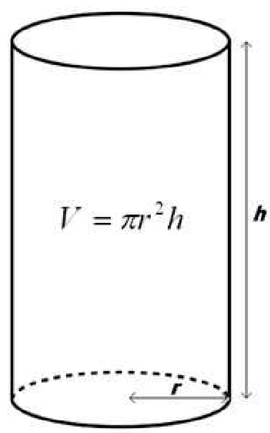
$V = l \cdot w \cdot h$ units cubed

sphere



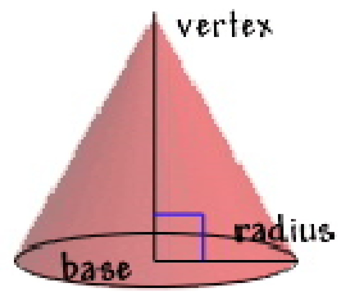
$V = \frac{4}{3}\pi r^3$ units cubed

right circular cylinder



$V = \pi r^2 h$ units cubed

right circular cone



$V = \frac{1}{3}\pi r^2 h$ units cubed

Example 3: Solve.

1. Solve the formula for the volume of a cone for *h*.

$$3(V) = \left(\frac{1}{\cancel{3}}\pi r^2 h\right)\cancel{3}$$

$$\frac{3V}{\pi r^2} = \frac{\pi r^2 \cdot h}{\pi r^2}$$

$$\frac{3V}{\pi r^2} = h$$

$$\boxed{h = \frac{3V}{\pi r^2}}$$

2. A cylinder with radius 2 inches and height 3 inches has its radius quadrupled. How many times greater is the volume of the larger cylinder than the smaller cylinder?

3. Find the volume of a shoebox with dimensions 6 in x 12 in x 5 in.

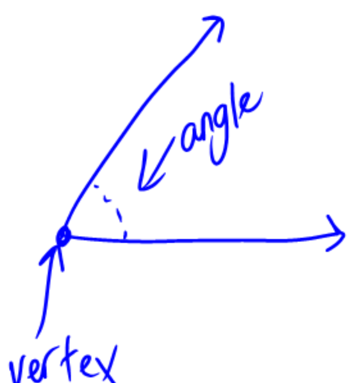
$V = l \cdot w \cdot h$

$V = 12 \cdot 6 \cdot 5$

$\boxed{V = 360 \text{ in}^3}$

THE ANGLES OF TRIANGLES

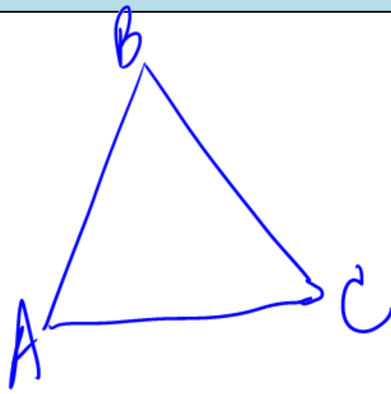
An ___angle___, symbolized by ___$\angle$___, is made up of two ___rays___ that have a common ___endpoint___. The common endpoint is called the ___vertex___. The two rays that form the angle are called its ___sides___.



One way to measure angles is in degrees, symbolized by a small, raised circle °. There are 360° in a circle. 1° is $\frac{1}{360}$ of a complete rotation.

THE ANGLES OF A TRIANGLE

The sum of the measures of the three angles of any triangle is 180°.

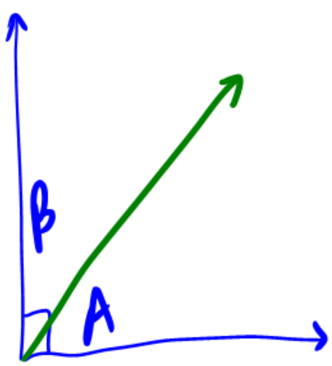


$A + B + C = 180°$

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

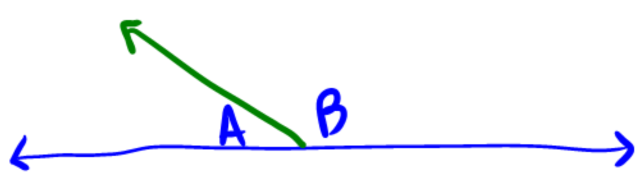
Two angles with measures having a sum of 90° are called complementary angles. Two angles with measures having a sum of 180° are called supplementary angles.

complementary angles



$A + B = 90°$

Supplementary angles



$A + B = 180°$

Example 4: Solve.

1. One angle of a triangle is three times as large as another. The measure of the third angle is 40º more than that of the smallest angle. Find the measure of each angle.

① Analysis

Let x be the smallest $\measuredangle$

② Translate

$x + 3x + x + 40 = 180$

③ Solve

$x + 3x + x + 40 = 180$

$5x + 40 = 180$

$-40 \quad -40$

$5x = 140$

$\frac{5x}{5} = \frac{140}{5}$

$x = 28$

$3x = 3(28) = 84$

$x + 40 = 28 + 40 = 68$

④ Conclusion

The angles measure 28°, 84°, and 68°

2. Find the measure of the complement of each angle.

a. 56º

$A + B = 90$

$A + 56 = 90$

$A = \boxed{34°}$

b. 89.5º

$A + B = 90$

$A + 89.5 = 90$

$A = \boxed{0.5°}$

3. Find the measure of the supplement of each angle.

a. 177º

$A + B = 180$

$A + 177 = 180$

$A = \boxed{3°}$

b. 0.2º

$A + B = 180$

$A + 0.2 = 180$

$A = \boxed{179.8°}$

4. Find the measure of the angle described.

The measure of the angle's supplement is 52º more than twice that of its complement.

Let the angle be A

comp: $A + B = 90 \rightarrow B = 90 - A$

supp: $A + B = 180 \rightarrow B = 180 - A$

$180 - A = 2(90 - A) + 52$

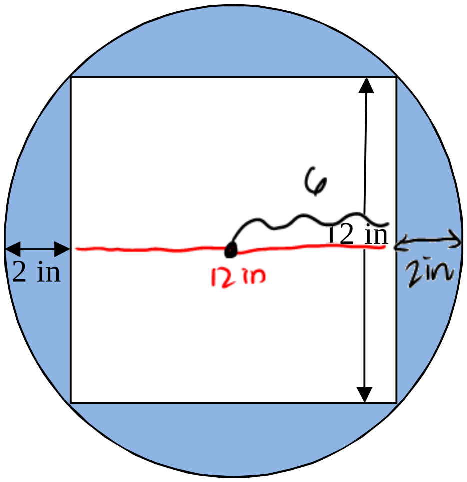
zero out

$\cancel{180} - A = \cancel{180} - 2A + 52$

$+2A \quad +2A$

$\boxed{A = 52°}$

Example 5: Find the area of the shaded region.



$r = 8$ in

$$A_{shaded} = A_{circle} - A_{square}$$

$$= \pi r^2 - s^2$$

$$= \pi(8)^2 - (12)^2$$

$$= \boxed{(64\pi - 144)\text{ in.}^2}$$

Section 2.7: SOLVING LINEAR INEQUALITIES

When you are done with your homework you should be able to…

- π Graph the solutions of an inequality on a number line
- π Use interval notation
- π Understand properties used to solve linear inequalities
- π Solve linear inequalities
- π Identify inequalities with no solution of infinitely many solutions
- π Solve problems using linear inequalities

WARM-UP:

Solve:

Find the volume of a sphere with diameter 11 meters.

VOCABULARY

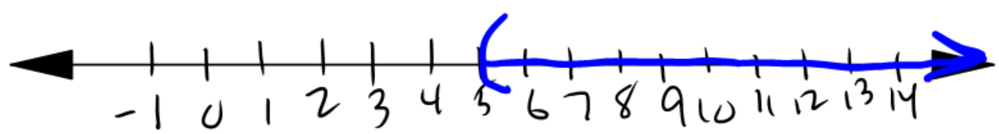
Linear inequality in one variable: An inequality in the form $ax+b<c$, $ax+b\le c$, $ax+b>c$, or $ax+b\ge c$ is a linear inequality in one variable. $<$ means less than, $\le$ means less than or equal to, $>$ means greater than, and $\ge$ means greater than or equal to.

Solving an inequality: The process of finding the set of numbers that will make the inequality a true statement. These numbers are called the **solutions** of the inequality, and we say they **satisfy** the inequality. The set of all solutions is called the **solution set** of the inequality.

GRAPHS OF INEQUALITIES

There are infinitely many solutions to the inequality $x > 5$. In other words, the solution set for this inequality is all real numbers which are greater than 5. Can we list all these numbers? What does the graph of the solution set look like? Hmmmm...

Set-builder: $\{x \mid x > 5\}$



interval: $(5, \infty)$

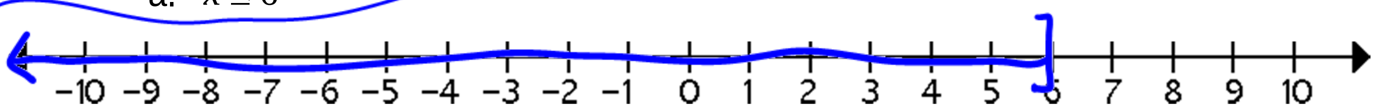
Graphs of solutions to linear inequalities are shown on a number line by shading all points representing numbers that are solutions. Square brackets, [ ], indicate endpoints that are solutions and parentheses, ( ), indicate endpoints that are not solutions.

set-builder: $\{x \mid x \leq 6\}$

interval: $(-\infty, 6]$

Example 1: Graph the solutions of each inequality.

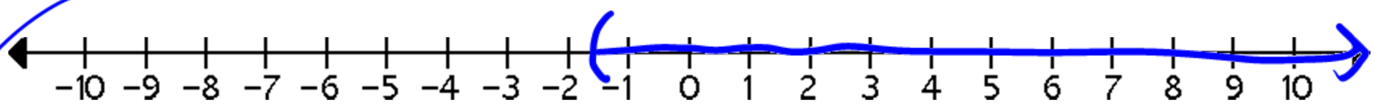
a. $x \leq 6$



b. $x > -\frac{3}{2}$

set-builder: $\{x \mid x > -\frac{3}{2}\}$

interval: $(-\frac{3}{2}, \infty)$

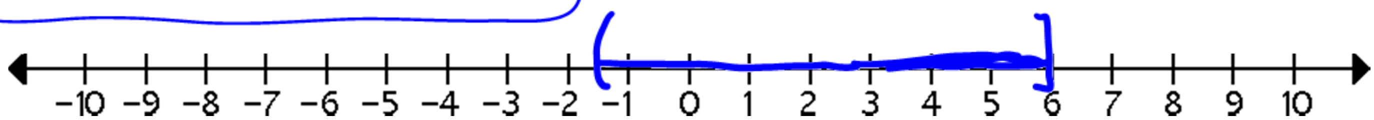


c. $-\frac{3}{2} < x \le 6$

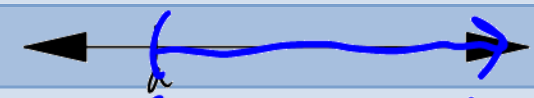
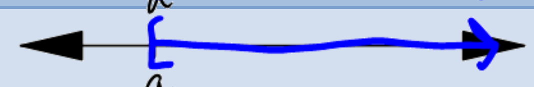
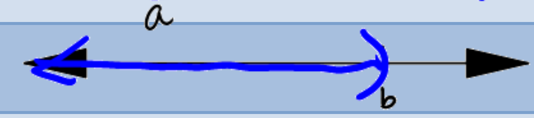
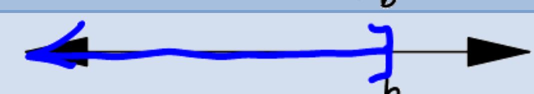
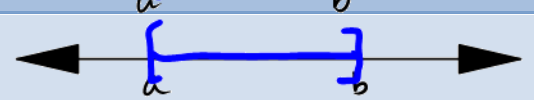
$x > -\frac{3}{2}$ and $x \le 6$

set-builder: $\{x \mid -\frac{3}{2} < x \le 6\}$

interval: $(-\frac{3}{2}, 6]$



SOLUTION SETS OF INEQUALITIES

INEQUALITY	INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
$x > a$	(a, ∞)	$\{x \mid x > a\}$	
$x \ge a$	$[a, \infty)$	$\{x \mid x \ge a\}$	
$x < b$	$(-\infty, b)$	$\{x \mid x < b\}$	
$x \le b$	$(-\infty, b]$	$\{x \mid x \le b\}$	
$a < x < b$	(a, b)	$\{x \mid a < x < b\}$	
$a \le x \le b$	$[a, b]$	$\{x \mid a \le x \le b\}$	
$a < x \le b$	$(a, b]$	$\{x \mid a < x \le b\}$	
$a \le x < b$	$[a, b)$	$\{x \mid a \le x < b\}$	

PARENTHESIS ARE ALWAYS USED WITH $-\infty$ **OR** ∞**!!!**

PROPERTIES OF INEQUALITIES

PROPERTY	THE PROPERTY IN WORDS	EXAMPLE
THE ADDITION PROPERTY OF INEQUALITY If $a<b$, then $a+c<b+c$. If $a<b$, then $a-c<b-c$.	If the same quantity is added to or subtracted from both sides of an inequality, the resulting inequality is equivalent.	$x+3<6$ $x+3-3<6-3$
THE POSITIVE MULTIPLICATION PROPERTY OF INEQUALITY If $a<b$ and c is positive, then $a\cdot c<b\cdot c$. If $a<b$ and c is positive, then $\frac{a}{c}<\frac{b}{c}$.	If we multiply or divide both sides of an inequality by a positive (nonzero) number, the resulting inequality is equivalent.	$5x\geq 10$ $\frac{5x}{5}\geq\frac{10}{5}$
THE NEGATIVE multiplication PROPERTY OF INEQUALITY If $a<b$ and c is negative, then $a\cdot c>b\cdot c$. If $a<b$ and c is negative, then $\frac{a}{c}>\frac{b}{c}$.	If we multiply or divide both sides of an inequality by the same negative number AND reverse the inequality symbol, the resulting inequality is equivalent.	$-3x>12$ $\frac{-3x}{-3}<\frac{12}{-3}$

$-x<1$
$+x \quad +x$
$0<x+1$

$0<x+1$
$-1 \quad -1$
$-1<x \rightarrow x>-1$

STEPS FOR SOLVING A LINEAR INEQUALITY

1. Simplify the algebraic expression on each side.
2. Use the addition property of inequality to collect all the variable terms on one side and all the constant terms on the other side.
3. Use the multiplication property of inequality to isolate the variable and solve. Change the direction of the inequality when multiplying or dividing both sides by a negative number.
4. Express the solution set in interval or set-builder notation, and graph the solution set on a number line.

Example 2: Solve each inequality and graph the solution.

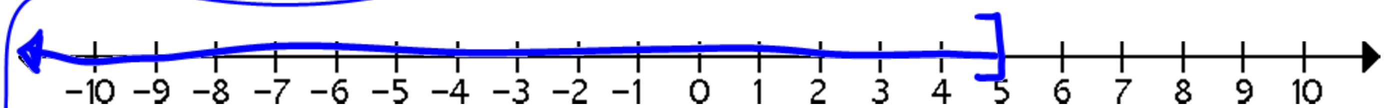
a. $x - 3 \leq 2$

$\quad +3 \quad +3$

$x \leq 5$

Set-builder: $\{x \mid x \leq 5\}$

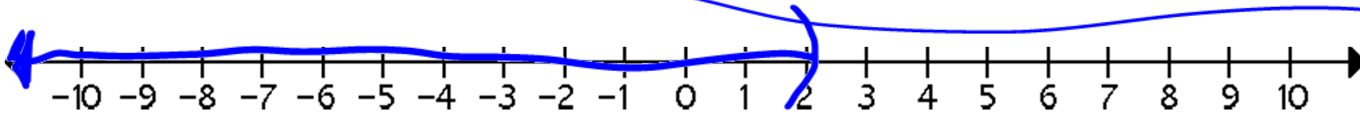
interval: $(-\infty, 5]$



b. ~~$5x+8>2x-7$~~

Change to: $-5x+8>2x-7$

$$\begin{array}{rcl} -5x+8 & > & 2x-7 \\ -8 & & -8 \\ \hline -5x & > & 2x-15 \\ -2x & & -2x \\ \hline -7x & > & -15 \\ \overline{-7} & & \overline{-7} \\ x & < & \frac{15}{7} \end{array}$$



Set-builder: $\{x \mid x < \frac{15}{7}\}$

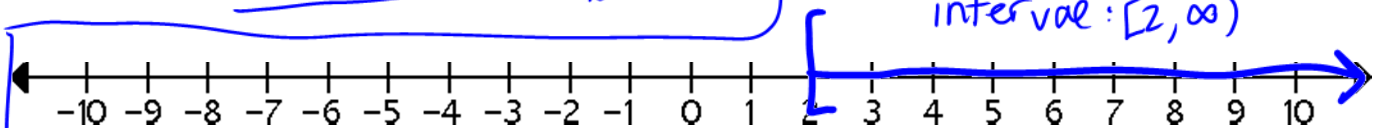
interval: $(-\infty, \frac{15}{7})$

c. $4(x+1) \geq 3x+6$

$$\begin{array}{rcl} 4x+4 & \geq & 3x+6 \\ -3x & & -3x \\ \hline x+4 & \geq & 6 \\ -4 & & -4 \\ \hline x & \geq & 2 \end{array}$$

set-builder: $\{x \mid x \geq 2\}$

interval: $[2, \infty)$



RECOGNIZING INEQUALITIES WITH NO SOLUTION OR INFINITELY MANY SOLUTIONS

If you attempt to solve an inequality with __no__ __Solution__ or one that is __true__ for __every__ __real__ number, you will __eliminate__ the __variable__.

- An inequality with __no__ __Solution__ results in a __false__ statement, such as __$0<0$__. The solution set is __$\{\ \}$__ or __$\emptyset$__, the __empty__ set, and the __graph__ is an __unshaded__ number line.

π An inequality that is true for every real number results in a true statement, such as $0 \le 0$. The solution set is $(-\infty, \infty)$ or $\{x \mid x \text{ is a real number}\}$, and the graph is a fully shaded number line.

Example 3: Solve each inequality and graph the solution.

a. $2(x+1)-1<2x+1$

$2x+2-1<2x+1$

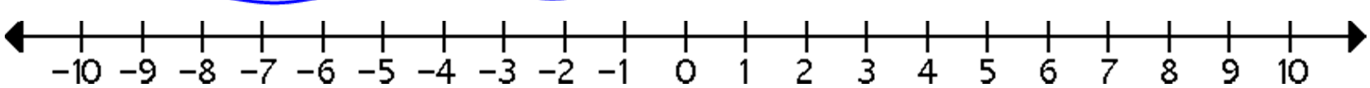
$2x+1<2x+1$

$-2x \quad -2x$

$1<1$ false! no solution

Set-builder: $\{\ \}$

interval: $\emptyset$



b. $5x>2(x-7)+3x$

$5x>2x-14+3x$

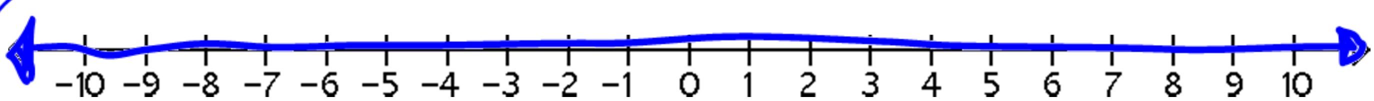
$5x>5x-14$

$-5x \quad -5x$

$0>-14$ true! infinitely many solutions

Set-builder: $\{x \mid x \text{ is a real number}\}$

interval: $(-\infty, \infty)$



APPLICATION

On three examinations, you have grades of 88, 78, and 86. There is still a final examination, which counts as one grade.

1. In order to get an A, your average must be at least 90. If you get 100 on the final, compute your average and determine if an A in the course is possible.

$$\begin{array}{r} 88 \\ 78 \\ 86 \\ \underline{100} \\ 352 \end{array}$$

$$\text{Average} = \frac{352}{4} = 88.$$

$\boxed{\text{An A is not possible.}}$

2. To earn a B in the course, you must have a final average of at least 80. What must you get on the final to earn a B in the course?

Let x be the grade earned on the final.

$$\cancel{4}\left(\frac{88+78+86+x}{\cancel{4}}\right) \geq 80 \cdot 4$$

$$88+78+86+x \geq 320$$

$$\begin{array}{rcl} 252+x & \geq & 320 \\ \underline{-252} & & \underline{-252} \\ x & \geq & 68 \end{array}$$

$\boxed{\text{You must earn at least a 68 on the final to get a B in the course.}}$

When you are done with your homework you should be able to…

- π Plot ordered pairs in the rectangular coordinate system
- π Find coordinates of points in the rectangular coordinate system
- π Determine whether an ordered pair is a solution of an equation
- π Find solutions of an equation in two variables
- π Use point plotting to graph linear equations
- π Use graphs of linear equations to solve problems

WARM-UP:

1. Find the volume of a box with dimensions ½ ft by 3 ft by 8 ft.

$V = l \cdot w \cdot h$

$V = (8)(3)\left(\frac{1}{2}\right)$

$\boxed{V = 12\ ft^3}$

2. Solve the following inequalities and graph the solution sets.

a. $x \le 6(3x-5)$

$x \le 18x - 30$

$-18x \quad -18x$

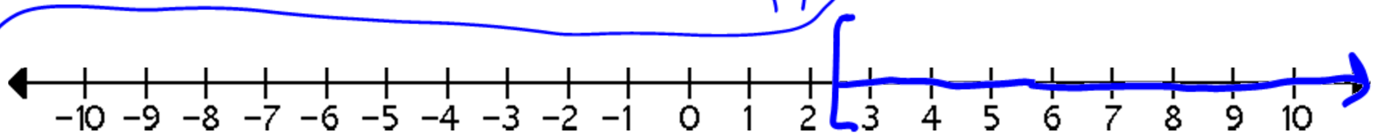
$-17x \le -30$

$\frac{-17x}{-17} \quad \frac{-30}{-17}$

$x \ge \frac{30}{17}$

Set-builder: $\{x \mid x \ge \frac{30}{17}\}$

interval: $\left[\frac{30}{17}, \infty\right)$



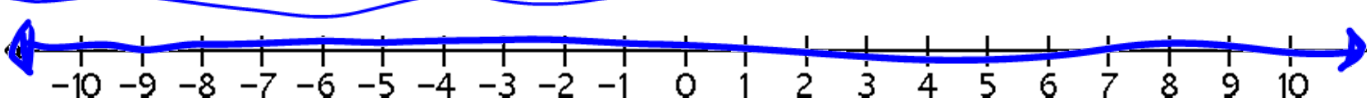
b. $2x - 1 \le 2x$

$-2x \quad -2x$

$-1 \le 0$ true! infinitely many solutions

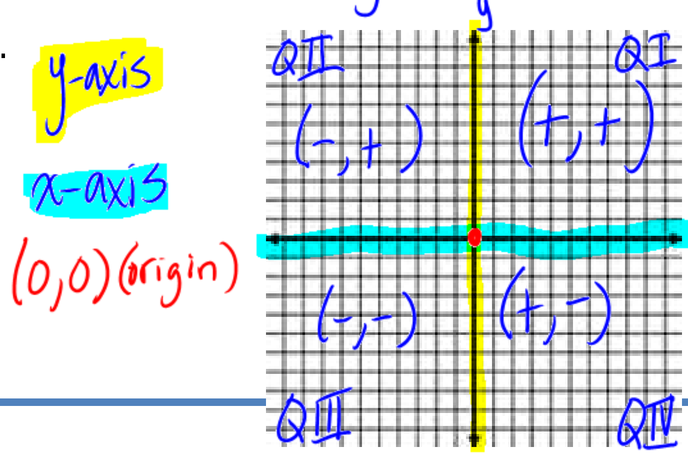
set-builder: $\{x \mid x$ is a real number$\}$

interval: $(-\infty, \infty)$



POINTS AND ORDERED PAIRS

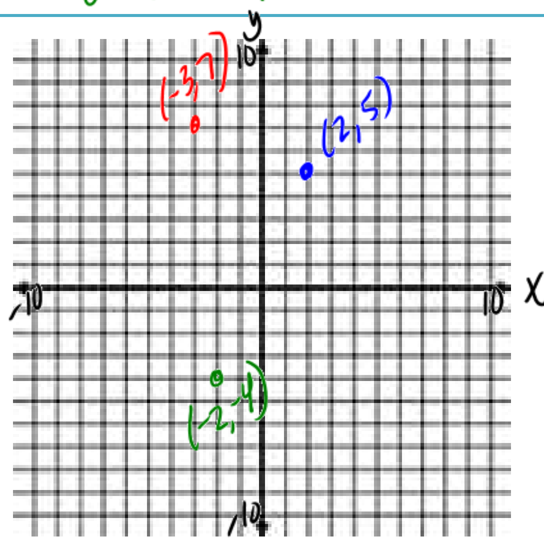
The idea of visualizing equations as geometric figures was developed by the French philosopher and mathematician Rene Descartes. This idea is the rectangular coordinate system or the Cartesian coordinate system. The rectangular coordinate system consists of 2 number lines that intersect at right angles at their zero points. The horizontal number line is the x-axis and the vertical number line is the y-axis. The point of intersection is a point called the origin. Positive numbers are to the right and up the origin. Negative numbers are to the left and down the origin. The axes divide the plane into 4 regions, called quadrants. The points located on the axes are not in any quadrant. Each point in the rectangular coordinate system corresponds to an ordered pair of real numbers, (x, y). The first number in each pair, called the x-coordinate, denotes the distance and direction from the origin along the x-axis. The second number, called the y-coordinate, denotes the vertical distance along a line parallel to the y-axis or along the y-axis itself.



Example 1: Plot the following ordered pairs.

$(2,5),\ (-3,7),\ (-2,-4)$

(2,5)	2 units to the right and 5 units up
(-3,7)	3 units to the left and 7 units up
(-2,-4)	2 units to the left and 4 units down



SOLUTIONS OF EQUATIONS IN TWO VARIABLES

A __Solution__ of an __equation__ in __2__ variables, __x__ and __y__, is an __ordered__ __pair__ of real numbers with the following property: When the __x-coordinate__ is substituted for __x__ and the __y-coordinate__ is substituted for __y__ in the equation, we obtain a __true__ statement.

Example 2: Determine whether each of the given points is a solution of the equation $8x+y=1$.

a. $(0,1)$

$8x+y=1$

$8(0)+(1)\stackrel{?}{=}1$

$0+1\stackrel{?}{=}1 \rightarrow 1=1$ ✓

$\boxed{\text{yes}}$

b. $(-1,3)$

$8x+y=1$

$8(-1)+(3)\stackrel{?}{=}1$

$-8+3\stackrel{?}{=}1$

$-5\neq 1$

$\boxed{\text{NO}}$

c. $(2,-15)$

$8x+y=1$

$8(2)+(-15)\stackrel{?}{=}1$

$16-15\stackrel{?}{=}1$

$1=1$

$\boxed{\text{yes}}$

Example 3: Find three solutions of $2y = -x - 1$.

① Let $x = 1$: $2y = -(1) - 1$, $2y = -2$, $y = -1$ $\boxed{(1, -1)}$

② Let $x = 5$: $2y = -(5) - 1$, $2y = -6$, $y = -3$ $\boxed{(5, -3)}$

③ Let $y = 0$: $2(0) = -x - 1$, $0 = -x - 1$, $x = -1$ $\boxed{(-1, 0)}$

GRAPHING LINEAR EQUATIONS IN THE FORM $y = mx + b$

The __graph__ of the __equation__ is the __set__ of all __points__ whose __coordinates__ satisfy the equation.

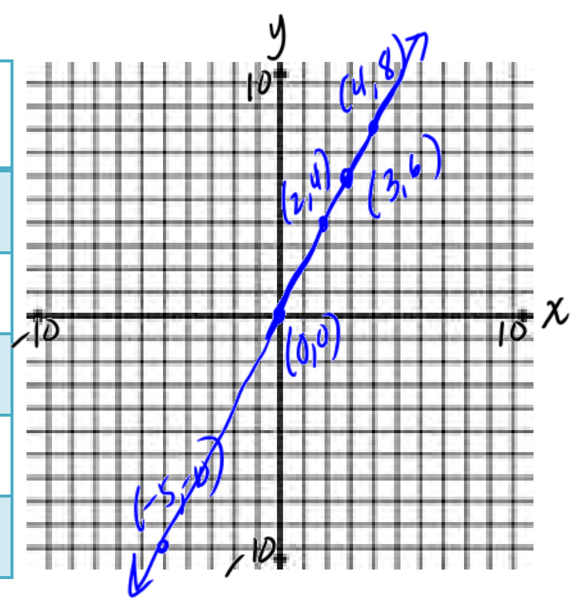
STEPS FOR USING THE POINT-PLOTTING METHOD FOR GRAPHING AN EQUATION IN TWO VARIABLES

1. Find several __ordered__ __pairs__ that are __solutions__ of the equation.
2. Plot these ordered pairs as __points__ in the __rectangular__ coordinate system.
3. __Connect__ the points with a __smooth__ curve or __line__, depending on the type of equation.

Example 3: Graph the following equations by plotting points.

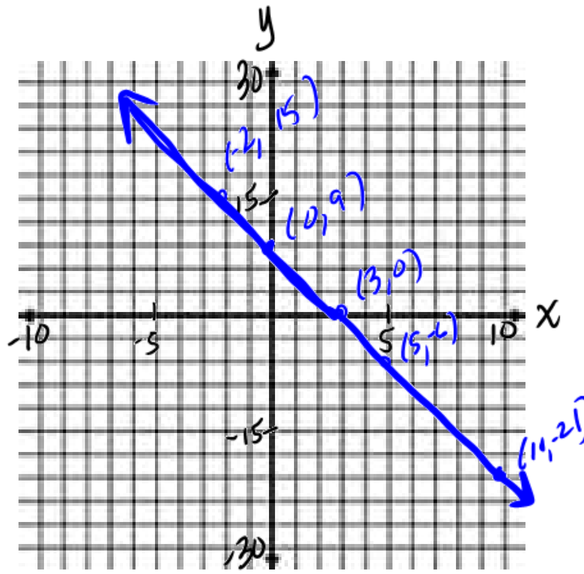
a. $y = 2x$

x	$y = 2x$	(x, y)
4	$y = 2(4) \to y = 8$	$(4, 8)$
3	$y = 2(3) \to y = 6$	$(3, 6)$
2	$y = 2(2) \to y = 4$	$(2, 4)$
0	$y = 2(0) \to y = 0$	$(0, 0)$
-5	$y = 2(-5) \to y = -10$	$(-5, -10)$



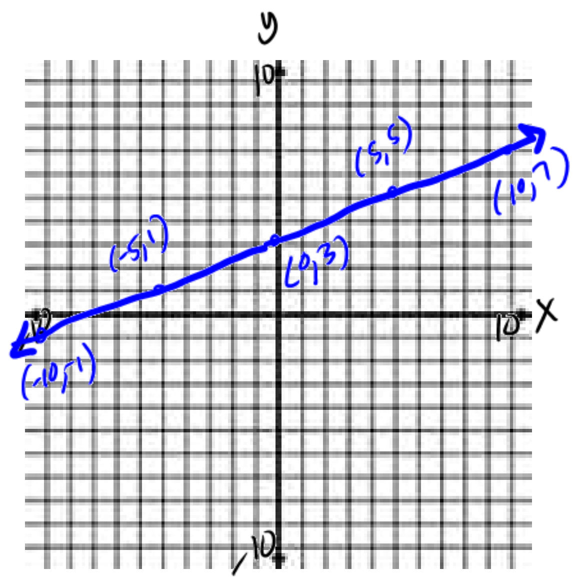
b. $y=-3x+9$

x	$y=-3x+9$	(x,y)
10	$y=-3(10)+9 \rightarrow y=-21$	$(10,-21)$
5	$y=-3(5)+9 \rightarrow y=-6$	$(5,-6)$
3	$y=-3(3)+9 \rightarrow y=0$	$(3,0)$
0	$y=-3(0)+9 \rightarrow y=9$	$(0,9)$
-2	$y=-3(-2)+9 \rightarrow y=15$	$(-2,15)$



c. $y=\frac{2}{5}x+3$

x	$y=\frac{2}{5}x+3$	(x,y)
-10	$y=\frac{2}{5}(-10)+3 \rightarrow y=-4+3 \rightarrow y=-1$	$(-10,-1)$
-5	$y=\frac{2}{5}(-5)+3 \rightarrow y=-2+3 \rightarrow y=1$	$(-5,1)$
0	$y=\frac{2}{5}(0)+3 \rightarrow y=0+3 \rightarrow y=3$	$(0,3)$
5	$y=\frac{2}{5}(5)+3 \rightarrow y=2+3 \rightarrow y=5$	$(5,5)$
10	$y=\frac{2}{5}(10)+3 \rightarrow y=4+3 \rightarrow y=7$	$(10,7)$



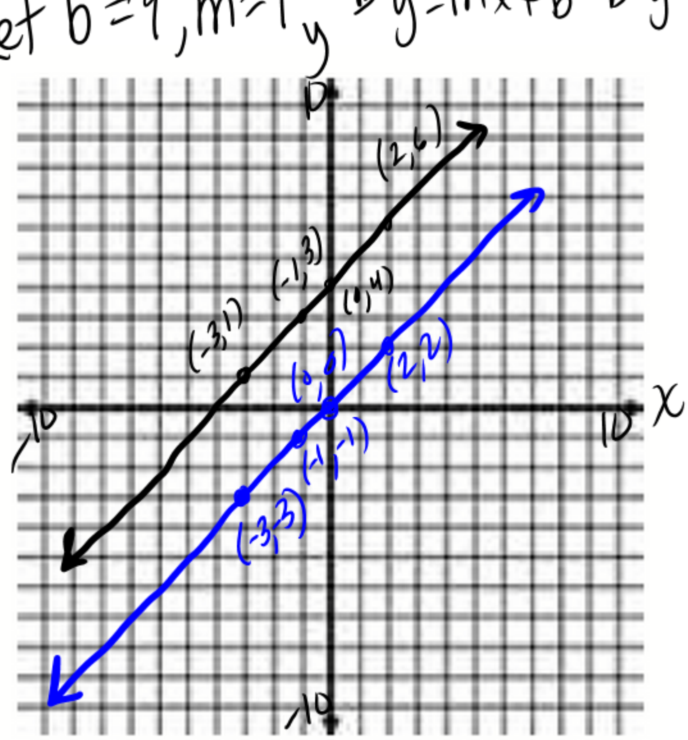
COMPARING GRAPHS OF LINEAR EQUATIONS

If the value of __m__ does not change,

- The graph of __$y=mx+b$__ is the graph of __$y=mx$__ shifted __b__ units __up__ when __b__ is a positive number.
- The graph of __$y=mx+b$__ is the graph of __$y=mx$__ shifted __b__ units __down__ when __b__ is a negative number.

Let $m=1$ so $y=mx \rightarrow y=x$

Let $b=4, m=1 \rightarrow y=mx+b \rightarrow y=x+4$



x	$y=x$
-3	-3
-1	-1
0	0
2	2

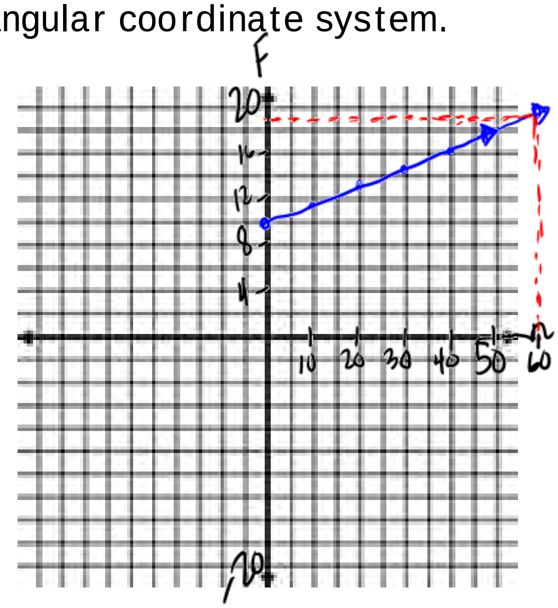
APPLICATION

In 1960, per capita fish consumption was 10 pounds. This increased by approximately 0.15 pound per year from 1960 through 2005. These conditions can be described by the mathematical model $F = 0.15n + 10$, where F is per capita fish consumption n years after 1960.

a. Let n = 0, 10, 20, 30, and 40. Make a table of values showing five solutions of the equation.

n	$F = 0.15n + 10$	(n, F)
0	$F = 0.15(0) + 10 \rightarrow F = 10$	$(0, 10)$
10	$F = 0.15(10) + 10 \rightarrow F = 11.5$	$(10, 11.5)$
20	$F = 0.15(20) + 10 \rightarrow F = 13$	$(20, 13)$
30	$F = 0.15(30) + 10 \rightarrow F = 14.5$	$(30, 14.5)$
40	$F = 0.15(40) + 10 \rightarrow F = 16$	$(40, 16)$

b. Graph the formula in a rectangular coordinate system.



c. Use the graph to estimate per capita fish consumption in 2020.

$F = \boxed{19}$

d. Use the formula to project per capita fish consumption in 2020.

$F = 0.15(n) + 10$

$F = 0.15(60) + 10$

$F = \boxed{19}$

Section 3.2: GRAPHING LINEAR EQUATIONS USING INTERCEPTS

When you are done with your homework you should be able to...

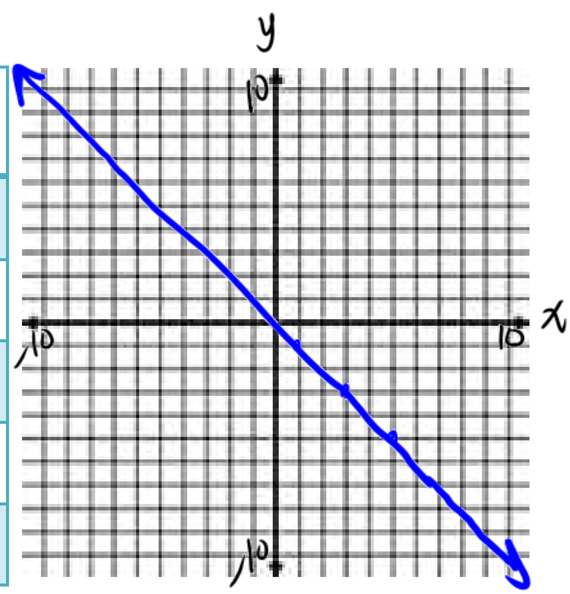
- π Use a graph to identify intercepts
- π Graph a linear equation in two variables using intercepts
- π Graph horizontal or vertical lines

WARM-UP:

Graph the following equations by plotting points.

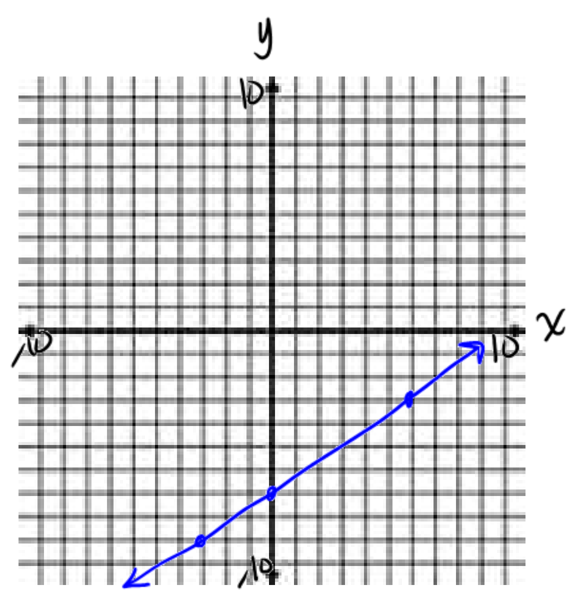
a. $y=-x$

x	$y=-x$	(x,y)
1	$y=-1$	$(1,-1)$
3	$y=-3$	$(3,-3)$
5	$y=-5$	$(5,-5)$



b. $y=\frac{2}{3}x-7$

x	$y=\frac{2}{3}x-7$	(x,y)
6	$y=\frac{2}{\cancel{3}_{1}}(\cancel{6}^{2})-7 \rightarrow y=-3$	$(6,-3)$
0	$y=\frac{2}{3}(0)-7 \rightarrow y=-7$	$(0,-7)$
-3	$y=\frac{2}{\cancel{3}_{1}}(-\cancel{3}^{1})-7 \rightarrow y=-9$	$(-3,-9)$



INTERCEPTS

An ___x- intercept___ of a graph is the ___x-coordinate___ of a point where the graph ___intersects___ the ___x-axis___. **The ___y-coordinate___ corresponding to an ___x-intercept___ is always ___0___!!!**

A ___y-intercept___ of a graph is the ___y-coordinate___ of a point where the graph ___intersects___ the ___y-axis___. **The ___x-coordinate___ corresponding to a ___y-intercept___ is always ___0___!!!**

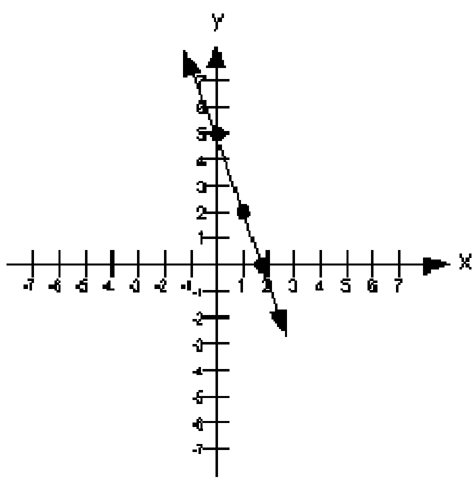
Example 1: Use the graph to identify the

a. *x*-intercept

Approx: (1.8, 0)

b. *y*-intercept

(0, 5)



GRAPHING USING INTERCEPTS

An equation of the form ___$Ax + By = C$___, where ___A___, ___B___, and ___C___ are integers, is called the ___Standard___ form of a line.

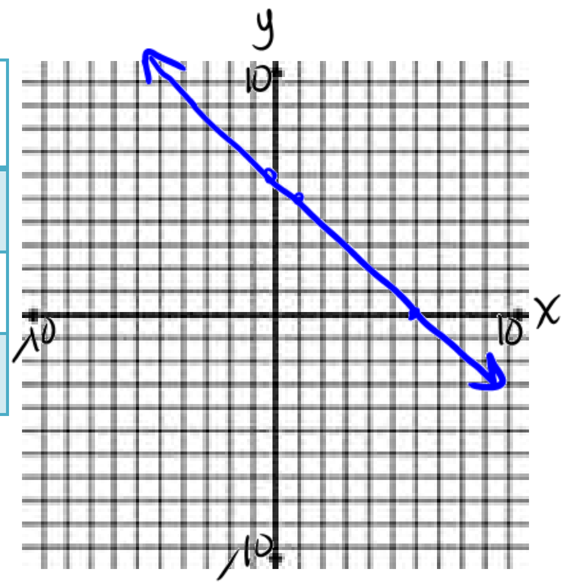
STEPS FOR USING INTERCEPTS TO GRAPH $Ax + By = C$

1. Find the x-intercept. Let $y=0$ and solve for x.
2. Find the y-intercept. Let $x=0$ and solve for y.
3. Find a checkpoint, a 3rd ordered-pair solution.
4. Graph the equation by drawing a line through the 3 points.

Example 2: Graph using intercepts and a checkpoint.

a. $x + y = 6$

	$x + y = 6$	(x, y)
x-int $y=0$	$x + 0 = 6 \rightarrow x = 6$	$(6,0)$
y-int $x=0$	$0 + y = 6 \rightarrow y = 6$	$(0,6)$
check point $y=5$	$x + 5 = 6 \rightarrow x = 1$	$(1,5)$

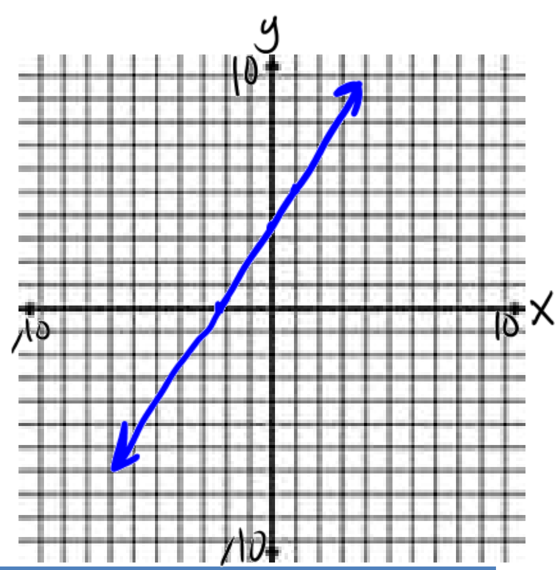


b. $3x - 2y = -7$

	$3x - 2y = -7$	(x, y)
x-int $y=0$	$3x - 2(0) = -7 \rightarrow 3x = -7 \rightarrow x = -\frac{7}{3}$	$(-\frac{7}{3}, 0)$
y-int $x=0$	$3(0) - 2y = -7 \rightarrow -2y = -7 \rightarrow y = \frac{7}{2}$	$(0, \frac{7}{2})$
check point $x=1$	$3(1) - 2y = -7 \rightarrow 3 - 2y = -7 \rightarrow -2y = -10 \rightarrow y = 5$	$(1,5)$

$$\frac{3x}{3} = \frac{-7}{3} \rightarrow x = -\frac{7}{3}$$

$1x = -7/3$



EQUATIONS OF HORIZONTAL AND VERTICAL LINES

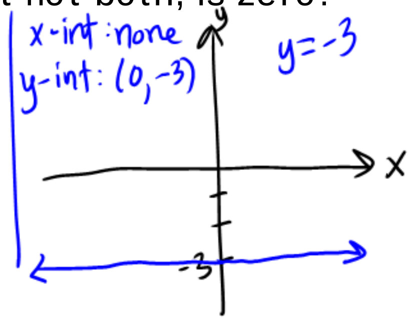
We know that the graph of any equation of the form $Ax+By=C$ is a line as long as A and B are not both 0. What happens if A or B, but not both, is zero?

$A=0 \to 0\cdot x+By=C$

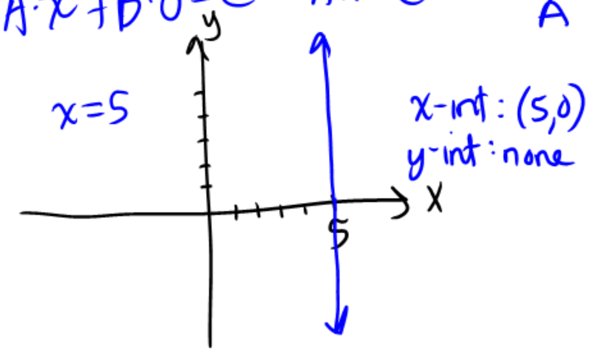
$By=C$

$y=\frac{C}{B}$

Horizontal line

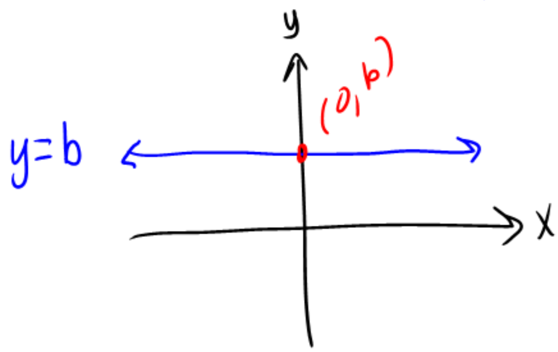


$B=0 \to A\cdot x+B\cdot 0=C \to Ax=C \to x=\frac{C}{A}$

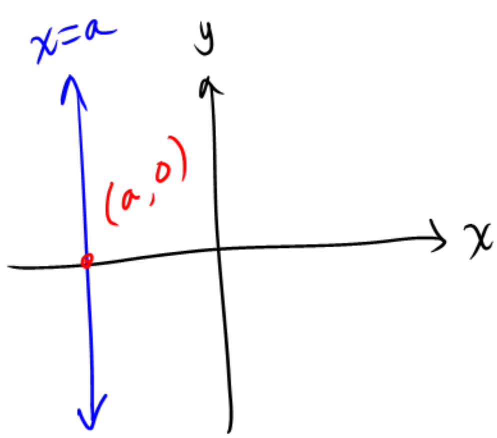


HORIZONTAL AND VERTICAL LINES

The graph of $y=b$ is a horizontal line. The y-intercept is $(0,b)$.

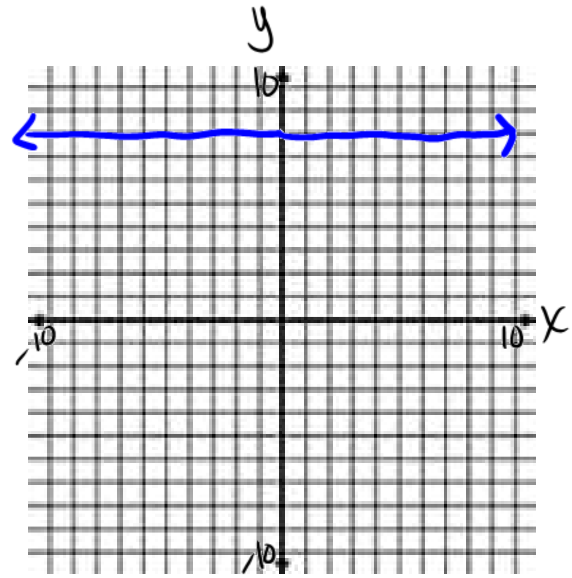


The graph of $x=a$ is a vertical line. The x-intercept is $(a,0)$.



Example 3: Graph.

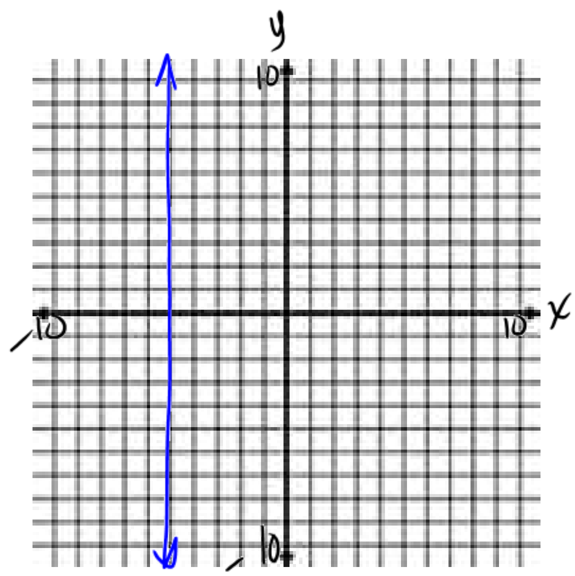
a. $y = 8$



b. $12x = -60$

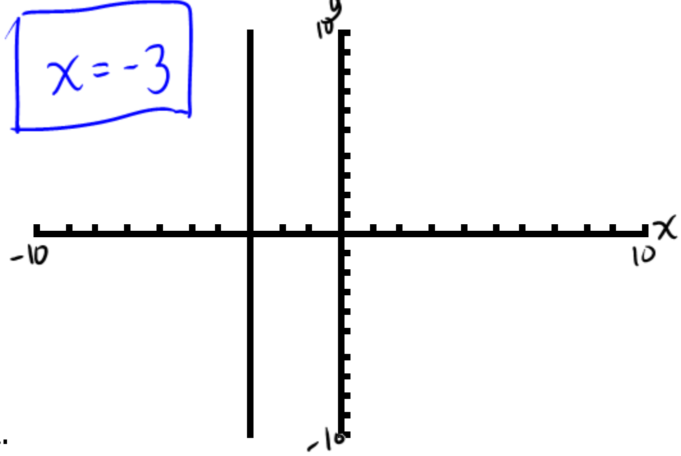
$\frac{12x}{12} = \frac{-60}{12}$

$x = -5$



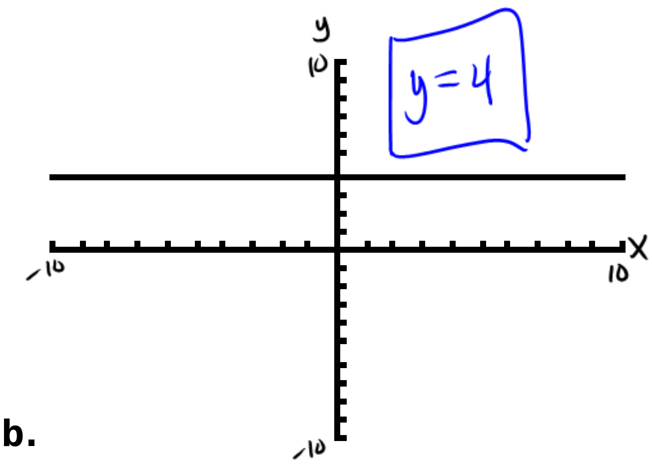
Example 4: Write an equation for each graph.

$\boxed{x = -3}$



a.

$\boxed{y = 4}$



b.

APPLICATION

A new car worth \$24,000 is depreciating in value by \$3000 per year. The mathematical model $y = -3000x + 24000$ describes the car's value, *y,* in dollars, after *x* years.

a. Find the *x*-intercept. Describe what this means in terms of the car's value.

Let $y=0$ $\quad 0 = -3000x + 24000$

$3000x = 24000$

$x = 8$

After 8 years the car has no monetary value.

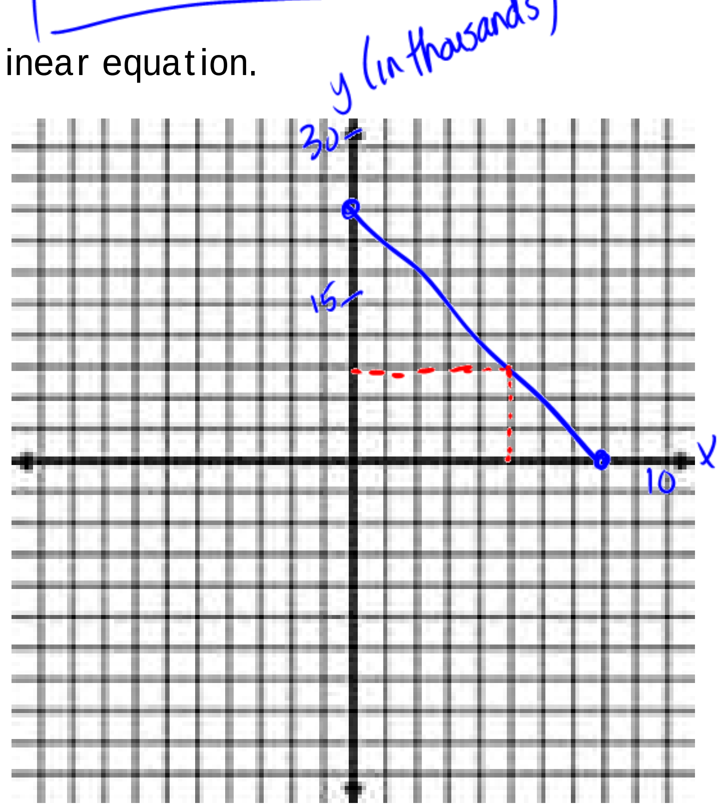
b. Find the *y*-intercept. Describe what this means in terms of the car's value.

Let $x=0$ $\quad y = -3000(0) + 24000$

$y = 24000$

When the car is brand new, it is worth \$24000.

c. Use the intercepts to graph the linear equation.



d. Use your graph to estimate the car's value after five years.

\$9,000

Section 3.3: SLOPE

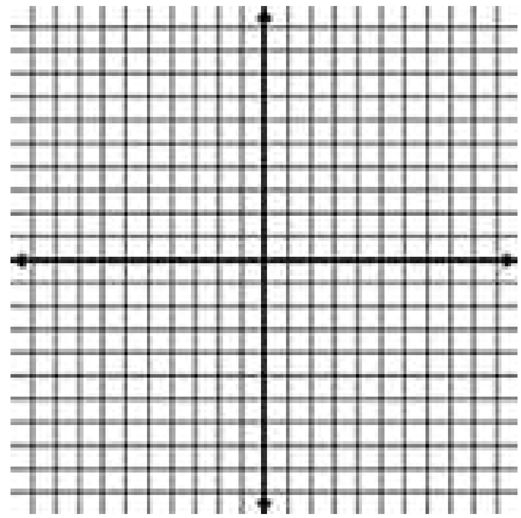
When you are done with your homework you should be able to…

- π Compute a line's slope
- π Use slope to show that lines are parallel
- π Use slope to show that lines are perpendicular
- π Calculate rate of change in applied situations

WARM-UP:

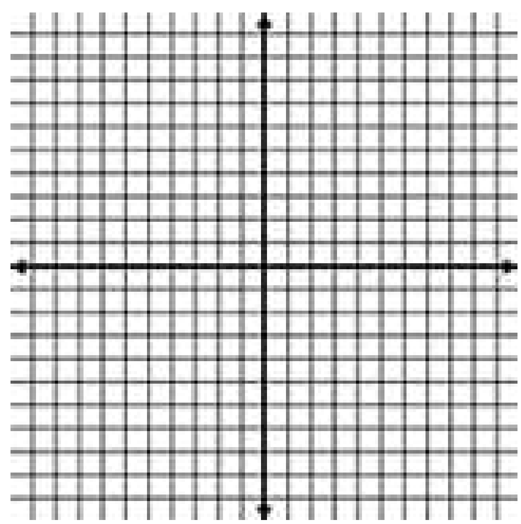
Graph each equation.

a. $y-2=0$



b. $-2x-3y=9$

	$-2x-3y=9$	(x,y)

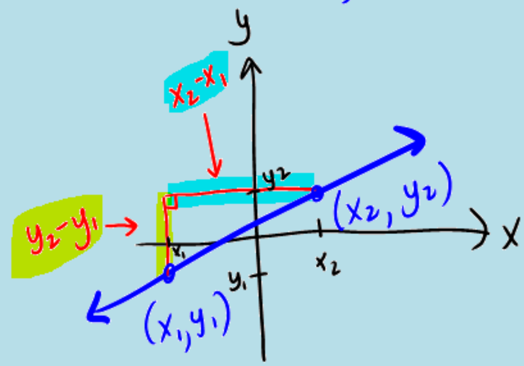


THE SLOPE OF A LINE

Mathematicians have developed a useful measure of the steepness of a line, called the Slope of the line. Slope compares the vertical change (the rise) to the horizontal change (the run) when moving from one fixed point to another along the line.

DEFINITION OF SLOPE

The Slope of the line through the distinct points (x_1, y_1) and (x_2, y_2) is



$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_2 - x_1 = 0$. It is common to use the letter m to represent the slope of a line. This letter is used because it is the first letter of the French verb monter, meaning to rise, or to ascend.

Example 1: Find the slope of the line passing through each pair of points:

a. $(-1, 4)$ and $(3, -6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6 - 4}{3 - (-1)}$$

$$m = \frac{-10}{4}$$

$$\boxed{m = -\frac{5}{2}}$$

b. $\left(8, \frac{3}{2}\right)$ and $\left(-\frac{5}{2}, 7\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\frac{2}{2} \cdot 7 - \frac{3}{2}}{-\frac{5}{2} - 8 \cdot \frac{2}{2}}$$

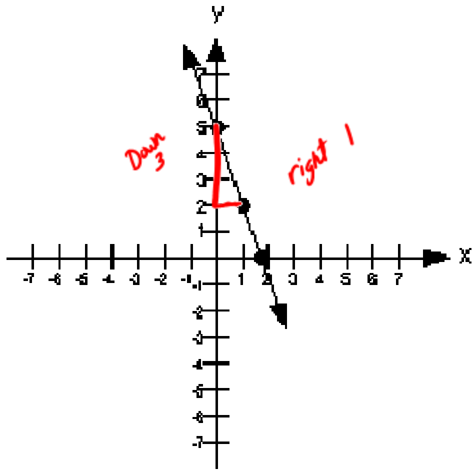
$$m = \frac{\frac{14 - 3}{2}}{\frac{-5 - 16}{2}}$$

$$m = \frac{\frac{11}{2}}{\frac{-21}{2}}$$

$$m = \frac{11}{\cancel{2}} \cdot \frac{\cancel{2}}{-21}$$

$$\boxed{m = -\frac{11}{21}}$$

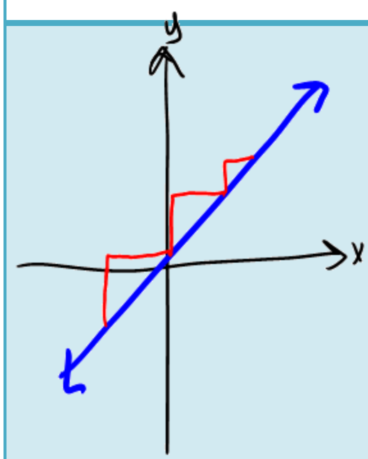
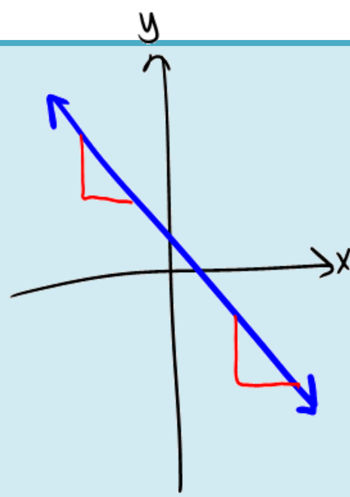
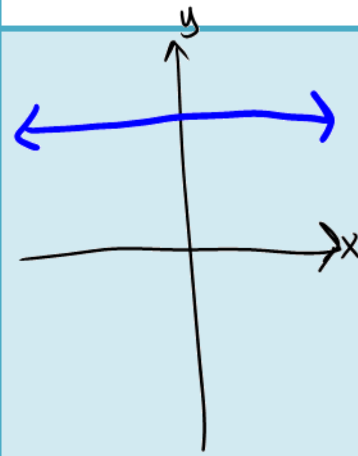
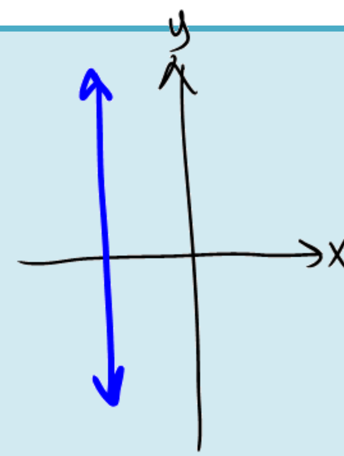
Example 2: Use the graph to find the slope of the line



$m = \frac{-3}{+1}$

$\boxed{m = -3}$

POSSIBILITIES FOR A LINE'S SLOPE

POSITIVE SLOPE	NEGATIVE SLOPE	ZERO SLOPE	UNDEFINED SLOPE
		 $y_2 - y_1 = 0$	 $x_2 - x_1 = 0$

SLOPE AND PARALLEL LINES

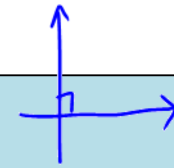
Two nonintersecting lines that lie in the same plane are parallel. If two lines do not intersect, the ratio of the vertical change to the horizontal change is the same for each line. Because two parallel lines have the same steepness, they must have the same slope.

1. If two nonvertical lines are parallel, then they have the same slope.
2. If two distinct nonvertical lines have the same slope, then they are parallel.
3. Two distinct vertical lines, each with undefined slope, are parallel.

SLOPE AND PERPENDICULAR LINES

Two lines that intersect at a right angle (90°) are said to be perpendicular.

1. If two nonvertical lines are perpendicular, then the product of their slopes is -1.
2. If the product of the slopes of two lines is -1, then the lines are perpendicular.

3. A __horizontal__ line having __zero__ slope is 

__perpendicular__ to a vertical line having __undefined__ slope.

Example 3: Determine whether the lines through each pair of points are parallel, perpendicular, or neither.

a. $(-2,-15)$ and $(0,-3)$; $(-12,6)$ and $(6,3)$

$m_1 = \frac{-3-(-15)}{0-(-2)}$

$m_1 = \frac{12}{2}$

$m_1 = 6$

$m_2 = \frac{3-6}{6-(-12)}$

$m_2 = \frac{-3}{18}$

$m_2 = -\frac{1}{6}$

- not parallel since $m_1 \neq m_2$
- Is $m_1 m_2 \stackrel{?}{=} -1$

$6\left(-\frac{1}{6}\right) \stackrel{?}{=} -1$

$-1 = -1$

yes so the lines are perpendicular

b. $(-2,-7)$ and $(3,13)$; $(-1,-9)$ and $(5,15)$

$m_1 = \frac{13-(-7)}{3-(-2)}$

$m_1 = \frac{20}{5}$

$m_1 = 4$

$m_2 = \frac{15-(-9)}{5-(-1)}$

$m_2 = \frac{24}{6}$

$m_2 = 4$

$m_1 = m_2$

So the lines are parallel

c. $(-1,-11)$ and $(0,-5)$; $(0,-8)$ and $(12,-6)$

$m_1 = \frac{-5-(-11)}{0-(-1)}$

$m_1 = \frac{6}{1}$

$m_1 = 6$

$m_2 = \frac{-6-(-8)}{12-0}$

$m_2 = \frac{2}{12}$

$m_2 = \frac{1}{6}$

$m_1 \neq m_2 \rightarrow$ not parallel

$m_1 m_2 \neq -1 \rightarrow$ not perpendicular

neither

APPLICATION

Construction laws are very specific when it comes to access ramps for the disabled. Every vertical rise of 1 foot requires a horizontal run of 12 feet. What is the grade of such a ramp? Round to the nearest tenth of a percent.

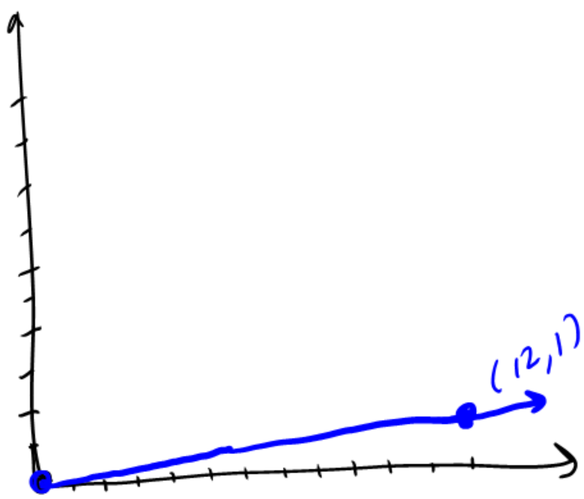
$m = \frac{rise}{run}$

$m = \frac{1 \text{ ft}}{12 \text{ ft}}$

$m = \frac{1}{12}$

$m \approx 0.0833$

$m \approx 8.3\%$



$\boxed{\text{The grade of the ramp should be } 8.3\%.}$

Section 3.4: THE SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

- π Find a line's slope and *y*-intercept from its equation
- π Graph lines in slope-intercept form
- π Use slope and *y*-intercept to graph $Ax+By=C$
- π Use slope and *y*-intercept to model data

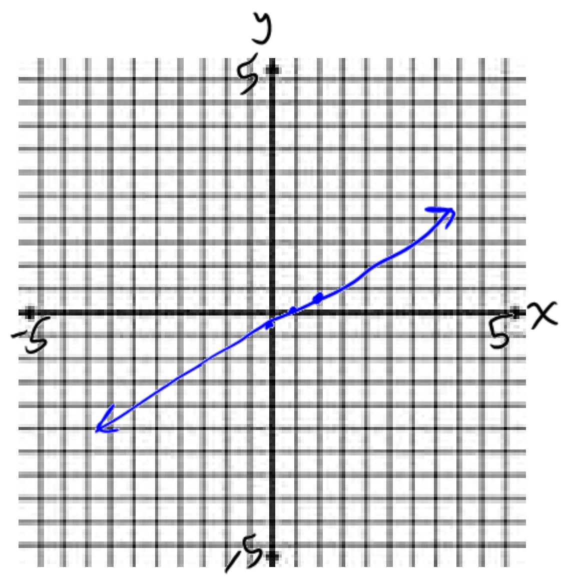
WARM-UP:

Graph each equation.

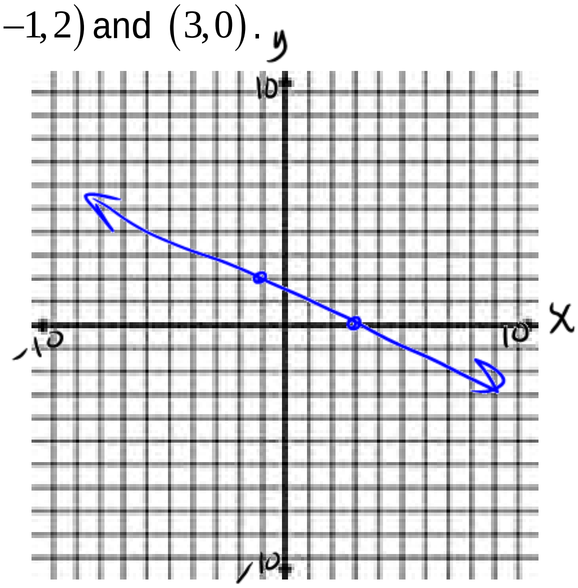
a. $4x-8y-2=0$

	$4x-8y-2=0$	(x,y)
$x=1$	$4(1)-8y-2=0 \Rightarrow 4-8y-2=0$, $2-8y=0$, $2=8y$, $\frac{1}{4}=y$	$\left(1,\frac{1}{4}\right)$
$x=0$	$4(0)-8y-2=0$, $-8y-2=0$, $-2=8y$, $-\frac{1}{4}=y$	$\left(0,-\frac{1}{4}\right)$
$y=0$	$4x-8(0)-2=0$	$\left(\frac{1}{2},0\right)$

$4x-2=0$
$4x=2$
$x=\frac{1}{2}$



b. The line which passes through the points $(-1,2)$ and $(3,0)$.



SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

The Slope - intercept form of the equation of a nonvertical line with slope m and y-intercept b is

$$y = mx + b$$

slope

(0,b) is the y-intercept

Example 1: Find the slope and the *y*-intercept of the line with the given equation:

a. $y = -4x - 1$

$y = -4x + (-1)$

$m = -4$
y-int: $(0, -1)$

b. $6x - y = -1$

$(-1)(-y) = (-6x - 1)(-1)$

$y = 6x + 1$

$m = 6$, y-int: $(0, 1)$

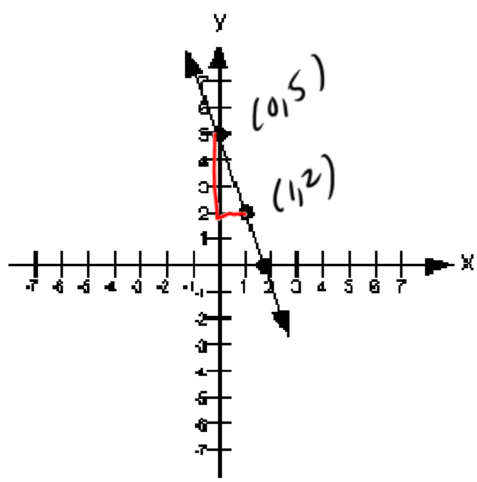
c. $y = \frac{5}{7}x + 2$

$m = \frac{5}{7}$, y-int: $(0, 2)$

d. $y = -\frac{x}{3} + \frac{2}{3}$

$m = -\frac{1}{3}$, y-int: $(0, \frac{2}{3})$

Example 2: Use the graph to find the equation of the line in slope-intercept form.



$m = \frac{-3}{+1}$

$m = -3$

y-int: $(0, 5)$

$y = mx + b$

$y = -3x + 5$

GRAPHING BY USING $y = mx + b$ SLOPE AND *Y*-INTERCEPT

1. Plot the point containing the y-intercept on the y axis.

   This is the point (0,b).

2. Obtain a second point using the slope, m. Write m as a fraction, and use rise over run, starting at the y-intercept.

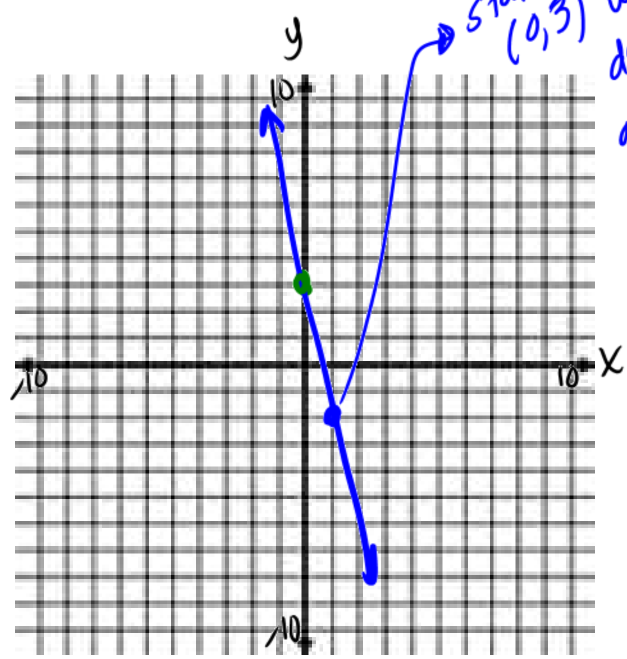
3. Use a straightedge to draw a line through the two points. Draw arrows at the ends of the line to show that the line continues indefinitely in both directions.

Example 3: Graph using the slope and *y*-intercept.

a. $y = -5x + 3$

$m = -5 \rightarrow m = \frac{-5}{+1}$

y-int: (0,3)



starting at (0,3) we went down 5 and right 1

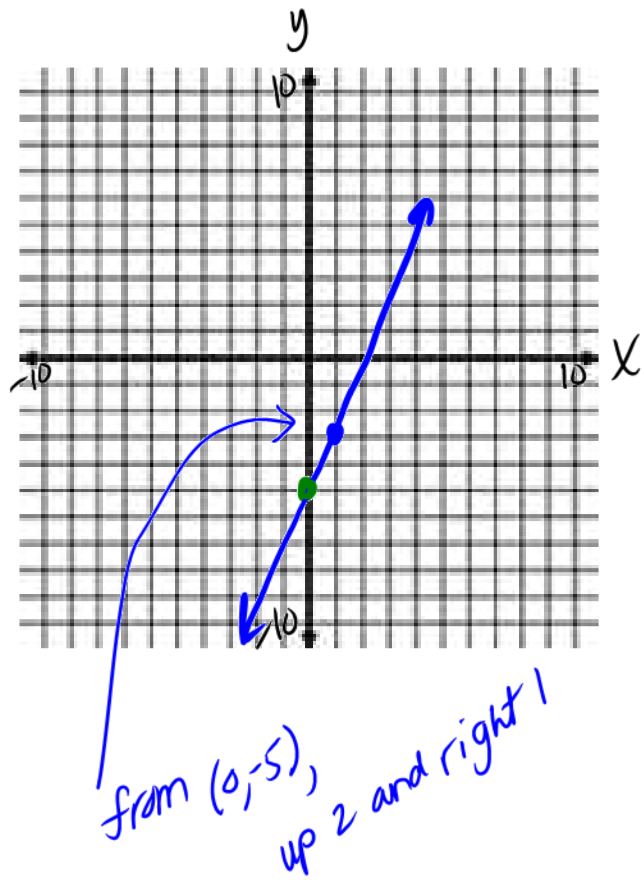
b. $10x - 5y = 25$

$$\frac{-5y}{-5} = \frac{-10x + 25}{-5}$$

$$y = 2x - 5$$

$$m = 2 \rightarrow m = \frac{+2}{+1}$$

y-int: $(0, -5)$



c. $x = 2y - 3$

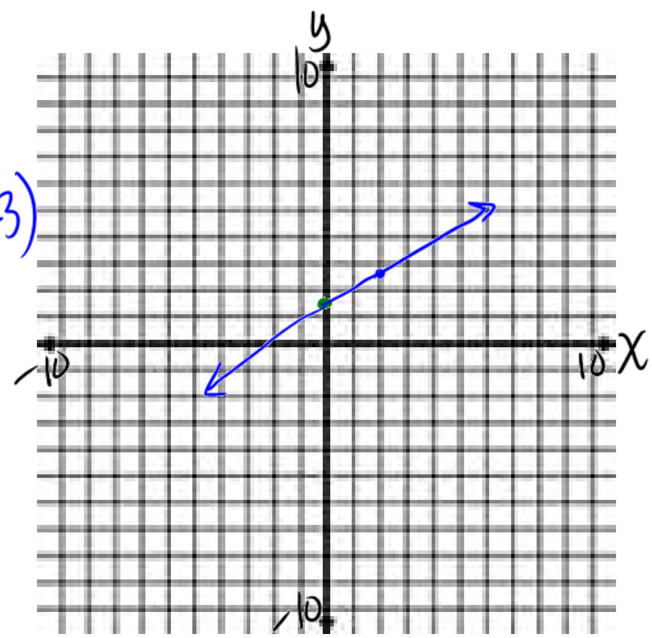
$$-2y + x = -3$$

$$\frac{-2y}{-2} = \frac{-x - 3}{-2} \Rightarrow -\frac{1}{2}(-2y) = -\frac{1}{2}(-x-3)$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

$$m = \frac{1}{2} \rightarrow m = \frac{+1}{+2}$$ up 1 right 2

y-int: $\left(0, \frac{3}{2}\right)$

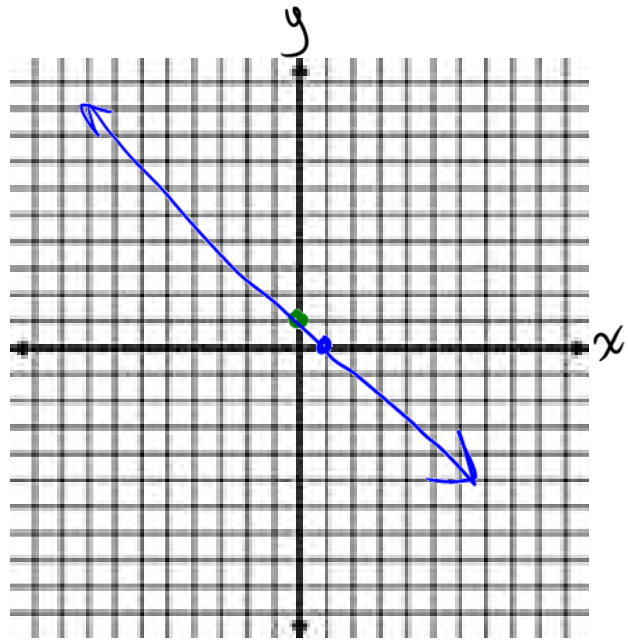


d. $(-1)(-y) = (x-1)(-1)$

$y = -x + 1$

$m = -1 \rightarrow m = \frac{-1}{+1}$

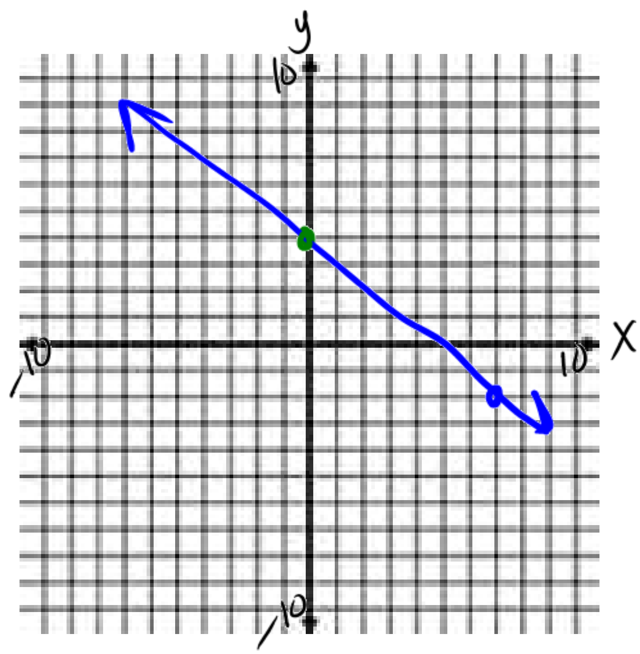
y-int: $(0,1)$



e. $y = -\frac{6}{7}x + 4$

$m = -\frac{6}{7} \rightarrow m = \frac{-6}{+7}$

y-int: $(0,4)$



APPLICATION

Write an equation in the form of $y = mx + b$ of the line that is described.

1. The y-intercept is -4 and the line is parallel to the line whose equation is $2x + y = 8$.

Use this line to find out the slope.

$2x+y=8$

$y=-2x+8$

$m=-2$

parallel lines have the same slope. So we use $m=-2$ and $b=-4$ to make the equation for our line.

$y=mx+b$

$y=-2x+(-4)$

$\boxed{y=-2x-4}$

2. The line falls from left to right. It passes through the origin and a second point with opposite x- and y-coordinates.

negative slope

$(0,0)$

$m=\frac{-1-0}{1-0}$

$m=-1$

$b=0$

$(1,-1)$

$(2,-2)$

$(-1,1)$

$y=-x+0$

$\boxed{y=-x}$

Section 3.5: THE POINT-SLOPE FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

- π Use the point-slope form to write equations of a line
- π Find slopes and equations of parallel and perpendicular lines
- π Write linear equations that model data and make predictions

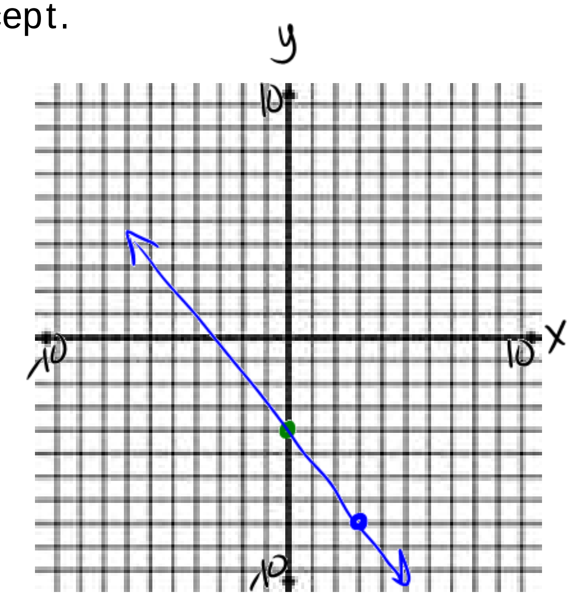
WARM-UP:

1. Simplify.

$$2-5\left[2-(7x+2)\right] = 2-5[2-7x-2]$$
$$= 2-5[0-7x]$$
$$= 2-5[-7x]$$
$$= \boxed{2+35x}$$

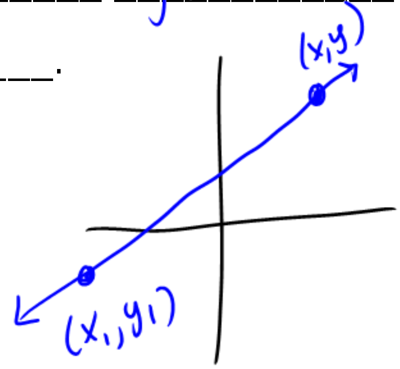
2. Graph the equation using the slope and y-intercept.

$$-\frac{x}{3}-\frac{y}{4}=1$$
$$+\frac{x}{3} \qquad +\frac{x}{3}$$
$$(-4)\left(-\frac{y}{4}\right)=\left(\frac{x}{3}+1\right)(-4)$$
$$y=-\frac{4}{3}x-4$$
$$m=-\frac{4}{3} \rightarrow m=\frac{-4}{+3}$$
$$y\text{-int}: (0,-4)$$



POINT-SLOPE FORM

We can use the slope of a line to obtain another useful form of the line's equation. Consider a nonvertical line that has slope m and contains the point (x_1, y_1). Now let (x, y) represent any other point on the line. Keep in mind that the point (x, y) is arbitrary and is not in one fixed position. The point (x_1, y_1) is fixed.



$$(x-x_1)m = \left(\frac{y-y_1}{\cancel{x-x_1}}\right)\cancel{(x-x_1)}$$

$$m(x-x_1) = y-y_1$$

$$y-y_1 = m(x-x_1)$$

POINT-SLOPE FORM OF THE EQUATION OF A LINE

The point - slope form of the equation of a nonvertical line with slope m that passes through the point (x_1, y_1) is

$$y-y_1 = m(x-x_1)$$

Example 1: Write the point-slope form of the equation of the line with the given slope that passes through the given point.

a. $m=-2;\ (5,-11)$

$$y-y_1 = m(x-x_1)$$

$$y-(-11) = -2(x-5)$$

$$\boxed{y+11 = -2(x-5)}$$

b. $m=\frac{5}{8};\ \left(\frac{1}{4}, 7\right)$

$$y-y_1 = m(x-x_1)$$

$$\boxed{y-7 = \frac{5}{8}\left(x-\frac{1}{4}\right)}$$

$Ax+By=C$ standard form

$y=mx+b$ slope-intercept form

$y-y_1=m(x-x_1)$ point-slope form

$y=b$ horizontal line

$x=a$ vertical line

horizontal line

c. $m = 0;\ (-21, 5)$ $b=5$

$y = b$

$\boxed{y = 5}$

$y - 5 = 0(x - (-21))$

$y - 5 = 0(x + 21)$

d. $m = \text{undefined};\ (0, 0)$

vertical line

a

$x = a$

$\boxed{x = 0}$

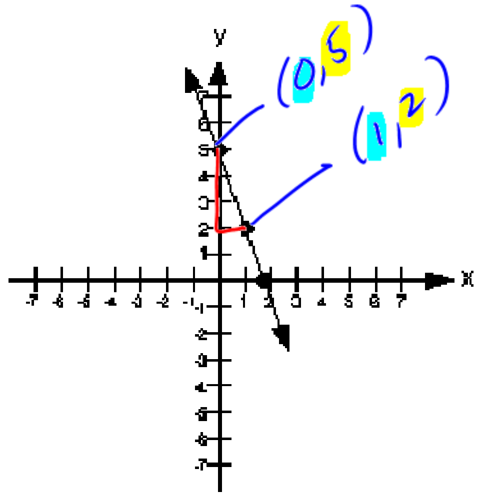
Example 2: Use the graph to find two equations of the line in point-slope form.

1. $y - y_1 = m(x - x_1)$

$y - 5 = -3(x - 0)$

2. $y - y_1 = m(x - x_1)$

$y - 2 = -3(x - 1)$



$m = \frac{-3}{+1}$

$m = -3$

Now write the slope-intercept form:

1. $y - 5 = -3(x - 0)$

$y - 5 = -3x$

$y = -3x + 5$

2. $y - 2 = -3(x - 1)$

$y - 2 = -3x + 3$

$y = -3x + 5$

They're the same!!!

EQUATIONS OF LINES

FORM	WHAT YOU SHOULD KNOW
Standard Form $Ax+By=C$	Graph equations in this form using _intercepts_ and a _checkpoint_.
$y=b$	Graph equations in this form as _horizontal_ lines with _(0,b)_ as the _y-intercept_.
$x=a$	Graph equations in this form as _vertical_ lines with _(a,0)_ as the _x-intercept_.
Slope-Intercept Form $y=mx+b$	Graph equations in this form using the _y-intercept_, _(0,b)_ and the slope, _m_. *Start with this form when writing a _linear_ equation if you know a line's _Slope_ and _y-intercept_.
Point-Slope Form $y-y_1=m(x-x_1)$	Start with this form when writing a linear equation if you know the _Slope_ of the line and a _point_ on the _line_ NOT containing the _y-intercept_ OR _2_ points on the line, _neither_ of which contains the _y-intercept_. Calculate the _Slope_ using $m=\frac{y_2-y_1}{x_2-x_1}$

PARALLEL AND PERPENDICULAR LINES

Recall that parallel lines have the ___Same___ ___Slope___ and perpendicular lines have ___Slopes___ which are ___Negative___ ___reciprocals___.

Example 3: Use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

a. Passing through $(-2,-7)$ and parallel to the line whose equation is $y=-5x+4$.

① Find slope using the given line $y=-5x+4$

$m=-5$

our slope is also -5

② Use $m=-5$ and $(-2,-7)$ in $y-y_1=m(x-x_1)$

$y-(-7)=-5(x-(-2))$

$\boxed{y+7=-5(x+2)}$ point-slope form

③ Isolate y to find slope-intercept form

$y+7=-5(x+2)$

$y+7=-5x-10$

$\boxed{y=-5x-17}$ slope-intercept form

b. Passing through $(-4,2)$ and perpendicular to the line whose equation is $y=-\frac{1}{3}x+7$.

① Find slope using the given line $y=-\frac{1}{3}x+7$

$m_1=-\frac{1}{3}$

$m_1 m_2=-1$

$(-\frac{1}{3})(m_2)=-1$

$m_2=3$

② Use $m=3$ and $(-4,2)$ in $y-y_1=m(x-x_1)$

$y-(2)=3(x-(-4))$

$\boxed{y-2=3(x+4)}$ point-slope form

③ Isolate y to find slope-intercept form

$y-2=3(x+4)$

$y-2=3x+12$

$\boxed{y=3x+14}$ slope-intercept form

c. Passing through $(5,-9)$ and parallel to the line whose equation is $x+7y=12$.

① Find slope using the given line $x+7y=12$

$x+7y=12$

$\frac{7y}{7}=\frac{-x+12}{7}$

$y=-\frac{1}{7}x+\frac{12}{7}$

$m=-\frac{1}{7}$

② Use $m=-\frac{1}{7}$ and $(5,-9)$ in $y-y_1=m(x-x_1)$

$y-(-9)=-\frac{1}{7}(x-(5))$

$\boxed{y+9=-\frac{1}{7}(x-5)}$ point-slope form

③ Isolate y to find slope-intercept form

$y+9=-\frac{1}{7}(x-5)$

$y+9\frac{\cdot 7}{7}=-\frac{1}{7}x+\frac{5}{7}$

$y=-\frac{1}{7}x+\frac{5}{7}-\frac{63}{7}$

$\boxed{y=-\frac{1}{7}x-\frac{58}{7}}$

Section 4.1: SOLVING SYSTEMS OF LINEAR EQUATIONS BY GRAPHING

When you are done with your homework you should be able to…

- π Decide whether an ordered pair is a solution of a linear system
- π Solve systems of linear equations by graphing
- π Use graphing to identify systems with no solution or infinitely many solutions
- π Use graphs of linear systems to solve problems

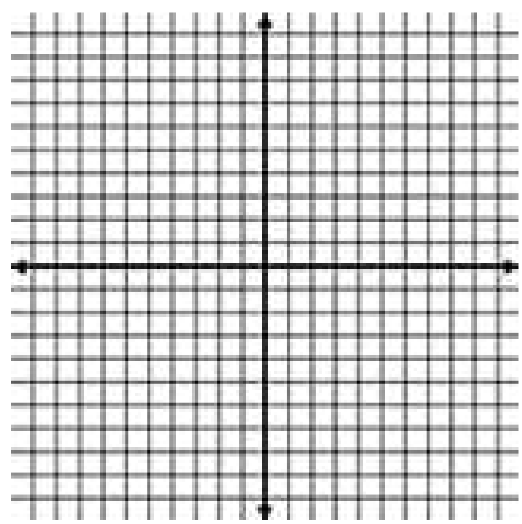
WARM-UP:

1. Determine if the given number or ordered pair is a solution to the given equation.

a. $5x+3=21;\ \frac{18}{5}$

b. $-x+2y=0;\ (4,1)$

2. Graph the line which passes through the points (0,1) and (-5,3).



SYSTEMS OF LINEAR EQUATIONS AND THEIR SOLUTIONS

We have seen that all __________________ in the form ________________ are straight _____________ when graphed. ___________ such equations are called a __________________ of ________________ __________________ or a __________________ __________________. A __________________ to a system of two ________________ equations in two _______________ is an __________________ _____________ that _______________ _______________ equations in the ________________.

Example 1: Determine whether the given ordered pair is a solution of the system.

a.

$$(-2,-5)$$
$$6x-2y=-2$$
$$3x+\ y=-11$$

b.

$$(10,7)$$
$$6x-\ 5y=25$$
$$4x+15y=13$$

SOLVING LINEAR SYSTEMS BY GRAPHING

The ______________ of a __________________ of two linear equations in _______ variables can be found by ________________ ______________ of the __________________ in the _______________ rectangular __________________ system. For a system with ____________ solution, the ___________________ of the point of ____________________ give the _____________________ solution.

STEPS FOR SOLVING SYSTEMS OF TWO LINEAR EQUATIONS IN TWO VARIABLES, *x* AND *y*, BY GRAPHING

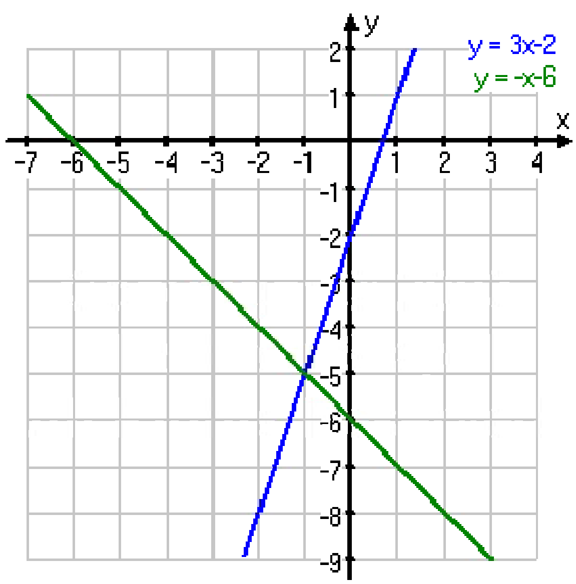
1. Graph the first ______________________.

2. ________________ the second equation on the _______________ set of ____________.

3. If the _______________ representing the __________ graphs ______________ at a _______________, determine the __________________ of this point of intersection. The ______________ ___________ is the __________________ of the ________________.

4. __________________ the __________________ in ______________ equations.

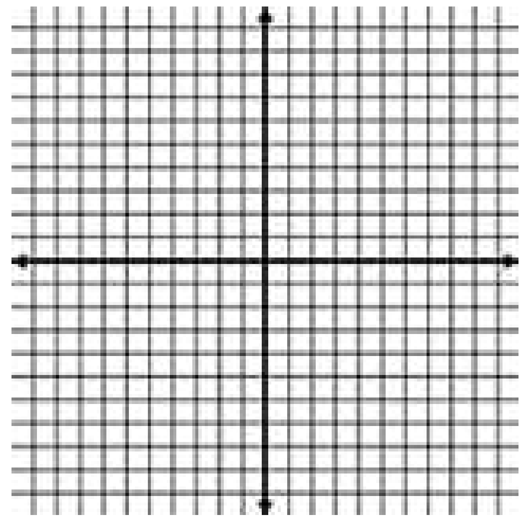
Example 2: Use the graph below to find the solution of the system of linear equations.



Example 3: Solve each system by graphing. Use set notation to express solution sets.

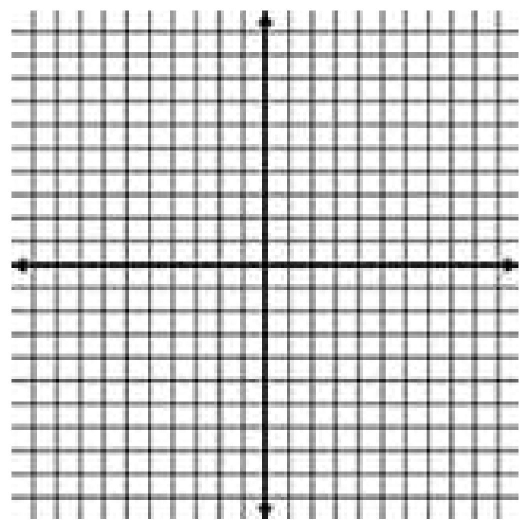
a.

$$x + y = 2$$
$$x - y = 4$$



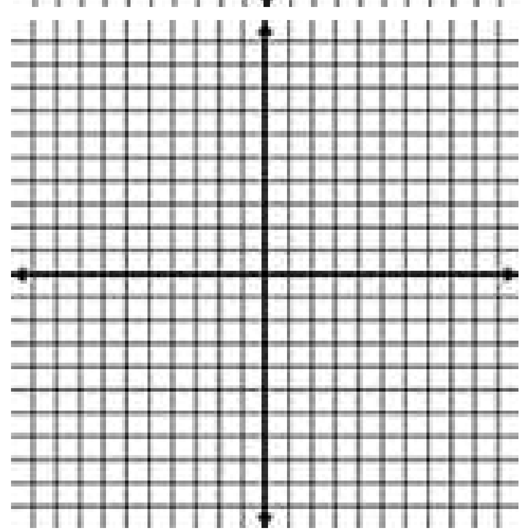
b.

$$y = 3x - 4$$
$$y = -2x + 1$$



c.

$$x + y = 6$$
$$y = -3$$



LINEAR SYSTEMS HAVING NO SOLUTION OR INFINITELY MANY SOLUTIONS

We have seen that a ____________________ of linear equations in __________ variables represents a _______________ of _______________. The lines either __________________ at ___________ point, are _______________, or are ______________________. Thus, there are ________________ possibilities for the _______________ of solutions to a system of two linear equations.

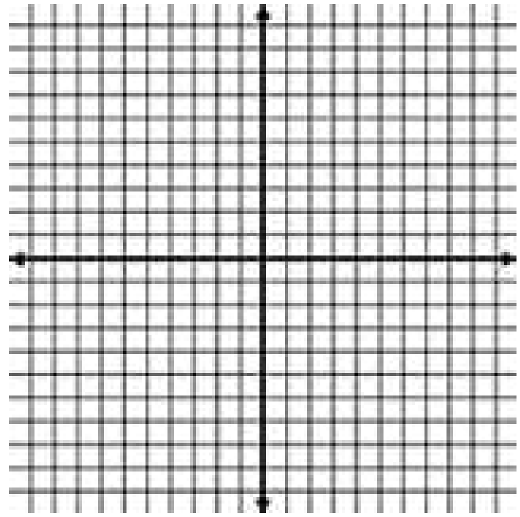
THE NUMBER OF SOLUTIONS TO A SYSTEM OF TWO LINEAR EQUATIONS

NUMBER OF SOLUTIONS	WHAT THIS MEANS GRAPHICALLY
Exactly _____________ ordered pair solution.	The two lines _________________ at _________ point. This is a __________________ system.
________ Solution	The two lines are __________________. This is an __________________ system.
___________________ many solutions	The two lines are __________________. This is a system with ________________ equations.

Example 4: Solve each system by graphing. If there is no solution or infinitely many solutions, so state. Use set notation to express solution sets.

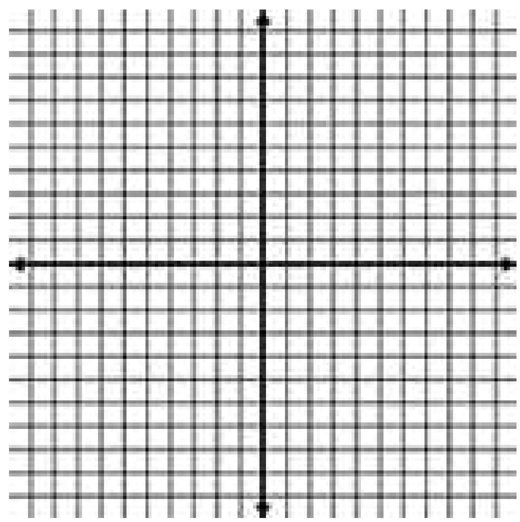
a.

$$x + y = 4$$
$$2x + 2y = 8$$



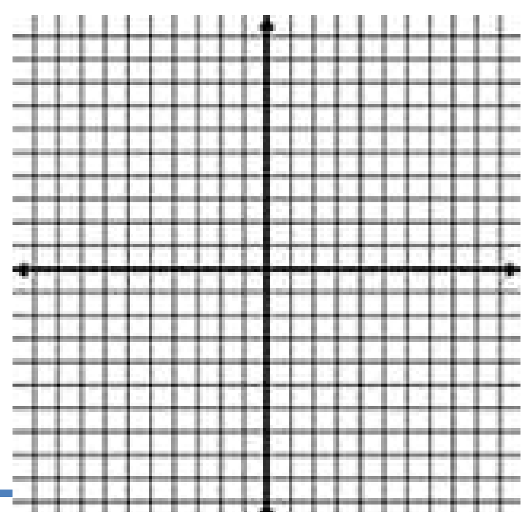
b.

$$y = 3x - 1$$
$$y = 3x + 2$$



c.

$$2x - y = 0$$
$$y = 2x$$

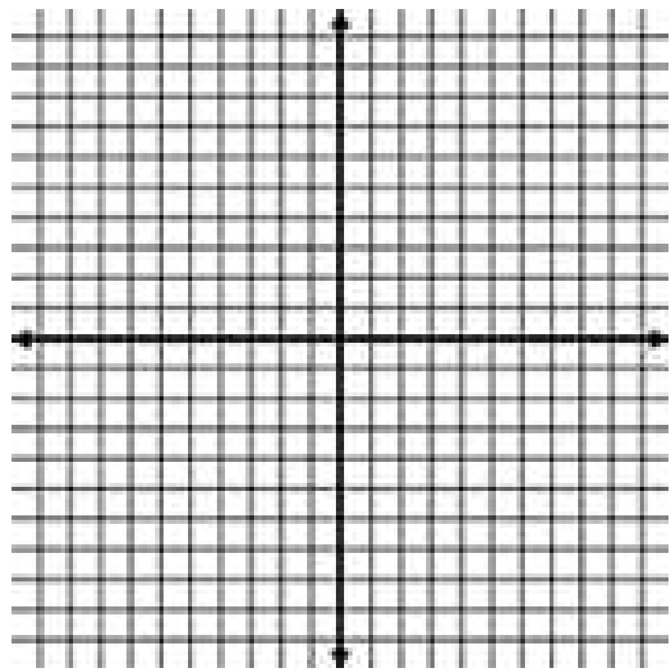


APPLICATION

A band plans to record a demo. Studio A rents for \$100 plus \$50 per hour. Studio B rents for \$50 plus \$75 per hour. The total cost, *y*, in dollars, of renting the studios for *x* hours can be modeled by the linear system

$$y = 50x + 100$$
$$y = 75x + 50$$

a. Use graphing to solve the system. Extend the *x*-axis from 0 to 4 and let each tick mark represent 1 unit (one hour in a recording studio). Extend the *y*-axis from 0 to 400 and let each tick mark represent 100 units (a rental cost of \$100).



b. Interpret the coordinates of the solution in practical terms.

When you are done with your 4.2 homework you should be able to…

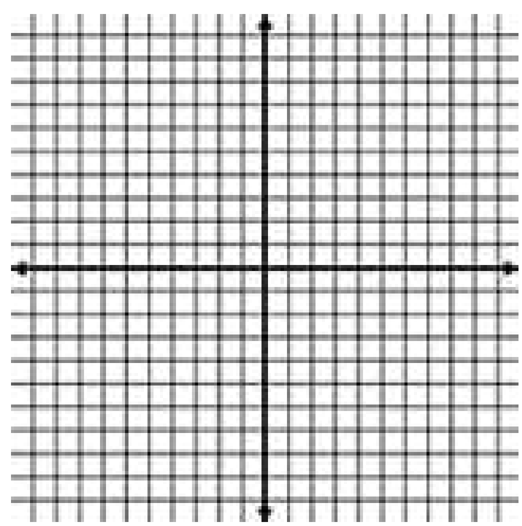
- π Solve linear systems by the substitution method
- π Use the substitution method to identify systems with no solution or infinitely many solutions
- π Solve problems using the substitution method

WARM-UP:

1. Solve.
   $-5x+3(2x-7)=x-21$

2. Solve the following system of linear equations by graphing. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

   $y=-4x+6$
   $y=-2x$



Steps for Solving a System of Two Linear Equations Containing Two Variables by Substitution

1. Solve one of the equations for one of the unknowns.
2. Substitute the expression solved for in Step 1 into the **<u>other</u>** equation. The

   result will be a _________________ equation in _______ variable.

3. ______________ the linear equation in one variable found in Step 2.

4. _____________________ the value of the variable found in Step 3 into one of

   the **<u>original</u>** equations to find the _________________ of the other

   _________________.
5. Check your answer by ___________________________ the ________________

   ______________ into ________________ of the original equations.

Example 1: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

a.
$$\begin{aligned} 5x+2y&=-5 \\ 3x-\ \ y&=-14 \end{aligned}$$

b.

$$y = 5x - 3$$
$$y = 2x - \frac{21}{5}$$

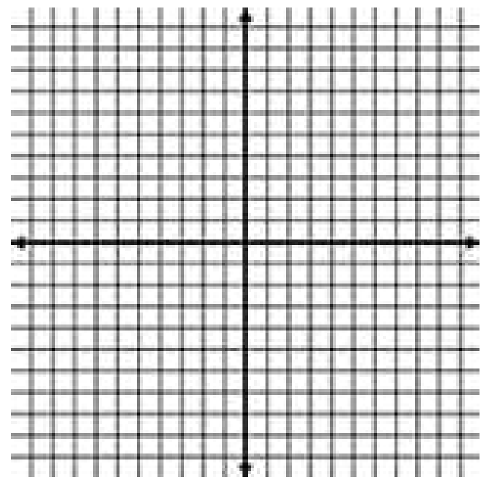
π Suppose you are solving a system of equations and you end up with 5 = 0. This is a _______________________ and yields a result of ________ or ________. This system consists of two __________________ lines which never ___________________.

π Suppose you are solving a system of equations and you end up with 5 = 5 or x = x. This is an ________________ and yields a result of all ______________ ____________ which are on the ______________. In other words, the system would have ______________________ ________________ solutions. This system consists of two lines which are _______________________.

Example 2: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Graph the system.

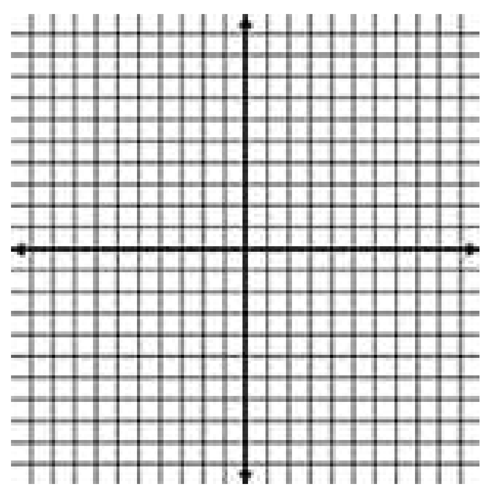
a.

$$-x+3y=4$$
$$2x-6y=-8$$



b.

$$x - 5y=3$$
$$-2x+10y=8$$



Example 3: Write a system of equations that has infinitely many solutions.

APPLICATIONS

1. Christa is a waitress and collects her tips at the table. At the end of the shift she has 68 bills in her tip wallet, all ones and fives. If the total value of her tips is $172, how many of each bill does she have?

(1) Analysis

Let x be the # of ones

Let y be the # of fives

(2) Translate

$x + y = 68$

$1x + 5y = 172$

(3) Solve

$x + y = 68$ (A)

$x + 5y = 172$ (B)

i) Isolate y in eq. A

$x + y = 68$

$y = 68 - x$

ii) Sub. $y = 68 - x$ into eq. B

$x + 5y = 172$

$x + 5(68 - x) = 172$

$x + 340 - 5x = 172$

$-4x = -168$

$x = 42$

iii) Sub. $x = 42$ into eq. A

$x + y = 68$

$42 + y = 68$

$y = 26$

(4) Conclusion

Christa has 42 ones and 26 fives.

2. Melody wishes to enclose a rectangular garden with fencing, using the side of her garage as one side of the rectangle. A neighbor gave her 30 feet of fencing, and Melody wants the length of the garden along the garage to be 3 feet more than the width. What are the dimensions of the garden?

(1) Analysis

Let w be the width

Let l be the length

$P = l + 2w$

$P = 30$

(2) Translate

$l + 2w = 30$

$l = w + 3$

(3) Solve

$l + 2w = 30$ (A)

$l = w + 3$ (B)

i) Sub. $l = w + 3$ into eq. A

$l + 2w = 30$

$(w + 3) + 2w = 30$

$w + 3 + 2w = 30$

$3w + 3 = 30$

$3w = 27$

$w = 9$

ii) Sub. $w = 9$ into eq. B

$l = w + 3$

$l = 9 + 3$

$l = 12$

(4) Conclusion

The width of the garden is 9 ft and the length is 12 ft.

When you are done with your 4.3 homework you should be able to…

- π Solve linear systems by the addition method
- π Use the addition method to identify systems with no solution or infinitely many solutions
- π Determine the most efficient method for solving a linear system

WARM-UP:

1. Solve the following system of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

$$y = \frac{7}{2}x - 3$$
$$y = -4x + 2$$

ELIMINATING A VARIABLE USING THE ADDITION METHOD

The substitution method is most useful if one of the equations has an isolated variable. A third method for solving a linear system is the addition method. The addition method eliminates a variable by adding the equations. When we use the addition method, we want to obtain two equations whose sum is an equation containing only _____1_____ variable. The key step is to obtain, for one of the variables, ___coefficients___ that differ only in ___sign___.

Steps for Solving a System of Two Linear Equations Containing Two Variables by Addition

1. If necessary, ___rewrite___ both equations in the form ___$Ax+By=C$___.
2. If necessary, ___multiply___ either equation or both equations by appropriate nonzero numbers so that the ___sum___ of the *x*-coefficients or *y*-coefficients is ___zero___.
3. ___Add___ the equations in step 2. The ___sum___ is an ___equation___ in ___1___ variable.
4. ___Solve___ the equation in one variable.
5. ___Back___-___substitute___ the value obtained in step 4 into either of the ___original___ equations and ___Solve___ for the other variable.
6. ___Check___ the solution in ___BOTH___ of the original equations.

Example 1: Solve the following systems of linear equations by the addition method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x + y = 6 \quad (A)$$
$$x - y = -2 \quad (B)$$

i) A+B, elim. y

$$x+y=6$$
$$\underline{x-y=-2}$$
$$2x = 4$$
$$x = 2$$

ii) Sub. x=2 into eq. A

$$x + y = 6$$
$$2 + y = 6$$
$$y = 4$$

iii) Conclusion

$\{(2,4)\}$

consistent system with independent equations.

$5(3x-y)=(11)5$

b.

$$3x - y = 11 \quad (A)$$
$$2x + 5y = 13 \quad (B)$$

i) 5A+B, elim. y

$$15x-5y=55$$
$$\underline{2x+5y=13}$$
$$17x = 68$$
$$x = 4$$

ii) Sub. x=4 into eq. A

$$3x-y=11$$
$$3(4)-y=11$$
$$12-y=11$$
$$-y=-1$$
$$y=1$$

iii) Conclusion

$\{(4,1)\}$

Consistent system with independent equations.

COMPARING SOLUTION METHODS

METHOD	ADVANTAGES	DISADVANTAGES

METHOD	ADVANTAGES	DISADVANTAGES
GRAPHING	You can see the solution(s).	If the solutions do not involve integers or are too large or small to be seen on the graph, it's impossible to tell exactly what the solutions are.
SUBSTITUTION	Gives exact solutions. Easy to use if a variable is on one side by itself.	Solutions cannot be seen. Can introduce extensive work with fraction when no variable has a coefficient of 1 or -1.
ADDITION	Gives exact solutions. Easy to use!	Solutions cannot be seen.

Example 2: Solve the following systems of linear equations by any method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.
$$2x+5y=-6 \quad (A)$$
$$7x-2y=11 \quad (B)$$

i) 2A + 5B, elim. y

$$4x+10y=-12$$
$$35x-10y=55$$
$$39x = 43$$
$$x=\frac{43}{39}$$

ii) Sub. $x=\frac{43}{39}$ into eq. A

$$7x-2y=11$$
$$7\left(\frac{43}{39}\right)-2y=11$$
$$\frac{301}{39}-2y=11\cdot\frac{39}{39}$$
$$-2y=\frac{429}{39}-\frac{301}{39}$$
$$\left(-\frac{1}{2}\right)(-2y)=\frac{\cancel{128}^{64}}{39}\left(-\frac{1}{\cancel{2}_{1}}\right)$$
$$y=\frac{64}{39}$$

iii) Conclusion

$$\left\{\left(\frac{43}{39},\frac{64}{39}\right)\right\}$$

Consistent system with independent equations.

b.

$$4x - y = 1 \quad (A)$$
$$y = 7x - 15 \quad (B)$$

i) Sub. $y=7x-15$ into eq. A

$4x - y = 1$

$4x - (7x-15) = 1$

$4x - 7x + 15 = 1$

$-3x + 15 = 1$

$-3x = -14$

$x = \frac{14}{3}$

ii) Sub. $x = \frac{14}{3}$ into eq. B

$y = 7x - 15$

$y = 7\left(\frac{14}{3}\right) - 15 \left(\frac{3}{3}\right)$

$y = \frac{98}{3} - \frac{45}{3}$

$y = \frac{53}{3}$

iii) Conclusion

$\left\{\left(\frac{14}{3}, \frac{53}{3}\right)\right\}$

consistent system with independent equations.

c.

$$4x - 2y = 2 \quad (A)$$
$$2x - y = 1 \quad (B)$$

i) $A + (-2B)$, elim. y

$4x - 2y = 2$

$-4x + 2y = -2$

$0x + 0x = 0$

$0 = 0$ true! identical lines!

ii) Conclusion

$\{(x,y) \mid 4x - 2y = 2\}$

consistent system with dependent equations.

d.

$$3x = 4y + 1$$
$$4x + 3y = 1$$

e.

$$2x + 4y = 5$$
$$3x + 6y = 6$$

Section 4.4: PROBLEM USING SOLVING SYSTEMS OF EQUATIONS

When you are done with your homework you should be able to…

- π Solve problems using linear systems
- π Solve simple interest problems
- π Solve mixture problems
- π Solve motion problems

WARM-UP:

1. Solve the system of linear equations using the substitution or the addition method. Determine if the system is consistent or inconsistent, and if the equations are dependent or independent. Give your result in set notation.

a.

$$2x-3y=4$$
$$3x+4y=0$$

b.

$$x-y=3$$
$$2x=4+2y$$

A STRATEGY FOR SOLVING WORD PROBLEMS USING SYSTEMS OF EQUATIONS

When we solved problems in chapter 2, we let x represent a __quantity__ that was __unknown__. Problems in this section involve __2__ unknown __quantities__. We will let __x__ and __y__ represent the __unknown__ quantities and __translate__ the English words into a __system__ of __linear__ equations.

Example 1: The sum of two numbers is five. If one number is subtracted from the other, their difference is thirteen. Find the numbers.

① Analysis
Let x be the 1st number
Let y be the 2nd number

② Translate
$x+y=5$
$x-y=13$

③ Solve
$x+y=5$ (A)
$x-y=13$ (B)

i) A+B, elim. y
$x+y=5$
$x-y=13$
$2x=18$
$x=9$

ii) Sub x=9 into eq. A
$x+y=5$
$9+y=5$
$y=-4$

④ Conclusion
The numbers are 9 and -4.

Example 2: Each day, the sum of the average times spent on grooming for 15- to 19-year-old women and men is 96 minutes. The difference between grooming times for 15- to 19-year-old women and men is 22 minutes. How many minutes per day do 15- to 19-year-old women and men spend on grooming?

① Analysis
Let x be the # of minutes per day the 15-19 yr old women spend on grooming
Let y be the number of minutes per day 15-19 yr old men spend on grooming

② Translate
$x+y=96$
$x-y=22$

③ Solve
$x+y=96$ (A)
$x-y=22$ (B)

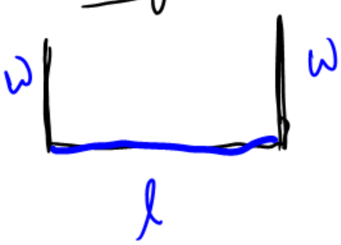
i) A+B, elim. y
$x+y=96$
$x-y=22$
$2x=118$
$x=59$

ii) Sub. x=59 into eq. A
$x+y=96$
$59+y=96$
$y=37$

④ Conclusion
The women spend 59 minutes and the men spend 37 minutes.

Example 3: A rectangular lot whose perimeter is 1600 feet is fenced along three sides. An expensive fencing along the lot's length costs $20 per foot. An inexpensive fencing along the two side widths costs only $5 per foot. The total cost of the fencing along the three sides comes to $13000. What are the lot's dimensions?

① Analysis



$P = 1600$ and

$P = 2w + 2l$

② Translate

$2w + 2l = 1600 \rightarrow w + l = 800$

$5 \cdot (2w) + 20l = 13000 \rightarrow 10w + 20l = 13000 \rightarrow w + 2l = 1300$

③ Solve

$w + l = 800 \quad (A)$

$w + 2l = 1300 \quad (B)$

i) $-A + B$, elim w

$$\begin{array}{r} -w - l = -800 \\ w + 2l = 1300 \\ \hline l = 500 \end{array}$$

ii) sub $l = 500$ into eq A

$w + 500 = 800 \rightarrow w = 300$

④ Conclusion

The lot's dimensions are 300ft x 500ft

Example 4: On a special day, tickets for a minor league baseball game cost $5 for adults and $1 for students. The attendance that day was 1281 and $3425 was collected. Find the number of each type of ticket sold.

① Analysis

Let x be the # of adult tickets sold

Let y be the # of student tickets sold

② Translate

$x + y = 1281$

$5x + 1y = 3425$

③ Solve

$x + y = 1281 \quad (A)$

$5x + y = 3425 \quad (B)$

i) $-A + B$, elim y

$$\begin{array}{r} -x - y = -1281 \\ 5x + y = 3425 \\ \hline 4x = 2144 \\ x = 536 \end{array}$$

ii) sub $x = 536$ into eq. A

$x + y = 1281$

$536 + y = 1281$

$y = 745$

④ Conclusion

536 adult and 745 student tickets were sold.

Example 5: You invested $11000 in stocks and bonds, paying 5% and 8% annual interest. If the total interest earned for the year was $730, how much was invested in stocks and how much was invested in bonds?

① Analysis

Let x be the amount invested in stocks and let y be the amount invested in bonds

② Translate

$x + y = 11000$

$0.05x + 0.08y = 730$

③ Solve

$x + y = 11000$ (A)

$.05x + .08y = 730$ (B)

i) Isolate x in eq. A

$x + y = 11000$

$x = 11000 - y$

ii) Sub. $x = 11000 - y$ into eq. B

$0.05x + .08y = 730$

$.05(11000 - y) + .08y = 730$

$550 - .05y + .08y = 730$

$\frac{.03y}{.03} = \frac{180}{.03}$

$y = 6000$

iii) Sub. $y = 6000$ into eq. A

$x + y = 11000 \rightarrow x + 6000 = 11000$

$x = 5000$

④ Conclusion

$5000 was invested in stocks and $6000 was invested in bonds.

Example 6: A jeweler needs to mix an alloy with a 16% gold content and an alloy with a 28% gold content to obtain 32 ounces of a new alloy with a 25% gold content. How many ounces of each of the original alloys must be used?

① Analysis

Let x be the # of oz of alloy with 16% gold content

Let y be the # of oz of alloy with 28% gold content

② Translate

$x + y = 32$

$.16x + .28 = .25(32)$

③ Solve

$x + y = 32$ (A)

$.16x + .28y = 8$ (B)

i) Isolate x in eq. A

$x + y = 32$

$x = 32 - y$

ii) Sub. $x = 32 - y$ into eq. B

$.16x + .28y = 8$

$.16(32 - y) + .28y = 8$

$5.12 - .16y + .28y = 8$

$.12y = 2.88$

$y = 24$

iii) Sub. $y = 24$ into eq. A

$x + y = 32$

$x + 24 = 32$

$x = 8$

④ Conclusion

We need to use 8 oz of the alloy w/16% gold content and 24 oz of the alloy w/28% gold content

A FORMULA FOR MOTION

$d = rt$

Distance equals ___rate___ times ___time___.

Example 7: When a plane flies with the wind, it can travel 4200 miles in 6 hours. When the plane flies in the opposite direction, against the wind, it takes 7 hours to fly the same distance. Find the rate of the plane in still air and the rate of the wind.

① Analysis

	r	t	d
with the wind	$x+y$	6	4200
against the wind	$x-y$	7	4200

Let x be the rate of the plane in still air
Let y be the rate of the wind

② Translate

$(x+y)\cdot 6 = 4200 \rightarrow x+y = 700$

$(x-y)\cdot 7 = 4200 \rightarrow x-y = 600$

③ Solve

$x+y = 700 \quad (A)$

$x-y = 600 \quad (B)$

i) A+B, elim. y

$$\begin{aligned} x+y &= 700 \\ x-y &= 600 \\ \hline 2x &= 1300 \\ x &= 650 \end{aligned}$$

ii) Sub. x=650 into eq. A

$x+y = 700$

$650+y = 700$

$y = 50$

④ Conclusion

The rate of the plane in still air is 650 mph and the rate of the wind is 50 mph.

Example 8: With the current, you can row 24 miles in 3 hours. Against the same current, you can row only 2/3 of this distance in 4 hours. Find your rowing rate in still water and the rate of the current.

① Analysis

Let x be the rowing rate in still water and let y be the rate of the current

	r	t	d
with current	$x+y$	3	24
against current	$x-y$	4	$\frac{2}{3}\cdot 24 = 16$

② Translate

$(x+y)\cdot\cancel{3} = \frac{24}{3} \rightarrow x+y=8$

$(x-y)\cdot\cancel{4} = \frac{16}{4} \rightarrow x-y=4$

③ Solve

$x+y=8$ (A)

$x-y=4$ (B)

i) A+B, elim. y

$$\begin{aligned} x+y&=8\\ x-y&=4\\ \hline 2x&=12\\ x&=6 \end{aligned}$$

ii) Sub. x=6 into eq. A

$x+y=8$

$6+y=8$

$y=2$

④ Conclusion

The rowing rate in still water is 6mph and the rate of the current is 2mph.

Section 5.1: ADDING AND SUBTRACTING POLYNOMIALS

When you are done with your homework you should be able to…

- π Understand the vocabulary used to describe polynomials
- π Add polynomials
- π Subtract polynomials
- π Graph equations defined by polynomials of degree 2

WARM-UP:

Simplify:

$-6x+5y-2x^2-2y+x^2$

$= -6x+3y-x^2$

$= \boxed{-x^2-6x+3y}$

DESCRIBING POLYNOMIALS

A __polynomial__ is a __single__ term or the __sum__ of two or more __terms__ containing __variables__ with __whole__ number __exponents__. It is customary to write the __terms__ in the order of __descending__ powers of the __variable__. This is the __standard__ form of a __polynomial__. We begin this chapter by limiting discussion to polynomials containing __one__ variable. Each term of such a __polynomial__ in __x__ is of the form __$a\cdot x^n$__. The __degree__ of __ax^n__ is __n__.

THE DEGREE OF ax^n

If $a \neq 0$ and n is a whole number, the degree of ax^n is n. The degree of a nonzero constant term is 0. The constant zero has no defined degree.

Example 1: Identify the terms of the polynomial and the degree of each term.

a. $-4x^5 - 13x^3 + 5$

terms	$-4x^5$	$-13x^3$	5
degree	5	3	0

b. $-x^2 + 3x - 7$

terms	$-x^2$	$3x$	-7
degree	2	1	0

A polynomial is simplified when it contains no grouping symbols and no like terms. A simplified polynomial that has exactly one term is called a monomial. A simplified polynomial that has two terms is called a binomial and a simplified polynomial with three terms is called a trinomial. Simplified polynomials with four or more terms have no special names. The degree of a polynomial is the greatest degree of all the terms of a polynomial.

Example 2: Find the degree of the polynomial.

a. $\underbrace{5x^2}_{2} - \underbrace{x^8}_{8} + \underbrace{16x^4}_{4}$

$\boxed{\text{degree of the polynomial is 8}}$

b. -2

$\boxed{\text{degree of the polynomial is 0.}}$

ADDING POLYNOMIALS

Recall that like terms are terms containing exactly the same variable to the same powers. Polynomials are added by combining like terms.

Example 3: Add the polynomials.

a. $(8x-5)+(-13x+9)$ $= 8x - 5 + -13x + 9$

$= 8x - 13x - 5 + 9$

$= \boxed{-5x + 4}$

b. $(7y^3+5y-1)+(2y^2-6y+3)$ $= 7y^3 + 5y - 1 + 2y^2 - 6y + 3$

$= 7y^3 + 2y^2 + 5y - 6y - 1 + 3$

$= \boxed{7y^3 + 2y^2 - y + 2}$

c. $\left(\frac{2}{5}x^4+\frac{2}{3}x^3+\frac{5}{8}x^2+7\right)+\left(-\frac{4}{5}x^4+\frac{1}{3}x^3-\frac{1}{4}x^2-7\right)$

$= \frac{2}{5}x^4 - \frac{4}{5}x^4 + \frac{2}{3}x^3 + \frac{1}{3}x^3 + \frac{5}{8}x^2 - \frac{1}{4}x^2 + 7 - 7$

$= \frac{2-4}{5}x^4 + \frac{2+1}{3}x^3 + \frac{5-2}{8}x^2 + 0$

$= -\frac{2}{5}x^4 + \frac{3}{3}x^3 + \frac{3}{8}x^2$

$= \boxed{-\frac{2}{5}x^4 + x^3 + \frac{3}{8}x^2}$

d.

$$\begin{array}{r} 7x^2-5x-6 \\ \underline{-9x^2+4x+6} \\ -2x^2-x+0 \end{array}$$

$= \boxed{-2x^2 - x}$

SUBTRACTING POLYNOMIALS

We __Subtract__ real numbers by __adding__ the __opposite__ of the number being __Subtracted__. Subtraction of polynomials also involves __opposites__. If the sum of two polynomials is __zero__, the polynomials are __opposites__ of each other.

Example 4: Find the opposite of the polynomial.

a. $x+8$

$-x-8$ is the opposite of $x+8$

b. $-12x^3-x+1$

$12x^3+x-1$ is the opp. of $-12x^3-x+1$.

SUBTRACTING POLYNOMIALS

To __subtract__ two polynomials, __add__ the first polynomial and the __opposite__ of the second polynomial

Example 5: Subtract the polynomials.

a. $(x-2)-(7x+9) = x-2-7x-9$

$= x-7x-2-9$

$= \boxed{-6x-11}$

b. $(3x^2-2x)-(5x^2-6x) = 3x^2-2x-5x^2+6x$

$= 3x^2-5x^2-2x+6x$

$= \boxed{-2x^2+4x}$

c. $\left(\frac{3}{8}x^2-\frac{1}{3}x-\frac{1}{4}\right)-\left(-\frac{1}{8}x^2+\frac{1}{2}x-\frac{1}{4}\right)$

$=\frac{3}{8}x^2-\frac{1}{3}x-\frac{1}{4}+\frac{1}{8}x^2-\frac{1}{2}x+\frac{1}{4}$

$=\frac{3}{8}x^2+\frac{1}{8}x^2-\frac{1}{3}x-\frac{1}{2}x-\frac{1}{4}+\frac{1}{4}$

$=\frac{3+1}{8}x^2-\frac{2}{6}x-\frac{3}{6}x+0$

$=\frac{4}{8}x^2-\frac{5}{6}x$

$=\boxed{\frac{1}{2}x^2-\frac{5}{6}x}$

d.

$$\begin{array}{r} 3x^5-5x^3+6 \\ -\left(7x^5+4x^3-2\right) \\ \hline \boxed{-4x^5-9x^3+8} \end{array}$$

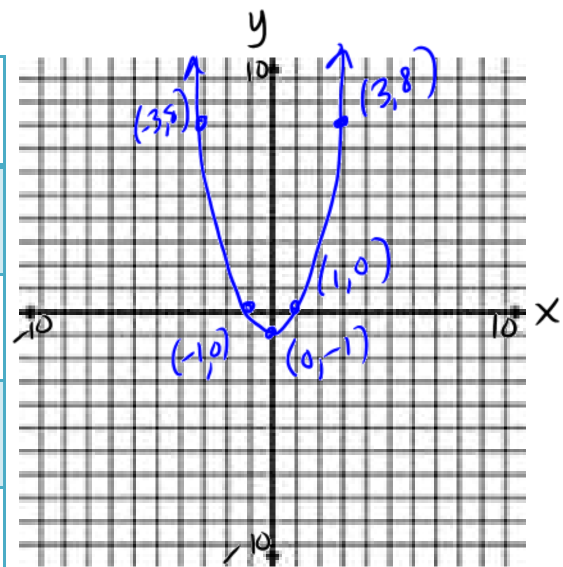
GRAPHING EQUATIONS DEFINED BY POLYNOMIALS

Graphs of equations defined by __polynomials__ of degree __2__ have a __mirrorlike__ quality. We can obtain their graphs, shaped like __bowls__ or __inverted__ bowls, using the __point__-__plotting__ method for graphing an equation in two variables.

Example 6: Graph the following equations by plotting points.

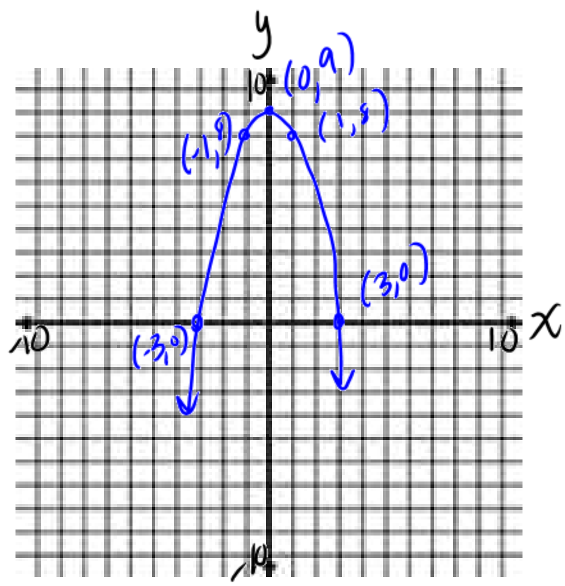
a. $y = x^2 - 1$

x	$y = x^2 - 1$	(x, y)
-3	$y=(-3)^2-1$ $y=9-1 \rightarrow y=8$	$(-3,8)$
-1	$y=(-1)^2-1$ $y=1-1 \rightarrow y=0$	$(-1,0)$
0	$y=(0)^2-1$ $y=-1$	$(0,-1)$
1	$y=(1)^2-1$ $y=1-1 \rightarrow y=0$	$(1,0)$
3	$y=(3)^2-1$ $y=9-1 \rightarrow y=8$	$(3,8)$



b. $y = 9 - x^2$

x	$y = 9 - x^2$	(x, y)
-3	$y=9-(-3)^2$ $y=9-9 \rightarrow y=0$	$(-3,0)$
-1	$y=9-(-1)^2$ $y=9-1 \rightarrow y=8$	$(-1,8)$
0	$y=9-(0)^2 \rightarrow y=9$	
1	$y=9-(1)^2$ $y=9-1 \rightarrow y=8$	$(1,8)$
3	$y=9-(3)^2$ $y=9-9 \rightarrow y=0$	$(3,0)$



Section 5.2: MULTIPLYING POLYNOMIALS

When you are done with your homework you should be able to…

- π Use the product rule for exponents
- π Use the power rule for exponents
- π Use the products-to-power rule
- π Multiply monomials
- π Multiply a monomial and a polynomial
- π Multiply polynomials when neither is a monomial

WARM-UP:

Add or subtract the following polynomials:

a. $\left(-22r^7+6r^3-r^2\right)-\left(2r^7+r^2-1\right)$

$= -22r^7+6r^3-r^2-2r^7-r^2+1$

$= -22r^7-2r^7+6r^3-r^2-r^2+1$

$= \boxed{-24r^7+6r^3-2r^2+1}$

b. $\left(8x^4-x^3-x^2\right)+\left(-8x^4+x^3\right)$

$= 8x^4-8x^4-x^3+x^3-x^2$

$= 0x^4-0x^3-x^2$

$= 0-0-x^2$

$= \boxed{-x^2}$

THE PRODUCT RULE FOR EXPONENTS

We have seen that __exponents__ are used to indicate __repeated__ multiplication. Recall that 3^4 = __$3\cdot3\cdot3\cdot3$__. Now consider $3^4\cdot3^2$:

$= 3\cdot3\cdot3\cdot3\cdot3\cdot3$

$= 3^6$

$= 3^{4+2}$

THE PRODUCT RULE

$b^m\cdot b^n = b^{m+n}$

When multiplying __exponential__ expressions with the __same__ base, __add__ the __exponents__. Use this __sum__ as the __exponent__ of the __common__ base.

Example 1: Simplify each expression.

a. $2^5 \cdot 2^3 = 2^{5+3}$
$= 2^8$
$= \boxed{256}$

b. $x^2 \cdot x^1 \cdot x^4 = x^{2+1+4}$
$= \boxed{x^7}$

$(4^2)^3 = (4^2)(4^2)(4^2)$
$= 4^{2+2+2}$
$= 4^6 \rightarrow 4^{2\cdot 3}$

THE POWER RULE (POWERS TO POWERS)

$(b^m)^n = b^{m\cdot n}$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

Example 2: Simplify each expression.

a. $(4^2)^3 = 4^{2\cdot 3}$
$= 4^6$
$= \boxed{4096}$

b. $(x^{12})^5 = x^{12\cdot 5}$
$= \boxed{x^{60}}$

THE PRODUCTS-TO-POWERS RULE FOR EXPONENTS

$(ab)^m = a^m b^m$

When a product is raised to a power, raise each factor to the power.

Example 3: Simplify each expression.

a. $(-2y)^5 = (-2)^5 \cdot y^5$

$= \boxed{-32y^5}$

b. $(10x^3)^2 = 10^2 (x^3)^2$

$= 100x^{3\cdot 2}$

$= \boxed{100x^6}$

MULTIPLYING MONOMIALS

To multiply monomials with the same variable base, multiply the coefficients and then multiply the variable. Use the product rule for exponents to multiply the variables.

Example 4: Multiply.

d. $(8x)(-11x^4)$

$= [8(-11)][x \cdot x^4]$

$= -88x^{1+4}$

$= \boxed{-88x^5}$

e. $(7y^3)(2y^2)$

$= (7 \cdot 2)(y^3 \cdot y^2)$

$= 14y^{3+2}$

$= \boxed{14y^5}$

f. $\left(\frac{2}{5}x^4\right)\left(-\frac{5}{6}x^7\right)$

$= \left[\frac{\cancel{2}^{1}}{\cancel{5}_{1}}\left(-\frac{\cancel{5}^{1}}{\cancel{6}_{3}}\right)\right]\left[x^4 \cdot x^7\right]$

$= -\frac{1}{3}x^{4+7}$

$= \boxed{-\frac{1}{3}x^{11}}$

MULTIPLYING A MONOMIAL AND A POLYNOMIAL THAT IS NOT A MONOMIAL

To multiply a monomial and a polynomial, use the distributive property to multiply each term of the polynomial by the monomial.

Example 5: Multiply.

a. $3x^2(2x-5)$

$= (3x^2)(2x) - (3x^2)(5)$

$= 6x^{2+1} - 15x^2$

$= \boxed{6x^3 - 15x^2}$

b. $-x(x^2+6x-5)$

$= (-x)(x^2) + (-x)(6x) - (-x)(5)$

$= -x^{1+2} - 6x^{1+1} + 5x$

$= \boxed{-x^3 - 6x^2 + 5x}$

MULTIPLYING POLYNOMIALS WHEN NEITHER IS A MONOMIAL

Multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

Example 6: Multiply.

a. $(x+2)(x+5)$

$= x(x+5) + 2(x+5)$

$= x(x) + x(5) + 2(x) + 2(5)$

$= x^2 + 5x + 2x + 10$

$= \boxed{x^2 + 7x + 10}$

b. $(2x+5)(x+3)$

$= 2x(x+3) + 5(x+3)$

$= 2x^2 + 6x + 5x + 15$

$= \boxed{2x^2 + 11x + 15}$

c. $\left(x^2-7x+9\right)\left(x+4\right)$ $= x^2(x+4)-7x(x+4)+9(x+4)$

$= x^3+4x^2-7x^2-28x+9x+36$

$= \boxed{x^3-3x^2-19x+36}$

Example 7: Simplify.

a. $3x^2\left(6x^3+2x-3\right)-4x^3\left(x^2-5\right)$

$= 18x^5+6x^3-9x^2-4x^5+20x^3$

$= \boxed{14x^5+26x^3-9x^2}$

b. $\left(y+6\right)^2-\left(y-2\right)^2$

$= (y+6)(y+6)-(y-2)(y-2)$

$= y(y+6)+6(y+6)-[y(y-2)-2(y-2)]$

$= y^2+6y+6y+36-[y^2-2y-2y+4]$

$= y^2+12y+36-[y^2-4y+4]$

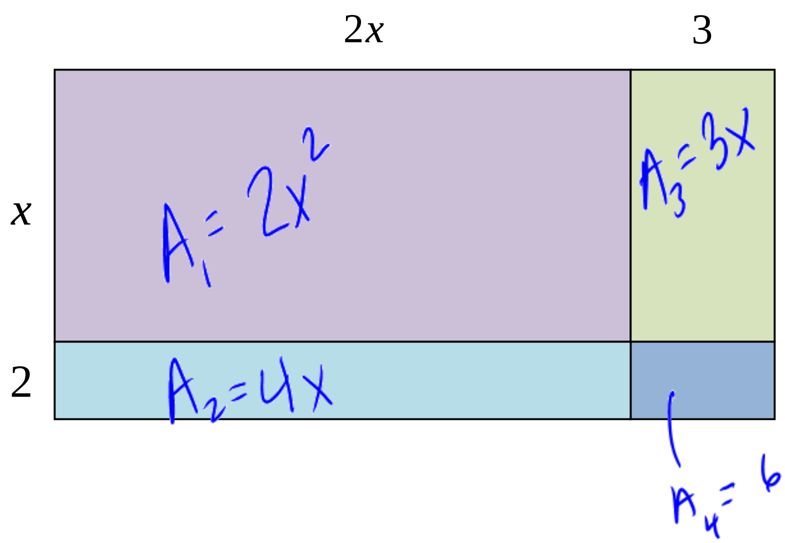
$= y^2+12y+36-y^2+4y-4$

$= 0y^2+16y+32$

$= 0+16y+32$

$= \boxed{16y+32}$

APPLICATION



a. Express the area of the large rectangle as the product of two binomials.

$$A = (2x+3)(x+2)$$

b. Find the sum of the areas of the four smaller rectangles.

$$A = 2x^2 + 3x + 4x + 6$$

c. Use polynomial multiplication to show that your expressions for area in parts (a) and (b) are equal.

Part a

$$A = (2x+3)(x+2)$$
$$A = 2x(x+2) + 3(x+2)$$
$$A = 2x^2 + 4x + 3x + 6$$
$$A = 2x^2 + 7x + 6$$

Part b

$$A = 2x^2 + 3x + 4x + 6$$
$$A = 2x^2 + 7x + 6$$

Section 5.3: SPECIAL PRODUCTS

When you are done with your homework you should be able to…

- π Use FOIL in polynomial multiplication
- π Multiply the sum and difference of two terms
- π Find the square of a binomial sum
- π Find the square of a binomial difference

$(xy)^2 \rightarrow x^2y^2$

$(x+y)^2 \neq x^2+y^2$

$= x^2+2xy+y^2$

WARM-UP:

Multiply the following polynomials:

a. $(x-1)^2$ $=(x-1)(x-1)$

$= x(x-1)-1(x-1)$

$= x^2-x-x+1$

$= x^2-2x+1$

b. $(x-5)(x+5)$

$= x(x+5)-5(x+5)$

$= x^2+5x-5x-25$

$= x^2+0x-25$

$= \boxed{x^2-25}$

THE PRODUCT OF TWO BINOMIALS: FOIL

F represents the __product__ of the __First__ terms in each __binomial__, **O** represents the __product__ of the __Outside__ terms, **I** represents the __product__ of the __Inner__ terms, and **L** represents the __product__ of the __Last__ terms.

USING THE FOIL METHOD TO MULTIPLY BINOMIALS

$(ax+b)(cx+d) = \underbrace{(ax)(cx)}_{\text{First}} + \underbrace{(ax)(d)}_{\text{outside}} + \underbrace{(b)(cx)}_{\text{inside}} + \underbrace{(b)(d)}_{\text{last}}$

Example 1: Multiply using FOIL.

a. $(5x+3)(3x+8)$

$= (5x)(3x) + (5x)(8) + (3)(3x) + (3)(8)$

$= 15x^2 + 40x + 9x + 24$

$= \boxed{15x^2 + 49x + 24}$

b. $(x-10)(x+9)$

$= (x)(x) + (x)(9) + (-10)(x) + (-10)(9)$

$= x^2 + 9x - 10x - 90$

$= \boxed{x^2 - x - 90}$

THE PRODUCT OF THE SUM AND DIFFERENCE OF TWO TERMS

$(A+B)(A-B) =$ $A^2 - B^2$

The product of the sum and the difference of the same two terms is the square of the first minus the square of the second.

Example 2: Multiply.

a. $(x+4)(x-4) = (x)^2 - (4)^2$

$A = x$

$B = 4$

$= \boxed{x^2 - 16}$

b. $(3x-7y)(3x+7y) = (3x)^2 - (7y)^2$

$A = 3x$

$B = 7y$

$= \boxed{9x^2 - 49y^2}$

THE SQUARE OF A BINOMIAL SUM

$(A+B)^2 =$ $A^2 + 2AB + B^2$

The square of a binomial sum is the first term squared plus 2 times the product of the terms plus the last term squared.

$(A+B)^2 = A^2 + 2AB + B^2$

Example 3: Multiply.

a. $(x+6)^2 = (x)^2 + 2(x)(6) + (6)^2$

$A = x$

$B = 6$

$= \boxed{x^2 + 12x + 36}$

b. $(x^2+9)^2 = (x^2)^2 + 2(x^2)(9) + (9)^2$

$A = x^2$

$B = 9$

$= \boxed{x^4 + 18x^2 + 81}$

THE SQUARE OF A BINOMIAL DIFFERENCE

$(A-B)^2 =$ $A^2 - 2AB + B^2$

The Square of a binomial difference is the first term squared minus 2 times the product of the terms plus the last term squared.

Example 4: Multiply. $(A-B)^2 = A^2 - 2AB + B^2$

a. $(5x-y)^2 = (5x)^2 - 2(5x)(y) + (y)^2$

$A = 5x$

$B = y$

$= \boxed{25x^2 - 10xy + y^2}$

b. $(x^3-11)^2 = (x^3)^2 - 2(x^3)(11) + (11)^2$

$A = x^3$

$B = 11$

$= \boxed{x^6 - 22x^3 + 121}$

Section 5.4: POLYNOMIALS IN SEVERAL VARIABLES

When you are done with your homework you should be able to…

- π Evaluate polynomials in several variables
- π Understand the vocabulary of polynomials in two variables
- π Add and subtract polynomials in several variables
- π Multiply polynomials in several variables

WARM-UP:

Evaluate the polynomial:

$x^3y + 2xy^2 + 5x - 2;\ x = -2$ and $y = 3$

$(-2)^3(3) + 2(-2)(3)^2 + 5(-2) - 2$

$= (-8)(3) - 4(9) - 10 - 2$

$= -24 - 36 - 12 = \boxed{-72}$

EVALUATING A POLYNOMIAL IN SEVERAL VARIABLES

1. Substitute the given value for each variable.
2. Perform the resulting computations using the order of operations.

DESCRIBING POLYNOMIALS IN TWO VARIABLES

In general, a polynomial in 2 variables, x and y, contains the sum of one or more monomials in the form ax^ny^m. The constant, a, is the coefficient. The exponents, m and n, represent whole numbers. The degree of the monomial ax^ny^m is $n+m$.

Example 1: Determine the coefficient of each term, the degree of each term, and the degree of the polynomial.

$8xy^4 - 17x^5y^3 + 4x^2y - 9y^3 + 7$

term	$8xy^4$	$-17x^5y^3$	$4x^2y$	$-9y^3$	7
degree	5	8	3	3	0

Degree of the polynomial is 8.

ADDING AND SUBTRACTING POLYNOMIALS IN SEVERAL VARIABLES

Polynomials in several variables are added by combining like terms.

Example 2: Add or subtract.

a. $(x^3 - y^3) - (-4x^3 - x^2y + xy^2 + 3y^3)$

$= x^3 - y^3 + 4x^3 + x^2y - xy^2 - 3y^3$

$= x^3 + 4x^3 + x^2y - xy^2 - y^3 - 3y^3$

$= \boxed{5x^3 + x^2y - xy^2 - 4y^3}$

b. $(7x^2y + 5xy + 13) + (-3x^2y + 6xy + 4)$

$= 7x^2y + 5xy + 13 - 3x^2y + 6xy + 4$

$= 7x^2y - 3x^2y + 5xy + 6xy + 13 + 4$

$= \boxed{4x^2y + 11xy + 17}$

MULTIPLYING POLYNOMIALS IN SEVERAL VARIABLES

The product of monomials forms the basis of polynomial multiplication. Multiplication can be done mentally by multiplying coefficients and adding exponents on variables with the same base.

Example 3: Multiply.

a. $(5xy^3)(-10x^2y^4)$

$= [5(-10)](x \cdot x^2)(y^3 \cdot y^4)$

$= -50x^{1+2}y^{3+4}$

$= \boxed{-50x^3y^7}$

c. $(x - 2y^4)(x + 2y^4)$

$A = x$

$B = 2y^4$

$(A+B)(A-B)$

$= A^2 - B^2$

$= (x)^2 - (2y^4)^2$

$= \boxed{x^2 - 4y^8}$

b. $-x^7y^2(x^2 + 7xy - 4)$

$= (-x^7y^2)(x^2) + (-x^7y^2)(7xy) - (-x^7y^2)(4)$

$= -x^7 \cdot x^2 \cdot y^2 - 7x^7 \cdot x^1 \cdot y^2 \cdot y^1 + 4x^7y^2$

$= -x^{7+2}y^2 - 7x^{7+1}y^{2+1} + 4x^7y^2$

$= \boxed{-x^9y^2 - 7x^8y^3 + 4x^7y^2}$

d. $(x^2 - y)^2 = (x^2)^2 - 2(x^2)(y) + (y)^2$

$= \boxed{x^4 - 2x^2y + y^2}$

$A = x^2$

$B = y$

$(A-B)^2 = A^2 - 2AB + B^2$

Section 5.5: DIVIDING POLYNOMIALS

When you are done with your homework you should be able to…

- π Use the quotient rule for exponents
- π Use the zero-exponent rule for exponents
- π Use the quotients-to-power rule
- π Divide monomials
- π Check polynomial division
- π Divide a polynomial by a monomial

WARM-UP:

1. Find the missing exponent, designated by the question mark, in the final step:

$$\frac{x^8}{x^3} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = x^? \rightarrow ?=5 \rightarrow \frac{x^8}{x^3} = x^{8-3} = x^5$$

2. Simplify:

$$\frac{(2a^3)^5}{(b^4)^5} = \frac{2^5(a^3)^5}{b^{4\cdot 5}} = \frac{32a^{3\cdot 5}}{b^{20}} = \boxed{\frac{32a^{15}}{b^{20}}}$$

THE QUOTIENT RULE FOR EXPONENTS

$$\frac{b^m}{b^n} = b^{m-n}$$

When dividing _exponential_ expressions with the _same_ nonzero base, _Subtract_ the exponent in the _denominator_ from the _exponent_ in the _numerator_. Use this _difference_ as the _exponent_ of the _common_ base.

Example 1: Simplify each expression.

a. $\frac{2^5}{2^3} = 2^{5-3}$

$= 2^2$

$= \boxed{4}$

b. $\frac{x^{10}}{x^8} = x^{10-8}$

$= \boxed{x^2}$

THE ZERO-EXPONENT RULE

If __b____ is any __real___________ number other than __0____,

$b^0 = 1$

Example 2: Simplify each expression.

a. $(4^2)^0 = \boxed{1}$

b. $-7x^0 = -7(1)$

$= \boxed{-7}$

THE QUOTIENTS-TO-POWERS RULE FOR EXPONENTS

If ___a___ and ___b___ are real numbers and ___b___ is nonzero, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

When a ___quotient___ is ___raised___ to a ___power___, ___raise___ the ___numerator___ to the ___power___ and ___divide___ by the ___denominator___ raised to the ___power___.

Example 3: Simplify each expression.

a. $\left(\frac{x}{3}\right)^5 = \frac{x^5}{3^5} = \boxed{\frac{x^5}{243}}$

b. $\left(\frac{4x^3}{5y}\right)^2 = \frac{(4x^3)^2}{(5y)^2} = \frac{4^2(x^3)^2}{5^2y^2} = \frac{16x^{3\cdot 2}}{25y^2} = \boxed{\frac{16x^6}{25y^2}}$

DIVIDING MONOMIALS

To ___divide___ ___monomials___, ___divide___ the ___coefficients___ and then divide the ___variables___.

Use the ___quotient___ rule for ___exponents___ to divide the ___variables___.

Example 4: Divide.

a. $\frac{16x^4}{2x^4} = \left(\frac{16}{2}\right) \cdot x^{4-4}$

$= 8x^0$

$= 8(1)$

$= \boxed{8}$

b. $\frac{6x^2y^5}{21xy^3} = \left(\frac{6}{21}\right) x^{2-1} y^{5-3}$

$= \frac{2}{7} x^1 y^2$

$= \boxed{\frac{2}{7}xy^2 \text{ or } \frac{2xy^2}{7}}$

c. $\frac{35r^8}{14r^7} = \left(\frac{35}{14}\right) r^{8-7}$

$= \frac{5}{2} r^1$

$= \boxed{\frac{5}{2}r \text{ or } \frac{5r}{2}}$

DIVIDING A POLYNOMIAL THAT IS NOT A MONOMIAL BY A MONOMIAL

To __divide__ by a __monomial__, __divide__ each __term__ of the __numerator__ by the __denominator__.

$\frac{3+6}{3} = \frac{3}{3} + \frac{6}{3}$

$= 1 + 2 = \boxed{3}$

Example 5: Find the quotient.

a. $\left(24x^6 - 12x^4 + 8x^3\right) \div \left(4x^3\right)$

$= \frac{24x^6 - 12x^4 + 8x^3}{4x^3}$

$= \frac{24x^6}{4x^3} - \frac{12x^4}{4x^3} + \frac{8x^3}{4x^3}$

$= 6x^{6-3} - 3x^{4-3} + 2x^{3-3}$

$= 6x^3 - 3x^1 + 2x^0$

$= 6x^3 - 3x + 2(1)$

$= \boxed{6x^3 - 3x + 2}$

b. $\frac{459x^{10}y^9 + 18x^5y^3 - 9x^4y}{-9x^3y}$

$= \frac{459x^{10}y^9}{-9x^3y} + \frac{18x^5y^3}{-9x^3y} - \frac{9x^4y}{-9x^3y}$

$= -51x^{10-3}y^{9-1} - 2x^{5-3}y^{3-1} + 1x^{4-3}y^{1-1}$

$= -51x^7y^8 - 2x^2y^2 + x^1y^0$

$= -51x^7y^8 - 2x^2y^2 + x(1)$

$= \boxed{-51x^7y^8 - 2x^2y^2 + x}$

Section 5.6: LONG DIVISION OF POLYNOMIALS AND SYNTHETIC DIVISION

When you are done with your homework you should be able to…

- π Use long division to divide by a polynomial containing more than one term
- π Divide polynomials using synthetic division

WARM-UP:

a. Divide using long division:

22045 ← quotient

56)1234567 ← dividend

divisor

$$
\begin{array}{r}
-(112) \\
\hline
114 \\
-(112) \\
\hline
25 \\
-(0) \\
\hline
256 \\
-(224) \\
\hline
327 \\
-(280) \\
\hline
47
\end{array}
$$

remainder

$$1234567 \div 56$$

$$\underbrace{= 22045 + \frac{47}{56}}$$

$$= 22045\tfrac{47}{56}$$

side work

How many…

56's in 123 → 2

56's in 114 → 2

56's in 25 → 0

56's in 256 → 4

56's in 327 → 5

Dividend ÷ divisor

$$= \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

b. Simplify:

$$\frac{5x^5 - 8x^3 + x^2}{2x^2} = \frac{5x^5}{2x^2} - \frac{8x^3}{2x^2} + \frac{x^2}{2x^2}$$

$$= \frac{5}{2}x^{5-2} - 4x^{3-2} + \frac{1}{2}x^{2-2}$$

$$= \frac{5}{2}x^3 - 4x^1 + \frac{1}{2}x^0$$

$$= \frac{5}{2}x^3 - 4x + \frac{1}{2}(1)$$

$$= \boxed{\frac{5}{2}x^3 - 4x + \frac{1}{2}}$$

STEPS FOR DIVIDING A POLYNOMIAL BY A BINOMIAL

1. __Arrange__ the terms of __both__ the __dividend__ and the __divisor__ in __descending__ powers of the variable.
   * If any powers are missing, write in 0 times the missing power
   $x^3 - x + 1 \rightarrow x^3 + 0x^2 - x + 1$
2. __Divide__ the __first__ term in the __dividend__ by the __first__ term in the __divisor__. The result is the __first__ term of the __quotient__.
3. __Multiply__ every term in the __divisor__ by the __first__ term in the __quotient__. Write the resulting __product__ beneath the __dividend__ with __like__ terms lined up.
4. __Subtract__ the __product__ from the __dividend__.
5. __Bring__ down the next term in the __dividend__ dividend and write it next to the __remainder__ to form a new __dividend__.
6. Use this new expression as the __dividend__ and repeat the process until the __remainder__ can no longer be __divided__. This will occur when the __degree__ of the __remainder__ is __less__ than the __degree__ of the __divisor__.

Example 1: Divide.

a. $\dfrac{x^2+7x+10}{x+5} = \boxed{x+2}$

$$\begin{array}{r l} & x+2 \\ (x+5)\overline{)} & x^2+7x+10 \\ & -(x^2+5x)\downarrow \\ & 2x+10 \\ & -(2x+10) \\ & 0 \end{array}$$

side work

① $\dfrac{x^2}{x} = x$

② $\dfrac{2x}{x} = 2$

b. $\dfrac{2y^2-13y+21}{y-3} = \boxed{2y-7}$

$$\begin{array}{r l} & 2y-7 \\ (y-3)\overline{)} & 2y^2-13y+21 \\ & -(2y^2-6y)\downarrow \\ & -7y+21 \\ & -(-7y+21) \\ & 0 \end{array}$$

side work

① $\dfrac{2y^2}{y} = 2y$

② $\dfrac{-7y}{y} = -7$

c. $\dfrac{x^3+2x^2-3}{x-2}$

$$= \frac{x^3+2x^2+0x-3}{x-2}$$

$$\boxed{x^2+4x+8+\frac{13}{x-2}}$$

$$
\begin{array}{r|l}
(x-2) & x^3+2x^2+0x-3 \\
 & -(x^3-2x^2) \\
\hline
 & \quad 4x^2+0x \\
 & \quad -(4x^2-8x) \\
\hline
 & \qquad 8x-3 \\
 & \qquad -(8x-16) \\
\hline
 & \qquad\quad 13
\end{array}
$$

Side work

① $\dfrac{x^3}{x}=x^2$

② $\dfrac{4x^2}{x}=4x$

③ $\dfrac{8x}{x}=8$

d. $\left(8y^3+y^4+16+32y+24y^2\right)\div(y+2)$

$$= \frac{y^4+8y^3+24y^2+32y+16}{y+2}$$

$$\boxed{y^3+6y^2+12y+8}$$

$$
\begin{array}{r|l}
(y+2) & y^4+8y^3+24y^2+32y+16 \\
 & -(y^4+2y^3) \\
\hline
 & \quad 6y^3+24y^2 \\
 & \quad -(6y^3+12y^2) \\
\hline
 & \qquad 12y^2+32y \\
 & \qquad -(12y^2+24y) \\
\hline
 & \qquad\quad 8y+16 \\
 & \qquad\quad -(8y+16) \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Side work

① $\dfrac{y^4}{y}=y^3$

② $\dfrac{6y^3}{y}=6y^2$

③ $\dfrac{12y^2}{y}=12y$

④ $\dfrac{8y}{y}=8$

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

We can use ___Synthetic___ division to divide ___polynomials___ if the ___divisor___ is of the form ___x-c___. This method provides a ___quotient___ more quickly than ___long___ division.

STEPS FOR SYNTHETIC DIVISION

1. Arrange the ___polynomial___ in ___descending___ powers, with a ___0___ coefficient for any ___missing___ term.
2. Write ___c___ for the ___divisor___, ___x-c___. To the ___right___, write the ___coefficients___ of the ___dividend___.
3. Write the ___leading___ ___coefficient___ of the ___dividend___ on the ___bottom___ row.
4. ___Multiply___ ___c___ times the ___Value___ just written on the ___bottom___ row. Write the ___product___ in the next ___column___ in the ___second___ row.
5. ___Add___ the values in this new column, writing the ___sum___ in the ___bottom___ row.
6. Repeat this series of ___multiplications___ and ___additions___ until all ___columns___ are filled in.

7. Use the numbers in the last row to write the quotient plus the remainder above the divisor. The degree of the first term of the quotient will be one less than the degree of the first term of the dividend. The final value in this row is the remainder.

Example 2: Divide using synthetic division.

a. $(1x^2 + 1x - 2) \div (x - 1) = \boxed{x+2}$

$c = 1$

1	1	1	−2
		1	2
	1	2	0

$$1x^1 + 2 + \frac{0}{x-1}$$

b. $(x^2 - 6x - 6x^3 + x^4) \div (6 + x)$

$$= \frac{x^4 - 6x^3 + x^2 - 6x + 0}{x+6}$$

$x + 6 = x - (-6)$

$c = -6$

−6	1	−6	1	−6	0
		−6	72	−438	2664
	1	−12	73	−444	2664

$$1x^3 - 12x^2 + 73x - 444 + \frac{2664}{x+6}$$

$$\boxed{x^3 - 12x^2 + 73x - 444 + \frac{2664}{x+6}}$$

c. $\dfrac{x^7-128}{x-2} = \dfrac{x^7+0x^6+0x^5+0x^4+0x^3+0x^2+0x-128}{x-2}$

$c=2$

$$\begin{array}{r|rrrrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -128 \\ & & 2 & 4 & 8 & 16 & 32 & 64 & 128 \\ \hline & 1 & 2 & 4 & 8 & 16 & 32 & 64 & 0 \end{array}$$

$=0$

$1x^6+2x^5+4x^4+8x^3+16x^2+32x+64+\dfrac{0}{x-2}$

$\boxed{x^6+2x^5+4x^4+8x^3+16x^2+32x+64}$

d. $\left(y^5-2y^4-y^3+3y^2-y+1\right)\div(y-2) = \boxed{y^4-y^2+y+1+\dfrac{3}{y-2}}$

$$\begin{array}{r|rrrrrr} 2 & 1 & -2 & -1 & 3 & -1 & 1 \\ & & 2 & 0 & -2 & 2 & 2 \\ \hline & 1 & 0 & -1 & 1 & 1 & 3 \end{array}$$

$1y^4+0y^3-1y^2+1y+1+\dfrac{3}{y-2}$

APPLICATION

You just signed a contract for a new job. The salary for the first year is $30,000 and there is to be a percent increase in your salary each year. The algebraic expression

$$\frac{30000x^n - 30000}{x-1}$$

describes your total salary over *n* years, where *x* is the sum of 1 and the yearly percent increase, expressed as a decimal.

a. Use the given expression and write a quotient of polynomials that describes your total salary over four years.

b. Simplify the expression in part (a) by performing the division.

c. Suppose you are to receive an increase of 8% per year. Thus, *x* is the sum of 1 and 0.08, or 1.08. Substitute 1.08 for *x* in the expression in part (a) as well as the simplified expression in part (b). Evaluate each expression. What is your total salary over the four-year period?

Section 5.7: NEGATIVE EXPONENTS AND SCIENTIFIC NOTATION

When you are done with your homework you should be able to…

- π Use the negative exponent rule
- π Simplify exponential expressions
- π Convert from scientific notation to decimal notation
- π Convert from decimal notation to scientific notation
- π Compute with scientific notation
- π Solve applied problems using scientific notation

WARM-UP:

1. Divide:

$\left(7x^4-8x\right)\div\left(x+3\right)$

$= \dfrac{7x^4+0x^3+0x^2-8x+0}{x+3}$

$= \boxed{7x^3-21x^2+63x-197+\dfrac{591}{x+3}}$

side work

① $\dfrac{7x^4}{x}=7x^3$

② $\dfrac{-21x^3}{x}=-21x^2$

③ $\dfrac{63x^2}{x}=63x$

④ $\dfrac{-197x}{x}=-197$

$$
\begin{array}{r}
7x^3-21x^2+63x-197+\dfrac{591}{x+3} \\
(x+3)\overline{)\,7x^4+0x^3+0x^2-8x+0} \\
-(7x^4+21x^3) \\
\hline
-21x^3+0x^2 \\
-(-21x^3-63x^2) \\
\hline
63x^2-8x \\
-(63x^2+189x) \\
\hline
-197x+0 \\
-(-197x-591) \\
\hline
591
\end{array}
$$

2. Simplify:

$\dfrac{1}{\left(6x^3\right)^2} = \dfrac{1}{6^2\left(x^3\right)^2}$

$= \dfrac{1}{36x^{3\cdot 2}}$

$= \boxed{\dfrac{1}{36x^6}}$

NEGATIVE INTEGERS AS EXPONENTS

A nonzero base can be raised to a __negative__ power. The __quotient__ rule can be used to help determine what a __negative__ __integer__ as an __exponent__ should mean.

$$\frac{1}{2^3} = \frac{2^0}{2^3} = 2^{0-3} = 2^{-3}$$

THE NEGATIVE EXPONENT RULE

If __b__ is any real number other than __0__ and __n__ is a natural number, then

$$b^{-n} = \frac{1}{b^n}$$

NEGATIVE EXPONENTS IN NUMERATORS AND DENOMINATORS

If __b__ is any real number other than __0__ and __n__ is a natural number, then

$$b^{-n} = \frac{1}{b^n} \quad \text{and} \quad \frac{1}{b^{-n}} = b^n \rightarrow \frac{1}{b^{-n}} = \frac{1}{\left(\frac{1}{b^n}\right)} = \frac{b^n}{1} = b^n$$

When a __negative__ number appears as an __exponent__, __switch__ the position of the __base__ (from __denominator__ to __numerator__ or from __numerator__ to __denominator__) and make the __exponent__ __positive__. The sign of the __base__ does __not__ change.

Example 1: Write each expression with positive exponents only. Then simplify, if possible.

a. $-7^{-2} = -\frac{1}{7^2} = \boxed{-\frac{1}{49}}$

b. $(-7)^{-2} = \frac{1}{(-7)^2} = \boxed{\frac{1}{49}}$

c. $3^{-1} - 6^{-1} = \frac{1}{3^1} - \frac{1}{6^1}$

$= \frac{1}{3} \cdot \frac{2}{2} - \frac{1}{6}$

$= \frac{2-1}{6} = \boxed{\frac{1}{6}}$

d. $\frac{x^{-12}}{y^{-1}} = \frac{\frac{1}{x^{12}}}{\frac{1}{y^1}}$

$= \frac{1}{x^{12}} \cdot \frac{y}{1} = \boxed{\frac{y}{x^{12}}}$

SIMPLIFYING EXPONENTIAL EXPRESSIONS

Properties of __exponents__ are used to __simplify__ exponential expressions. An exponential __expression__ is __simplified__ when

- Each __base__ occurs only __once__
- No __parentheses__ appear
- No __powers__ are raised to __powers__
- No __negative__ or __zero__ exponents appear

STEPS FOR SIMPLIFYING EXPONENTIAL EXPRESSIONS

1. If necessary, be sure that each base appears only once, using $b^m \cdot b^n = b^{m+n}$ or $\frac{b^m}{b^n} = b^{m-n}$.

2. If necessary, remove parentheses using $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

3. If necessary, simplify powers to powers using $(b^m)^n = b^{m \cdot n}$.

4. If necessary, rewrite exponential expressions with zero powers as 1 ($b^0 = 1$). Furthermore, write the answer with positive exponents using $b^{-n} = \frac{1}{b^n}$ or $\frac{1}{b^{-n}} = b^n$.

Example 2: Simplify. Assume that variables represent nonzero real numbers.

a. $\dfrac{45z^4}{15z^{12}} = 3z^{4-12} = 3z^{-8} = 3\left(\frac{1}{z^8}\right) = \boxed{\frac{3}{z^8}}$

b. $\dfrac{(3y^4)^3 y^{-7}}{y^7} = 3^3 (y^4)^3 y^{-7-7} = 27 y^{4 \cdot 3} y^{-14} = 27 y^{12} y^{-14} = 27 y^{12+(-14)} = 27 y^{-2} = 27\left(\frac{1}{y^2}\right) = \boxed{\frac{27}{y^2}}$

c. $\dfrac{(5x^3)^2}{x^7} = \dfrac{5^2 (x^3)^2}{x^7} = \dfrac{25 x^{3 \cdot 2}}{x^7} = \dfrac{25x^6}{x^7} = 25x^{6-7} = 25x^{-1} = 25\left(\frac{1}{x^1}\right) = \boxed{\frac{25}{x}}$

d. $\left(\dfrac{x^3}{y^2}\right)^{-4} = \dfrac{(x^3)^{-4}}{(y^2)^{-4}} = \dfrac{x^{3(-4)}}{y^{2(-4)}} = \dfrac{x^{-12}}{y^{-8}} = \boxed{\frac{y^8}{x^{12}}}$

SCIENTIFIC NOTATION

A __positive__ number is written in __scientific__ notation when it is expressed in the form $a \times 10^{n}$

↑ times

where __a__ is a number __greater__ than or equal to __1__ and __less__ than __10__ ($1 \le a < 10$) and __n__ is an __integer__.

It is customary to use the __multiplication__ symbol, __x__, rather than a dot, when writing a number in __scientific__ __notation__. We can use __n__, the exponent on the __10__ in __$a \times 10^{n}$__, to change a number in scientific notation to __decimal__ notation. If __n__ is __positive__, move the decimal point in __a__ to the __right__ __n__ places. If __n__ is __negative__, move the decimal point in __a__ to the __left__ __n__ places.

Example 3: Write each number in decimal notation.

a. 7.85×10^{8} $= \boxed{785{,}000{,}000}$

c. 1.001×10^{2} $= \boxed{100.1}$

b. 9×10^{-5} $= \boxed{.00009}$

d. 9.999×10^{-1} $= \boxed{.9999}$

CONVERTING FROM DECIMAL TO SCIENTIFIC NOTATION

Write the number in the form $a \times 10^n$.

- Determine a, the numerical factor. Move the decimal point in the given number to obtain a number greater than or equal to 1 and less than 10.
- Determine n, the exponent on 10^n. The absolute value of n is the number of places the decimal point was moved. The exponent n is positive if the given number is greater than or equal 10 and negative if the given number is between 0 and 1.

Example 4: Write each number in scientific notation.

a. 0.00000006589 $= \boxed{6.589 \times 10^{-8}}$

c. 0.234 $= \boxed{2.34 \times 10^{-1}}$

b. 6,789,000,000,000 $= \boxed{6.789 \times 10^{12}}$

d. 1,000,234,000 $= \boxed{1.000234 \times 10^{9}}$

COMPUTATIONS WITH NUMBERS IN SCIENTIFIC NOTATION

MULTIPLICATION $(a \times 10^n)\cdot(b \times 10^m) = (a\cdot b) \times 10^{n+m}$

DIVISION $\dfrac{a \times 10^n}{b \times 10^m} = \left(\dfrac{a}{b}\right) \times 10^{n-m}$

EXPONENTIATION

$$(a \times 10^n)^m = a^m \times (10^n)^m = a^m \times 10^{n\cdot m}$$

After the computation is __completed__, the __answer__ may require an additional __adjustment__ before it is expressed in __scientific__ notation.

Example 5: Perform the indicated operations, writing the answers in scientific notation.

a. $(3\times10^4)(4\times10^2)$

$= (3\cdot4) \times 10^{4+2}$

$= 12 \times 10^6$

$= 1.2 \times 10^1 \times 10^6$

$= 1.2 \times 10^{1+6}$

$= \boxed{1.2 \times 10^7}$

b. $(2\times10^{-3})^5$

$= 2^5 \times (10^{-3})^5$

$= 32 \times 10^{-15}$

$= 3.2 \times 10^1 \times 10^{-15}$

$= 3.2 \times 10^{1+(-15)}$

$= \boxed{3.2 \times 10^{-14}}$

c. $\frac{180\times10^{8}}{2\times10^{4}} = \left(\frac{180}{2}\right)\times 10^{8-4}$

$= 90 \times 10^{4}$

$= 9.0\times10^{1} \times 10^{4}$

$= \boxed{9.0\times10^{5}}$

d. $\left(5\times10^{4}\right)^{-1} = 5^{-1}\times\left(10^{4}\right)^{-1}$

$= \frac{1}{5}\times10^{-4}$

$= .2\times10^{-4}$

$= 2\times10^{-1}\times10^{-4}$

$= 2\times10^{-1+(-4)}$

$= \boxed{2\times10^{-5}}$

APPLICATIONS

1. A human brain contains 3×10^{10} neurons and a gorilla brain contains 7.5×10^{9} neurons. How many times as many neurons are in the brain of a human as in the brain of a gorilla?

$\frac{3\times10^{10}}{7.5\times10^{9}} = \left(\frac{3}{7.5}\right)\times10^{10-9}$

$= .4\times10^{1}$

$= \boxed{4}$

$4\times10^{-1}\times10^{1}$

$4\times10^{-1+1}$

4×10^{0}

2. If the sun is approximately 9.14×10^{7} miles from the earth, how many seconds, to the nearest tenth of a second does it take sunlight to reach Earth? Use the motion formula, $d = rt$, and the fact that light travels at the rate of 1.86×10^{5} miles per second.

$r = 1.86\times10^{5}$, $d = 9.14\times10^{7}$

$d = rt$

$t = \frac{d}{r}$

$t = \frac{9.14\times10^{7}}{1.86\times10^{5}}$

$t = \left(\frac{9.14}{1.86}\right)\times10^{7-5}$

$t = 4.9\times10^{2}$

It takes sunlight approximately 4.9×10^{2} seconds to reach the Earth.

Section 6.1: THE GREATEST COMMON FACTOR AND FACTORING BY GROUPING

When you are done with your homework you should be able to…

- π Find the greatest common factor (GCF)
- π Factor out the GCF of a polynomial
- π Factor by grouping

WARM-UP:

$a(b+c)=a\cdot b+a\cdot c$

$\underbrace{a\cdot b+a\cdot c=a(b+c)}_{\text{factoring}}$

1. Multiply:

$x^2\left(7x^4-8\right) = x^2\cdot 7x^4 - x^2\cdot 8$

$= \boxed{7x^6-8x^2}$

2. Divide:

$\dfrac{16x^4-8x^2}{4x^2} = \dfrac{16x^4}{4x^2} - \dfrac{8x^2}{4x^2}$

$= \boxed{4x^2-2}$

FACTORING A __polynomial__ **CONTAINING THE SUM OF** __monomials__ **MEANS FINDING AN** __equivalent__ **EXPRESSION THAT IS A** __product__.

FACTORING OUT THE GREATEST COMMON FACTOR (GCF)

We use the distributive property to multiply a monomial and a polynomial of 2 or more terms.

When we factor, we reverse this process, expressing the polynomial as a product.

MULTIPLICATION

$a(b+c) = ab + ac$

$2(x+3) = 2 \cdot x + 2 \cdot 3$

FACTORING

$ab + ac = a(b+c)$

$4x + 8$

$4 \cdot x + 4 \cdot 2 = 4(x+2)$

In any factoring problem, the first step is to look for the greatest common factor. The GCF is an expression of the highest degree that divides each term of the polynomial.

The variable part of the GCF always contains the smallest power of a variable that appears in ALL terms of the polynomial.

Example 1: Find the greatest common factor of each list of monomials:

a. 5 and $15x$ → $5 = 5$ and $15x = 3 \cdot 5 \cdot x$ → $\boxed{\text{GCF is } 5}$

b. $-3x^4$ and $6x^3$ → $-3x^4 = -3 \cdot x^3 \cdot x$ and $6x^3 = 2 \cdot 3 \cdot x^3$ → $\boxed{\text{GCF is } 3x^3 \text{ or } -3x^3}$

c. x^2y, $7x^3y$ and $14x^2$ → $x^2y = x^2y$ and $7x^3y = 7x^2 \cdot x \cdot y$ and $14x^2 = 2 \cdot 7 \cdot x^2$ → $\boxed{\text{GCF is } x^2}$

STEPS FOR FACTORING A MONOMIAL FROM A POLYNOMIAL

1. Determine the greatest common factor of ALL terms in the polynomial.
2. Express each term as the product of the GCF and its other factors.
3. Use the distributive property to factor out the GCF.

Example 2: Factor each polynomial using the GCF:

a. $9x+9$ $= 9\cdot x + 9\cdot 1$
$= \boxed{9(x+1)}$

b. $32x-24$ $= 8\cdot 4x - 8\cdot 3$
$= \boxed{8(4x-3)}$

c. $18x^3y^2-12x^3y-24x^2y$ $= 6x^2y\cdot 3xy - 6x^2y\cdot 2x - 6x^2y\cdot 4$
$= 6x^2y(3xy-2x-4)$

GCF: $6x^2y$

d. $7(x+1)+21x(x+1)$ $= 7(x+1)\cdot 1 + 7(x+1)\cdot 3x$
$= \boxed{7(x+1)(1+3x)}$

GCF: $7\cdot(x+1)$

FACTORING BY GROUPING

1. Group terms that have a common monomial factor. There will usually be 2 groups. Sometimes the terms must be rearrange.
2. Factor out the common monomial factor from each group.
3. Factor out the remaining common binomial factor (if one exists).

Example 3: Factor by grouping:

a. $x^2+3x+5x+15$

$= x(x+3)+5(x+3)$

$= \boxed{(x+3)(x+5)}$

c. $xy-6x+2y-12$

$= x(y-6)+2(y-6)$

$= \boxed{(y-6)(x+2)}$

b. $x^3-3x^2+4x-12$

$= x^2(x-3)+4(x-3)$

$= \boxed{(x-3)(x^2+4)}$

d. $10x^2-12xy+35xy-42y^2$

$= 2x(5x-6y)+7y(5x-6y)$

$= \boxed{(5x-6y)(2x+7y)}$

Example 4: Factor each polynomial:

a. $x^3 - 5 + 2x^3y - 10y$

$= 1(x^3-5) + 2y(x^3-5)$

$= \boxed{(x^3-5)(1+2y)}$

c. $8x^5(x+2) - 10x^3(x+2) - 2x^2(x+2)$

$= 2x^2(x+2)\cdot 4x^3 - 2x^2(x+2)\cdot 5x - 2x^2(x+2)\cdot 1$

$= \boxed{2x^2(x+2)(4x^3-5x-1)}$

b. $7x^5 - 7x^4 + x^3 - x^2 + 3x - 3$

$= 7x^4(x-1) + x^2(x-1) + 3(x-1)$

$= \boxed{(x-1)(7x^4+x^2+3)}$

d. $12x^2 - 25$

$\boxed{\text{prime}}$

APPLICATION

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The polynomial $72x - 16x^2$ describes the height of the debris above the ground, in feet, after *x* seconds.

a. Find the height of the debris after 4 seconds.

At $x=4$:

$72(4)-16(4)^2$

$=288-256$

$=\boxed{32 \text{ feet}}$

b. Factor the polynomial.

$72x-16x^2 = 8x\cdot 9 - 8x\cdot 2x$

$=\boxed{8x(9-2x)}$

c. Use the factored form of the polynomial in part (b) to find the height of the debris after 4 seconds. Do you get the same answer as you did in part (a)? If so, does this prove that your factorization is correct?

yea!! yes!!

At $x=4$:

$8(4)(9-2(4))$

$=32(1)$

$=\boxed{32}$

Section 6.2: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS 1

When you are done with your homework you should be able to…

- π Factor trinomials of the form x^2+bx+c

WARM-UP:

Multiply:

a. $(x+1)(x+8)$

$= x(x+8)+1(x+8)$

$= x^2+8x+1x+8$

$= x^2+9x+8$

When combining like terms, we ended up with 2 positive terms

↳ sum

last term is positive (mult. 2 positives)

b. $(x-1)(x-8)$

$= x(x-8)-1(x-8)$

$= x^2-8x-1x+8$

$= x^2-9x+8$

when combining like terms, we ended up with 2 negative terms

↳ sum

positive last term (mult. 2 negatives)

c. $(x+1)(x-8)$

$= x(x-8)+1(x-8)$

$= x^2-8x+1x-8$

$= x^2-7x-8$

d. $(x-1)(x+8)$

$= x(x+8)-1(x+8)$

$= x^2+8x-1x-8$

$= x^2+7x-8$

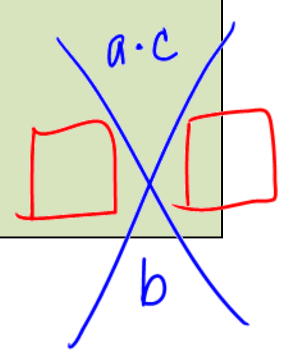
When combining like terms we had opposite signs

↳ difference

last term is negative (mult. a positive and a negative)

A STRATEGY FOR FACTORING ax^2+bx+c: USING GROUPING

1. Multiply the leading coefficient ($\underbrace{\text{in this case 1}}_{6.2}$) and the constant, __c___.
2. Find the __factors______ of __$a \cdot c$___ whose __sum____ is __b___.
3. Rewrite the __middle____ term, __b___, as a __sum____ or a __difference____ using the factors from step 2.
4. __Factor______ by __grouping_____.



ax^2+bx+c

Example 1: Factor each trinomial

a. x^2+9x+8

$a=1$, $c=8$

8 / 8, 1 / 9

$=x^2+8x+1x+8$

$=x(x+8)+1(x+8)$

$=\boxed{(x+8)(x+1)}$

b. $x^2+7x+10$

$a=1$, $c=10$

10 / 5, 2 / 7

$=x^2+5x+2x+10$

$=x(x+5)+2(x+5)$

$=\boxed{(x+5)(x+2)}$

c. $x^2-13x+40$

$a=1$, $c=40$

40 / −8, −5 / −13

$=x^2-8x-5x+40$

$=x(x-8)-5(x-8)$

$=\boxed{(x-8)(x-5)}$

d. $x^2+3x-28$

$a=1$, $c=-28$

−28 / −4, +7 / +3

$=x^2-4x+7x-28$

$=x(x-4)+7(x-4)$

$=\boxed{(x-4)(x+7)}$

e. x^2-4x-5

$a=1$, $c=-5$

−5 / −5, +1 / −4

$=x^2-5x+1x-5$

$=x(x-5)+1(x-5)$

$=\boxed{(x-5)(x+1)}$

f. $w^2+12w-64$

$= w^2 - 4w + 16w - 64$

$= w(w-4) + 16(w-4)$

$= \boxed{(w-4)(w+16)}$

$a=1$
$c=-64$

-64; -4, $+16$; $+12$

g. $y^2-15y+5$

$\boxed{\text{prime}}$

$a=1$
$c=5$

5; -15

impossible!

h. $x^2-9xy+14y^2$

$= x^2 - 7xy - 2xy + 14y^2$

$= x(x-7y) - 2y(x-7y)$

$= \boxed{(x-7y)(x-2y)}$

$a=1$
$c=14$

14; -7, -2; -9

Some ___polynomials___ can be ___factored___ using more than one ___method___. **Always begin by looking for the** ___greatest___ ___common___ ___factor___ **and, if there is one,** ___factor___ **it out!** A polynomial is ___completely___ ___factored___ when it is written as the ___product___ of ___prime___ ___polynomials___.

Example 4: Factor completely

a. $3x^2+21x+36$

$= 3[x^2 + 7x + 12]$

$= 3[x^2 + 4x + 3x + 12]$

$= 3[x(x+4) + 3(x+4)]$

$= \boxed{3(x+4)(x+3)}$

12; $+4$, $+3$; 7

b. $20x^2y-5xy-120y$

$= \boxed{5y[4x^2 - x - 24]}$

$4(-24)$
a, c

-96; -1

impossible

$$
\begin{array}{r|l}
2 & 96 \\
2 & 48 \\
2 & 24 \\
2 & 12 \\
2 & 6 \\
3 & 3 \\
 & 1
\end{array}
$$

c. $y^4 - 12y^3 + 35y^2$

$= y^2[y^2 - 12y + 35]$

$= y^2[y^2 - 7y - 5y + 35]$

$= y^2[y(y-7) - 5(y-7)]$

$= \boxed{y^2(y-7)(y-5)}$

35, −7, −5, −12

d. $(a+b)x^2 - 13(a+b)x + 36(a+b)$

$= (a+b)[x^2 - 13x + 36]$

$= (a+b)[x^2 - 9x - 4x + 36]$

$= (a+b)[x(x-9) - 4(x-9)]$

$= \boxed{(a+b)(x-9)(x-4)}$

36, −9, −4, −13

APPLICATION

You dive directly upward from a board that is 48 feet high. After *t* seconds, your height above the water is described by the polynomial $-16t^2 + 32t + 48$.

a. Factor the polynomial completely.

$-16t^2 + 32t + 48$

$= -16[t^2 - 2t - 3]$

$= -16[t^2 - 3t + 1t - 3]$

$= -16[t(t-3) + 1(t-3)]$

$= \boxed{-16(t-3)(t+1)}$

−3, −3, +1, −2

b. Evaluate both the original polynomial and its factored form for $t = 3$.

$-16t^2 + 32t + 48$

At $t = 3$: $-16(3)^2 + 32(3) + 48$

$= -144 + 96 + 48$

$= \boxed{0}$

$-16(t-3)(t+1)$

At $t = 3$: $-16(3-3)(3+1)$

$= -16(0)(4)$

$= \boxed{0}$

c. Do you get the same answer? Describe what this answer means?

yep! At 3 seconds, you'll reach the water.

Section 6.3: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS NOT 1

When you are done with your homework you should be able to…

- π Factor trinomials by trial and error
- π Factor trinomials by grouping

WARM-UP:

Factor:

a. $x^2y - xy^2$

$= \boxed{xy(x-y)}$

b. $x^2 - 14x - 51$

$= x^2 - 17x + 3x - 51$

$= x(x-17) + 3(x-17)$

$= \boxed{(x-17)(x+3)}$

-51

$-17 \quad +3$

-14

c. $2x^3 - 6x^2 + 4x$

$= 2x[x^2 - 3x + 2]$

$= 2x[x^2 - 2x - 1x + 2]$

$= 2x[x(x-2) - 1(x-2)]$

$= \boxed{2x(x-2)(x-1)}$

2

$-2 \quad -1$

-3

d. $z^2 + z - 72$

$= z^2 - 8z + 9z - 72$

$= z(z-8) + 9(z-8)$

$= \boxed{(z-8)(z+9)}$

-72

$-8 \quad +9$

$+1$

A STRATEGY FOR FACTORING $ax^2 + bx + c$: USING TRIAL AND ERROR

Assume, for the moment, that there is no __greatest__ __common__ factor other than __1__.

1. __Find__ two First __terms__ whose __product__ is __ax^2__.

2. __Find__ two Last __terms__ whose __product__ is __c__.

3. By __trial__ and __error__, perform steps 1 and 2 until the __sum__ of the Outside __product__ and the Inside __product__ is __bx__.

If __no__ such __combinations__ exist, the polynomial is __prime__.

Example 1: Factor using trial and error.

5: $5\cdot 1$ or $1\cdot 5$

8: $8\cdot 1$, $1\cdot 8$, $4\cdot 2$, $2\cdot 4$

a. $5x^2 - 14x + 8$

$= \cancel{(5x-8)(x-1)}$ $-13x$

$= \cancel{(5x-1)(x-8)}$ $-41x$

$= \boxed{(5x-4)(x-2)}$ $-14x$

b. $6x^2 + 19x - 7$

c. $3x^2 - 13xy + 4y^2$

d. $9z^2 + 3z + 2$

A STRATEGY FOR FACTORING $ax^2 + bx + c$: USING GROUPING

1. Multiply the leading coefficient and the constant, $a \cdot c$.
2. Find the factors of $a \cdot c$ whose sum is bx.
3. Rewrite the middle term, bx, as a sum or a difference using the factors from step 2.
4. Factor by grouping.

$a \cdot c$

b

Example 1: Factor using grouping.

a. $3x^2 - x - 10$

$= 3x^2 - 6x + 5x - 10$

$= 3x(x-2) + 5(x-2)$

$= \boxed{(x-2)(3x+5)}$

$a=3$
$c=-10$

-30

$-6 \quad +5$

-1

b. $8x^2 - 10x + 3$

$= 8x^2 - 6x - 4x + 3$

$= 2x(4x-3) - 1(4x-3)$

$= \boxed{(4x-3)(2x-1)}$

$a=8$
$c=3$

24

$-6 \quad -4$

-10

c. $9y^2+5y-4$

$a=9$
$c=-4$

-36
$-4 \quad +9$
$+5$

$= \underline{9y^2-4y} + \underline{9y-4}$

$= \underline{y(9y-4)} + \underline{1(9y-4)}$

$= \boxed{(9y-4)(y+1)}$

d. $12x^2+7xy-12y^2$

$a=12$
$c=-12$

-144
$-9 \quad +16$
$+7$

$= \underline{12x^2-9xy} + \underline{16xy-12y^2}$

$= \underline{3x(4x-3y)} + \underline{4y(4x-3y)}$

$= \boxed{(4x-3y)(3x+4y)}$

Example 4: Factor completely

a. $4x^2-18x-10$

$a=2$
$c=-5$

-10
$-10 \quad +1$
-9

$= 2[2x^2-9x-5]$

$= 2[\underline{2x^2-10x} + \underline{1x-5}]$

$= 2[\underline{2x(x-5)} + \underline{1(x-5)}]$

$= \boxed{2(x-5)(2x+1)}$

c. $24y^4+10y^3-4y^2$

$a=12$
$c=-2$

-24
$-3 \quad +8$
$+5$

$= 2y^2[12y^2+5y-2]$

$= 2y^2[\underline{12y^2-3y} + \underline{8y-2}]$

$= 2y^2[\underline{3y(4y-1)} + \underline{2(4y-1)}]$

$= \boxed{2y^2(4y-1)(3y+2)}$

b. $3x^3+14x^2+8x$

$a=3$
$c=8$

24
$+12 \quad +2$
14

$= x[3x^2+14x+8]$

$= x[\underline{3x^2+12x} + \underline{2x+8}]$

$= x[\underline{3x(x+4)} + \underline{2(x+4)}]$

$= \boxed{x(x+4)(3x+2)}$

d. $6(y+1)x^2+33(y+1)x+15(y+1)$

10
$+10 \quad +1$
11

$= 3(y+1)[2x^2+11x+5]$

$= 3(y+1)[\underline{2x^2+10x} + \underline{1x+5}]$

$= 3(y+1)[\underline{2x(x+5)} + \underline{1(x+5)}]$

$= \boxed{3(y+1)(x+5)(2x+1)}$

Section 6.4: FACTORING SPECIAL FORMS

When you are done with your homework you should be able to…

- Factor the difference of two squares
- Factor perfect square trinomials
- Factor the sum of two cubes
- Factor the difference of two cubes

WARM-UP:

Factor:

a. $3a^2 - ab - 14b^2$

$= 3a^2 - 7ab + 6ab - 14b^2$

$= a(3a-7b) + 2b(3a-7b)$

$= \boxed{(3a-7b)(a+2b)}$

-42

$-7 \quad +6$

-1

c. $80z^3 + 80z^2 - 60z$

$a=4,\ c=-3$

$= 20z(4z^2+4z-3)$

$= 20z[4z^2-2z+6z-3]$

$= 20z[2z(2z-1)+3(2z-1)]$

$= \boxed{20z(2z-1)(2z+3)}$

-12

$-2 \quad +6$

$+4$

b. $12x^2 - 33x + 21$

$= 3[4x^2-11x+7]$

$= 3[4x^2-7x-4x+7]$

$= 3[x(4x-7)-1(4x-7)]$

$= \boxed{3(4x-7)(x-1)}$

28

$-7 \quad -4$

-11

d. $-10x^2y^4 + 14xy^4 + 12y^4$

$= -2y^4[5x^2-7x-6]$

$= -2y^4[5x^2-10x+3x-6]$

$= -2y^4[5x(x-2)+3(x-2)]$

$= \boxed{-2y^4(x-2)(5x+3)}$

-30

$-10 \quad +3$

-7

THE DIFFERENCE OF TWO SQUARES

If __A__ and __B__ are real numbers, or __algebraic__ expressions, then

$$A^2 - B^2 = (A+B)(A-B)$$

The __difference__ of the __squares__ of __2__ __terms__ factors as the __product__ of a __sum__ and a __difference__ of those terms.

16 PERFECT SQUARES

$1 = 1^2$	$25 = 5^2$	$81 = 9^2$	$169 = 13^2$
$4 = 2^2$	$36 = 6^2$	$100 = 10^2$	$196 = 14^2$
$9 = 3^2$	$49 = 7^2$	$121 = 11^2$	$225 = 15^2$
$16 = 4^2$	$64 = 8^2$	$144 = 12^2$	$256 = 16^2$

new way

Example 1: Factor.

a. $x^2 - 144 = x^2 + 0x - 144$

6.3 way

$a=1, c=-144$

-144, -12, $+12$, 0

$= x^2 - 12x + 12x - 144$

$= x(x-12) + 12(x-12)$

$= (x-12)(x+12)$

new way:

$x^2 - 144$

$= (x)^2 - (12)^2$

$= (x+12)(x-12)$

$A = x$

$B = 12$

c. $25 - 4x^{10} = (5)^2 - (2x^5)^2$

$A = 5$

$B = 2x$

$= \boxed{(5+2x^5)(5-2x^5)}$

b. $16x^2 - 196y^2$

$= 4[4x^2 - 49y^2]$

$= 4[(2x)^2 - (7y)^2]$

$= \boxed{4(2x+7y)(2x-7y)}$

$A = 2x$

$B = 7y$

d. $18x^3 - 2x = 2x[9x^2 - 1]$

$= 2x[(3x)^2 - (1)^2]$

$= \boxed{2x(3x+1)(3x-1)}$

$A = 3x$

$B = 1$

$A^2 - B^2 = (A+B)(A-B)$

$9x^2 = A^2 \rightarrow (3x)^2 = (A)^2$

FACTORING PERFECT SQUARE TRINOMIALS

Let __A__ and __B__ be real numbers, __variables__, or __algebraic__ expressions.

1. $A^2 + 2AB + B^2 =$ __$(A+B)^2$__

2. $A^2 - 2AB + B^2 =$ __$(A-B)^2$__

- The __first__ and __Last__ terms are __squared__ of __monomials__ or __constants__.
- The __middle__ term is __twice__ the __product__ of the __expressions__ being __squared__ in the __first__ and __last__ terms.

Example 2: Factor. $A^2+2AB+B^2=(A+B)^2$ and $A^2-2AB+B^2=(A-B)^2$

a. $9x^2 + 6x + 1$ $= (3x)^2 + 2(3x)(1) + (1)^2$

$= \boxed{(3x+1)^2}$

$A = 3x$
$B = 1$
Is $6x = 2(3x)(1)$
yes!

b. $x^2 + 4x + 4$

$a=1$
$c=4$

4; $+2$, $+2$; 4

$= x^2+2x+2x+4$

$= x(x+2)+2(x+2)$

$= (x+2)(x+2)$

$= \boxed{(x+2)^2}$

c. $x^2 - 18xy + 81y^2$ $= (x)^2 - 2(x)(9y) + (9y)^2$

$A = x$
$B = 9y$
Is $18xy = 2(x)(9y)$
yes!

$= \boxed{(x-9y)^2}$

d. $2y^2 - 40y + 200$

$A = y$, $B = 10$
is $20y = 2(y)(10)$
yes!

$= 2(y^2 - 20y + 100)$

$= 2[(y)^2 - 2(y)(10) + (10)^2]$

$= \boxed{2(y-10)^2}$

FACTORING THE SUM OR DIFFERENCE OF TWO CUBES

Let _____ and _____ be real numbers, _________________, or _______________ expressions.

1. $A^3 + B^3 =$ ____________________________

2. $A^3 - B^3 =$ ____________________________

SKIP

see Movie if you're interested

Example 3: Factor.

a. $x^3 + 64$

c. $128 - 250y^3$

b. $8y^3 - 1$

d. $125x^3 + y^3$

Example 4: Factor completely

a. $25x^2 - \frac{4}{49}$

c. $(y+6)^2 - (y-2)^2$

b. $20x^3 - 5x$

d. $0.064 - x^3$

Section 6.5: A GENERAL FACTORING STRATEGY

When you are done with your homework you should be able to…

- π Recognize the appropriate method for factoring a polynomial
- π Use a general strategy for factoring polynomials

WARM-UP:

Multiply:

a. $(x+1)(x^2-x+1)$

b. $(2x-3y)(4x^2+6xy+9y^2)$

skip

A STRATEGY FOR FACTORING A POLYNOMIAL

1. If there is a ___common___ factor other than ___1___, factor out the ___GCF___.
2. Determine the ___number___ of ___terms___ in the polynomial and try factoring as follows:
   a. If there are ___2___ terms, can the ___binomial___ be factored by one of the following special forms?

   ___Difference___ of ___2___ ___squares___:

   $A^2-B^2=(A+B)(A-B)$

~~_____________ of ________ _____________:~~

~~________________ of __________ ____________:~~

b. If there are ___3___ terms, is the ___trinomial___ a ___perfect___ ___square___ ___trinomial___? If so, factor by one of the following special forms:

$A^2+2AB+B^2$ = $(A+B)^2$

$A^2-2AB+B^2$ = $(A-B)^2$

If the trinomial is ___not___ a ___perfect___ ___square___, try ___factor___ by ___trial___ and ___error___ or ___grouping___.

c. If there are ___4___ or ___more___ terms, try ___factoring___ by ___grouping___.

3. Check to see if any ___factors___ with more than one term in the ___factored___ ___polynomial___ can be factored ___further___. If so, ___factor___ completely.

4. ___Check___ by ___multiplying___.

$A^2 - B^2 = (A+B)(A-B)$

Example 1: Factor

a. $5x^4 - 45x^2$ $= 5x^2\left[x^2 - 9\right]$

$A = x$

$B = 3$

$= 5x^2\left[(x)^2 - (3)^2\right]$

$= \boxed{5x^2(x+3)(x-3)}$

$ax^2 + bx + c$

b. $4x^2 - 16x - 48$ $= 4\left[x^2 - 4x - 12\right]$

$a = 1$
$c = -12$

-12

$-6 \quad +2$

-4

$= 4\left[x^2 - 6x + 2x - 12\right]$

$= 4\left[x(x-6) + 2(x-6)\right]$

$= \boxed{4(x-6)(x+2)}$

c. $4x^5 - 64x$ $= 4x\left[x^4 - 16\right]$

Diff. of squares

$= 4x\left[(x^2)^2 - (4)^2\right]$

$= 4x(x^2+4)(x^2-4)$

$= 4x(x^2+4)\left[(x)^2 - (2)^2\right]$

$= \boxed{4x(x^2+4)(x+2)(x-2)}$

d. $x^3 - 4x^2 - 9x + 36$

$= x^2(x-4) - 9(x-4)$

$= (x-4)(x^2-9)$

$= (x-4)\left[(x)^2 - (3)^2\right]$

$= \boxed{(x-4)(x+3)(x-3)}$

e. $3x^3 - 30x^2 + 75x$

$= 3x\left[x^2 - 10x + 25\right]$

$= 3x\left[(x)^2 - 2(x)(5) + (5)^2\right]$

$= 3x(x - 5)^2$

f. $2w^5 + 54w^2$

Skip

g. $3x^4y - 48y^5$ $= 3y\left[x^4 - 16y^4\right]$

Diff of Squares

$= 3y\left[\underbrace{(x^2)^2 - (4y^2)^2}\right]$

$= 3y(x^2 + 4y^2)\underbrace{(x^2 - 4y^2)}$

$= 3y(x^2 + 4y^2)\left[(x)^2 - (2y)^2\right]$

$= \boxed{3y(x^2 + 4y^2)(x + 2y)(x - 2y)}$

h. $12x^3 + 36x^2y + 27xy^2$

$= 3x\left[4x^2 + 12xy + 9y^2\right]$

$= 3x\left[(2x)^2 + 2(2x)(3y) + (3y)^2\right]$

$= \boxed{3x(2x + 3y)^2}$

$A^2 + 2AB + B^2 = (A + B)^2$

$A = 2x$

$B = 3y$

Is $12xy = 2(2x)(3y)$

yep!

i. $12x^2(x-1)-4x(x-1)-5(x-1)$

$a=12$
$c=-5$

-60, -10, $+6$, -4

$= (x-1)\left[12x^2-4x-5\right]$

$= (x-1)\left[12x^2-10x+6x-5\right]$

$= (x-1)\left[2x(6x-5)+1(6x-5)\right]$

$= \boxed{(x-1)(6x-5)(2x+1)}$

j. $x^2+14x+49-16a^2$

$x^2+14x+49 = (x)^2+2(x)(7)+(7)^2$

$= (x+7)^2$

$= (x+7)^2-(4a)^2$ Diff. of squares

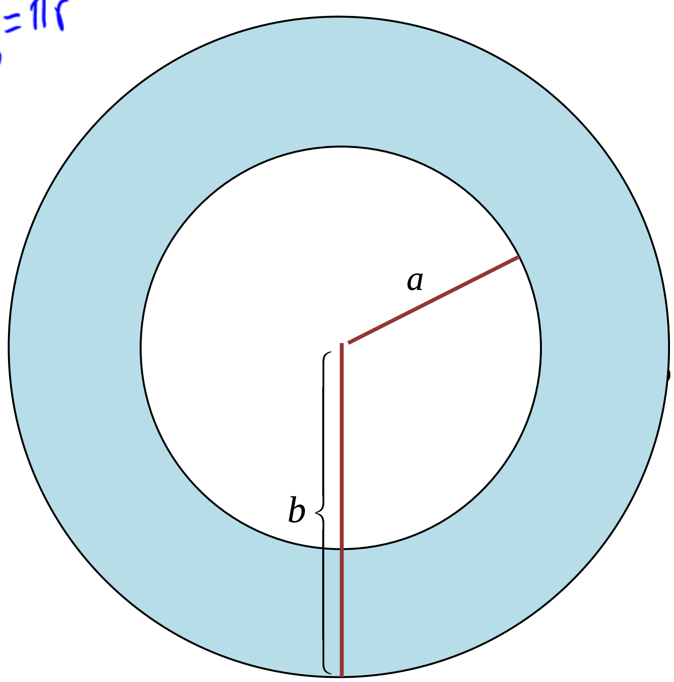
$= \left[(x+7)+4a\right]\left[(x+7)-4a\right]$

$= \boxed{(x+7+4a)(x+7-4a)}$

APPLICATION

Express the area of the shaded ring shown in the figure in terms of π. Then factor this expression completely.

$A_O = \pi r^2$



little circle
$A = \pi a^2$

big circle
$A = \pi \cdot b^2$

$A_{shaded} = \pi b^2 - \pi a^2$

$= \pi\left[b^2-a^2\right]$

$= \boxed{\pi(b-a)(b+a)}$

Section 6.6: SOLVING QUADRATIC EQUATIONS BY FACTORING

When you are done with your homework you should be able to…

- π Use the zero-product principle
- π Solve quadratic equations by factoring
- π Solve problems using quadratic equations

WARM-UP:

a. Factor:

x^2-8x+7

$= x^2-7x-1x+7$

$= x(x-7)-1(x-7)$

$= \boxed{(x-7)(x-1)}$

$a=1$
$c=7$

7

-7 -1

-8

b. Solve:

$x-7=0$

$+7 \quad +7$

$x = 7$

$\boxed{\{7\}}$

DEFINITION OF A QUADRATIC EQUATION

A __quadratic__ __equation__ in __x__ is an equation that can be written in the __standard__ __form__

$$ax^2+bx+c=0$$

where __a__, __b__, and __c__ are real numbers, with __$a \neq 0$__. A __quadratic__ __equation__ in __x__ is also called a __second__-__degree__ __polynomial__ equation in __x__.

SOLVING QUADRATIC EQUATIONS BY FACTORING

Consider the quadratic equation $x^2 - 8x + 7 = 0$. How is this different from the first warm-up?

algebraic

It is equal to 0 → warm-up is an expression, this is an equation

We can __factor__ the __left__ side of the __quadratic__ equation __$x^2 - 8x + 7$__ to get __$(x-7)(x-1)$__. If a quadratic equation has a zero on one side and a __factored__ __expression__ on the other side, it can be __solved__ using the __zero__-__product__ principle.

THE ZERO-PRODUCT PRINCIPLE

If the __product__ of two or more __algebraic__ expressions is __zero__, then __at__ __least__ one of them is __equal__ to __zero__.

If $A \cdot B = 0$ then $A = 0$ or $B = 0$

$$x^2 - 8x + 7 = 0$$

$$(x-7)(x-1) = 0$$

$$x - 7 = 0 \text{ or } x - 1 = 0$$

$$x = 7 \qquad x = 1$$

$$\boxed{\{1, 7\}}$$

Example 1: Solve the following equations:

a. $2x - 11 = 0$

$$\frac{+11 \quad +11}{}$$

$$\frac{2x}{2} = \frac{11}{2}$$

$$x = \frac{11}{2}$$

$$\boxed{\left\{\frac{11}{2}\right\}}$$

b. $x + 1 = 0$

$$x = -1$$

$$\boxed{\{-1\}}$$

c. $(2x - 11)(x + 1) = 0$

$2x - 11 = 0$ or $x + 1 = 0$

$x = \frac{11}{2}$ $\quad$ $x = -1$

$$\boxed{\left\{-1, \frac{11}{2}\right\}}$$

STEPS FOR SOLVING A QUADRATIC EQUATION BY FACTORING

1. If necessary, __rewrite__ the equation in __Standard__ form __$ax^2 + bx + c = 0$__, moving all __terms__ to one side, thereby obtaining __zero__ on the other side.
2. __Factor__.
3. Apply the __Zero__-__product__ principle, setting each __factor__ equal to __zero__.
4. __Solve__ the equations formed in step 3.
5. __Check__ the __solutions__ in the __original__ equation.

Example 2: Solve:

a. $x(x+9)=0$

$x=0$ or $x+9=0$

$x=-9$

$\boxed{\{-9,0\}}$

Check:

$x=-9$: $(-9)((-9)+9)\stackrel{?}{=}0$

$-9(0)\stackrel{?}{=}0$

$0=0$ ✓

$x=0$: $(0)((0)+9)\stackrel{?}{=}0$

$0=0$ ✓

b. $8(x-5)(3x+11)=0$

$\cancel{8=0}$ or $x-5=0$ or $3x+11=0$

$x=5$

$3x=-11$

$x=-\frac{11}{3}$

$\boxed{\left\{-\frac{11}{3},5\right\}}$

c. $x^2+x-42=0$

$a=1$
$c=-42$

-42

-6 $+7$

$+1$

$\left.\begin{array}{l} x^2-6x+7x-42=0 \\ x(x-6)+7(x-6)=0 \end{array}\right\}$ factor by grouping

$\left.\begin{array}{l} (x-6)(x+7)=0 \\ x-6=0 \text{ or } x+7=0 \end{array}\right\}$ apply zero-product principle

$x=6$ $\quad$ $x=-7$

$\boxed{\{-7,6\}}$

d. $x^2=8x$

$-8x$ $\quad$ $-8x$

$x^2-8x=0$ rewrite in standard form

$x(x-8)=0$ factor

$x=0$ or $x-8=0$ zero product principle

$x=8$

$\boxed{\{0,8\}}$

e. $4x^2 = 12x - 9$

$-12x+9 \quad -12x+9$

$4x^2-12x+9=0$ rewrite in standard form

$4x^2-6x-6x+9=0$

$2x(2x-3)-3(2x-3)=0$

$(2x-3)(2x-3)=0$

factor

$2x-3=0 \rightarrow x=\frac{3}{2}$ zero product principle

$a=4$
$c=9$

36

$-6 \quad -6$

-12

$\boxed{\left\{\frac{3}{2}\right\}}$

$2x-3=0$
$+3 \quad +3$
$2x=3$
$x=\frac{3}{2}$

f. $(x+3)(3x+5)=7$

$-7 \quad -7$

$(x+3)(3x+5)-7=0$

$x(3x+5)+3(3x+5)-7=0$

$3x^2+5x+9x+15-7=0$

$3x^2+14x+8=0$

rewrite in standard form

$3x^2+2x+12x+8=0$

$x(3x+2)+4(3x+2)=0$

$(3x+2)(x+4)=0$

factor

$a=3$
$c=8$

24

$+2 \quad +12$

14

$\rightarrow 3x+2=0$ or $x+4=0$ zero product principle

$3x=-2 \qquad x=-4$

$x=-\frac{2}{3}$

$\boxed{\left\{-4,-\frac{2}{3}\right\}}$

g. $x^3-4x=0$

$x\left[x^2-4\right]=0$

$x\left[(x)^2-(2)^2\right]=0$

$x(x+2)(x-2)=0$

factor

$\rightarrow x=0$ or $x+2=0$ or $x-2=0$ zero product principle

$x=-2 \qquad x=2$

$\boxed{\{-2,0,2\}}$

h. $(x-3)^2+2(x-3)-8=0$

$a=1$, $c=-8$

$u^2+2u-8=0$

$u^2-2u+4u-8=0$

$u(u-2)+4(u-2)=0$

$(u-2)(u+4)=0$

factor

$u-2=0$ or $u+4=0$ zero product principle

$u=2$ $\quad$ $u=-4$

Let $x-3=u$

Back substitute

$x-3=2$ or $x-3=-4$

$x=5$ $\quad$ $x=-1$

$\boxed{\{-1,5\}}$

APPLICATION

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The formula $h=-16t^2+72t$ describes the height of the debris above the ground, h, in feet, t seconds after the explosion.

a. How long will it take for the debris to hit the ground?

$h=-16t^2+72t$

$0=-16t^2+72t$

$0=-8t(2t-9)$

$-8t=0$ or $2t-9=0$

$t=0$ $\quad$ $2t=9$

$t=\frac{9}{2}$

It will take the debris $\frac{9}{2}$ seconds to hit the ground.

b. When will the debris be 32 feet above the ground?

$h=-16t^2+72t$

$32=-16t^2+72t$

$16t^2-72t+32=0$

$8[2t^2-9t+4]=0$

$8[2t^2-1t-8t+4]=0$

$8[t(2t-1)-4(2t-1)]=0$

$8(2t-1)(t-4)=0$

$\cancel{8=0}$ or $2t-1=0$ or $t-4=0$

$t=\frac{1}{2}$ $\quad$ $t=4$

$a=2$, $c=4$

The debris will be 32 feet above the ground at $\frac{1}{2}$ second and at 4 seconds.

Section 7.1: RATIONAL EXPRESSIONS AND THEIR SIMPLIFICATION

When you are done with your homework you should be able to…

- π Find numbers for which a rational expression is undefined
- π Simplify rational expressions
- π Solve applied problems involving rational expressions

WARM-UP:

a. Factor:

$$x^3 - 8x^2 + 2x - 16$$

$= x^2(x-8) + 2(x-8)$

$= \boxed{(x-8)(x^2+2)}$

b. Solve:

$$2x^2 - x - 10 = 0$$

$a = -10$
$c = 2$

-20; -5, $+4$; -1

$2x^2 - 5x + 4x - 10 = 0$

$x(2x-5) + 2(2x-5) = 0$

$(2x-5)(x+2) = 0$

$2x - 5 = 0$ or $x + 2 = 0$

$2x = 5$ $x = -2$

$x = \frac{5}{2}$

$\boxed{\left\{-2, \frac{5}{2}\right\}}$

EXCLUDING NUMBERS FROM RATIONAL EXPRESSIONS

A __rational__ expression is the __quotient__ of two __polynomials__. Rational expressions indicate __divison__ and division by __zero__ is __undefined__. This means that we __must__ __exclude__ any value or values of the __variable__ that makes a __denominator__ __zero__!

Example 1: Find all numbers for which the rational expression is undefined:

a. $\frac{5}{x}$

$x \neq 0$

$\boxed{\text{So when } x=0, \frac{5}{x} \text{ is undefined.}}$

b. $\frac{x+1}{x-4}$

$x-4 \neq 0, x \neq 4$

$\boxed{\text{when } x=4, \frac{x+1}{x-4} \text{ is undefined.}}$

c. $\frac{8x-40}{x^2+3x-28}$

$x^2+3x-28 \neq 0$

$x^2-4x+7x-28 \neq 0$

$x(x-4)+7(x-4) \neq 0$

$(x-4)(x+7) \neq 0$

-28

-4 +7

+3

$x-4 \neq 0$ or $x+7 \neq 0$

$x \neq 4$ $\quad$ $x \neq -7$

when $x=-7,4$ $\frac{8x-40}{x^2+3x-28}$ is undefined

d. $\frac{x-12}{x^2+4}$

x^2+4 is never zero, so $\frac{x-12}{x^2+4}$ is defined for all x.

SIMPLIFYING RATIONAL EXPRESSIONS

A __rational__ __expression__ is __simplified__ if its __numerator__ and __denominator__ have __no__ common __factors__ other than __1__ or __-1__.

FUNDAMENTAL PRINCIPLE OF RATIONAL EXPRESSIONS

If __P__, __Q__, and __R__ are __polynomials__ and __Q__ and __R__ are __not__ __0__,

$$\frac{P\cdot \cancel{R}^{1}}{Q\cdot \cancel{R}_{1}} = \frac{P}{Q}$$

STEPS FOR SIMPLIFYING RATIONAL EXPRESSIONS

1. Factor the numerator and the denominator completely.

2. Divide both the numerator and the denominator by any common factors.

Example 2: Simplify:

a. $\frac{4x-64}{16x} = \frac{\cancel{4}^{1}(x-16)}{\cancel{16}_{4}x} = \boxed{\frac{x-16}{4x}}$

$\frac{6}{8} = \frac{\cancel{2}^{1}\cdot 3}{\cancel{2}_{1}\cdot 4} = \boxed{\frac{3}{4}}$

Hypothetical

$\frac{x^2-16x}{4x} = \frac{\cancel{x}^{1}(x-16)}{4\cancel{x}_{1}}$

b. $\frac{6y+18}{11y+33} = \frac{6\cancel{(y+3)}^{1}}{11\cancel{(y+3)}_{1}} = \boxed{\frac{6}{11}}$

c. $\frac{x^2-12x+36}{4x-24} = \frac{x^2-6x-6x+36}{4(x-6)}$

36, −6, −6, −12

$= \frac{x(x-6)-6(x-6)}{4(x-6)}$

$= \frac{(x-6)\cancel{(x-6)}^{1}}{4\cancel{(x-6)}_{1}} = \boxed{\frac{x-6}{4}}$

d. $\dfrac{x^3+4x^2-3x-12}{x+4} = \dfrac{x^2(x+4)-3(x+4)}{x+4}$

$= \dfrac{\cancel{(x+4)}(x^2-3)}{\cancel{x+4}}$

$= \boxed{x^2-3}$

e. $\dfrac{x+5}{x-5}$ already simplified!

f. $\dfrac{x^3-1}{x^2-1}$

Skip

SIMPLIFYING RATIONAL EXPRESIONS WITH OPPOSITE FACTORS IN THE NUMERATOR AND DENOMINATOR

The __quotient__ of two __polynomials__ that have __opposite__ signs and are __additive__ __inverses__ is __-1__.

you can check to see if they are opposites: $(x-3)+(3-x)\stackrel{?}{=}0$

$x-x-3+3\stackrel{?}{=}0$

$0=0$

yes

Example 3: Simplify:

a. $\dfrac{x-3}{3-x} = \dfrac{x-3}{-1(-3+x)} = \dfrac{\cancel{x-3}}{-(\cancel{x-3})} = \boxed{-1}$

b. $\dfrac{9x-15}{5-3x} = \dfrac{3\cancel{(3x-5)}}{-1\cancel{(3x-5)}} = \boxed{-3}$

c. $\dfrac{x^2-4}{2-x} = \dfrac{(x+2)\cancel{(x-2)}}{-1\cancel{(x-2)}} = \boxed{-(x+2) \text{ or } -x-2}$

SIDE NOTE

$$
\begin{array}{r}
-x-2 \\
-x+2\,\overline{)\,x^2+0x-4} \\
-(x^2-2x) \\
\hline
2x-4 \\
-(2x-4) \\
\hline
0
\end{array}
$$

APPLICATION

A company that manufactures small canoes has costs given by the equation

$$C = \frac{20x + 20000}{x}$$

in which x is the number of canoes manufactured and C is the cost to manufacture each canoe.

a. Find the cost per canoe when manufacturing 100 canoes.

watch the movie :)

b. Find the cost per canoe when manufacturing 10000 canoes.

c. Does the cost per canoe increase or decrease as more canoes are manufactured?

Section 7.2: MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Multiply rational expressions
- π Divide rational expressions

WARM-UP:

Simplify:

a. $\dfrac{a^2-2ab+b^2}{a^2-b^2}$

b. $\dfrac{x^2-3x+2}{x-1}$

MULTIPLYING RATIONAL EXPRESSIONS

If P, Q, R, and S are polynomials, where $Q \neq 0$ and $S \neq 0$, then

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S}$$

The product of two rational expressions is the product of their numerators, divided by the product of their denominators.

STEPS FOR MULTIPLYING RATIONAL EXPRESSIONS

1. Factor all numerators and denominators.
2. Divide numerators and denominators by common factors.
3. Multiply the remaining factors in the numerator and multiply the remaining factors in the denominator.

Example 1: Multiply.

a. $\frac{x-5}{3}\cdot\frac{18}{x-8} = \frac{(x-5)(\cancel{18}^{6})}{(\cancel{3}_{1})(x-8)} = \boxed{\frac{6(x-5)}{x-8}}$

c. $\frac{9y+21}{y^2-2y}\cdot\frac{y-2}{3y+7} = \frac{3\cancel{(3y+7)}\cancel{(y-2)}}{y\cancel{(y-2)}\cancel{(3y+7)}} = \boxed{\frac{3}{y}}$

b. $\frac{x}{5}\cdot\frac{30}{x-4} = \frac{(x)(\cancel{30}^{6})}{(\cancel{5}_{1})(x-4)} = \boxed{\frac{6x}{x-4}}$

d. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$

$= \frac{x^2+2x+3x+6}{x^2-2x+3x-6}\cdot\frac{(x+3)(x-3)}{x^2-3x+2x-6}$

$= \frac{x(x+2)+3(x+2)}{x(x-2)+3(x-2)}\cdot\frac{(x+3)(x-3)}{x(x-3)+2(x-3)}$

$= \frac{\cancel{(x+2)}\cancel{(x+3)}(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x+3)}\cancel{(x-3)}\cancel{(x+2)}}$

$= \boxed{\frac{x+3}{x-2}}$

+6: +2, +3, +5

−6: −2, +3, +1

−6: −3, +2, −1

DIVIDING RATIONAL EXPRESSIONS

If P, Q, R, and S are polynomials, where $Q\neq 0$, $R\neq 0$, and $S\neq 0$, then $\frac{P}{Q}\div\frac{R}{S}=\frac{P}{Q}\cdot\frac{S}{R}=\frac{P\cdot S}{Q\cdot R}$

The quotient of two rational expressions is the product of the first expression and the reciprocal of the second.

Example 2: Divide.

a. $\frac{x}{3}\div\frac{3}{8}$ $=\frac{x}{3}\cdot\frac{8}{3}$ $=\boxed{\frac{8x}{9}}$

b. $\frac{x+5}{7}\div\frac{4x+20}{9}$

$=\frac{x+5}{7}\cdot\frac{9}{4(x+5)}$

$=\frac{(\cancel{x+5})(9)}{(7)(4)(\cancel{x+5})}$

$=\boxed{\frac{9}{28}}$

c. $\frac{y^2-2y}{15}\div\frac{y-2}{5}$

$=\frac{y(y-2)}{15}\cdot\frac{5}{y-2}$

$=\frac{(y)(\cancel{y-2})(\cancel{5})}{(\cancel{15})(\cancel{y-2})}$ 3

$=\boxed{\frac{y}{3}}$

d. $\frac{x^2-4y^2}{x^2+3xy+2y^2}\div\frac{x^2-4xy+4y^2}{x+y}$

$=\frac{(x+2y)(x-2y)}{x^2+2xy+1xy+2y^2}\cdot\frac{x+y}{x^2-2xy-2xy+4y^2}$

$=\frac{(x+2y)(x-2y)}{x(x+2y)+y(x+2y)}\cdot\frac{x+y}{x(x-2y)-2y(x-2y)}$

$=\frac{(\cancel{x+2y})(\cancel{x-2y})(\cancel{x+y})}{(\cancel{x+2y})(\cancel{x+y})(\cancel{x-2y})(x-2y)}$ $\rightarrow=\boxed{\frac{1}{x-2y}}$

2, +2, +1, +3   +4, -2, -2, -4

Example 3: Perform the indicated operation or operations.

e. $$\frac{5x^2 - x}{3x+2} \div \left(\frac{6x^2 + x - 2}{10x^2 + 3x - 1} \cdot \frac{2x^2 - x - 1}{2x^2 - x} \right)$$

f. $$\frac{5xy - ay - 5xb + ab}{25x^2 - a^2} \div \frac{y^3 - b^3}{15x + 3a}$$

Section 7.4: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH DIFFERENT DENOMINATORS

When you are done with your homework you should be able to…

- π Find the least common denominator
- π Add and subtract rational expressions with different denominators

WARM-UP: Perform the indicated operation and simplify.

1. $\frac{-3}{8}+\frac{5}{12}$

b. $\frac{x+2}{x^2+x}+\frac{-1}{x^2+x}$

FINDING THE LEAST COMMON DENOMINATOR (LCD)

The ________________ _________________ denominator of several

________________ __________________ is a __________________ consisting

Of the ________________ of all __________________ ________________ in

the __________________, with each _______________ raised to the greatest

________________ of its occurrence in any denominator.

FINDING THE LEAST COMMON DENOMINATOR

1. ________________ each ______________ completely.

2. List the factors of the first ________________________.

3. Add to the list in step 2 any _________________ of the second denominator that do not appear in the list. Repeat this step for all denominators.

4. Form the _________________ of the _________________ from the list in step 3. This product is the LCD.

Example 1: Find the LCD of the rational expressions.

a. $\frac{11}{25x^2}$ and $\frac{17}{35x}$

b. $\frac{7}{y^2-49}$ and $\frac{12}{y^2-14y+49}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS THAT HAVE DIFFERENT DENOMINATORS

1. Find the __________ of the ___________________ ___________________.

2. Rewrite each rational expression as an ___________________ expression whose ___________________ is the ____________.

3. Add or subtract _________________, placing the resulting expression over the LCD.

4. If possible, ________________ the resulting rational expression.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{5}{6x}+\frac{7}{8x}$

b. $3+\frac{1}{x}$

c. $\frac{2}{3x}+\frac{x}{x+3}$

d. $\frac{y}{y-5}-\frac{y-5}{y}$

e. $\frac{3x+7}{x^2-5x+6}-\frac{3}{x-3}$

f. $\frac{5}{x^2-36}+\frac{3}{(x+6)^2}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WHEN DENOMINATORS CONTAIN OPPOSITE FACTORS

When one denominator contains the _______________ factor of the other, first ___________________ either rational expression by ____________. Then apply the ___________________ for __________________ or __________________ rational expressions that have ___________________ ________________________.

Example 3: Add or subtract as indicated. Simplify the result, if possible.

a. $\dfrac{x+7}{4x+12}+\dfrac{x}{9-x^2}$

b. $\dfrac{5x}{x^2-y^2}-\dfrac{2}{y-x}$

c. $\dfrac{7y-2}{y^2-y-12}+\dfrac{2y}{4-y}+\dfrac{y+1}{y+3}$

Section 7.5: COMPLEX RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Simplify complex rational expressions by dividing
- π Simplify complex rational expressions by multiplying by the LCD

WARM-UP: Perform the indicated operation. Simplify, if possible.

1. $\frac{x+1}{x}+\frac{3x}{x+1}$

2. $\frac{x^2+x}{x^2-4}\div\frac{12x}{2x-4}$

SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY DIVIDING

1. If necessary, add or subtract to get a _____________ rational expression in the ___________________.
2. If necessary, add or subtract to get a _____________ rational expression in the ___________________.
3. Perform the ___________________ indicated by the main _______________ bar: ________________ the denominator of the complex rational expression and ___________________.
4. If possible, ________________.

Let's simplify the problem below using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let's replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 1: Simplify each complex rational expression.

a. $\dfrac{\frac{4}{5}-x}{\frac{4}{5}+x}$

b. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$

c. $$\frac{\frac{8}{x^2}-\frac{2}{x}}{\frac{10}{x}-\frac{6}{x^2}}$$

d. $$\frac{\frac{1}{x-2}}{1-\frac{1}{x-2}}$$

SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY MULTIPLYING BY THE LCD

1. Find the LCD of ALL _____________ expressions within the ______________ rational expression.
2. ___________________ both the _______________ and _______________ by this LCD.
3. Use the ___________________ property and multiply each ___________ in the numerator and denominator by this ____________. ______________ each term. No _______________ expressions should remain.
4. If possible, ________________ and _________________.

Let’s simplify the earlier problem using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let's replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 2: Simplify each complex rational expression.

a. $\dfrac{4-\frac{7}{y}}{3-\frac{2}{y}}$

b. $\dfrac{\dfrac{3}{x}+\dfrac{x}{3}}{\dfrac{x}{3}-\dfrac{3}{x}}$

c. $\dfrac{\dfrac{2}{x^3y}+\dfrac{5}{xy^4}}{\dfrac{5}{x^3y}-\dfrac{3}{xy}}$

d. $$\frac{\frac{1}{x-2}}{1-\frac{1}{x-2}}$$

Example 3: Simplify each complex rational expression using the method of your choice.

a. $$\frac{\frac{3}{x+2}-\frac{3}{x-2}}{\frac{5}{x^2-4}}$$

b. $\dfrac{y^{-1}-(y+2)^{-1}}{2}$

Application:

The average rate on a round-trip commute having a one-way distance d is given by the complex rational expression $\dfrac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}}$ in which r_1 and r_2 are the average rates on the outgoing and return trips, respectively.

a. Simplify the expression.

b. Find your average rate if you drive to the campus averaging 40 mph and return home on the same route averaging 30 mph.

Section 7.6: SOLVING RATIONAL EQUATIONS

When you are done with your homework you should be able to…

- π Solve rational equations
- π Solve problems involving formulas with rational expressions
- π Solve a formula with a rational expression for a variable

WARM-UP:

Solve.

$3x^2 - 2x - 8 = 0$

SOLVING RATIONAL EQUATIONS

1. List _____________ on the variable. (Remember—no _________ in the denominator!)

2. Clear the equation of fractions by multiplying __________ sides of the equation by the LCD of _________ rational expressions in the equation.

3. ___________ the resulting equation.

4. Reject any proposed solution that is in the list of _________________ on the variable. ________________ other proposed solutions in the ________________ equation.

Example 1: Solve each rational equation.

a. $\frac{7}{2x}=\frac{5}{3x}+\frac{22}{3}$

b. $\frac{10}{y+2}=3-\frac{5y}{y+2}$

c. $\dfrac{x-1}{2x+3}=\dfrac{6}{x-2}$

d. $\dfrac{2t}{t^2+2t+1}+\dfrac{t-1}{t^2+t}=\dfrac{6t+8}{t^3+2t^2+t}$

e. $3y^{-2}+1=4y^{-1}$

SOLVING A FORMULA FOR A VARIABLE

Formulas and ____________ models frequently contain rational expressions. The goal is to get the ________________ variable ___________ on one side of the equation. It is sometimes necessary to _____________ out the variable you are solving for.

Example 2: Solve each formula for the specified variable.

a. $\frac{V_1}{V_2}=\frac{P_2}{P_1}$ for V_2

b. $z=\frac{x-\overline{x}}{s}$ for x

c. $f=\frac{f_1 f_2}{f_1+f_2}$ for f_2

Section 7.7: APPLICATIONS USING RATIONAL EQUATIONS AND PROPORTIONS

When you are done with your homework you should be able to…

- π Solve problems involving motion
- π Solve problems involving work
- π Solve problems involving proportions
- π Solve problems involving similar triangles

WARM-UP:

A motorboat traveled 36 miles downstream, with the current, in 1.5 hours. The return trip upstream, against the current, covered the same distance, but took 2 hours. Find the boat's rate in still water and the rate of the current.

PROBLEMS INVOLVING MOTION

Recall that ____________. Rational expressions appear in _____________

problems when the conditions of the problem involve the ___________ traveled.

When we isolate time in the formula above, we get

Example 1: As part of an exercise regimen, you walk 2 miles on an indoor track. Then you jog at twice your walking speed for another 2 miles. If the total time spent walking and jogging is 1 hour, find the walking and jogging rates.

Example 2: The water's current is 2 mph. A canoe can travel 6 miles downstream, with the current, in the same amount of time that it travels 2 miles upstream, against the current. What is the canoe's average rate in still water?

PROBLEMS INVOLVING WORK

In __________ problems, the number ____ represents one ________ job _______________. Equations in work problems are based on the following condition:

Example 3: Shannon can clean the house in 4 hours. When she worked with Rory, it took 3 hours. How long would it take Rory to clean the house if he worked alone?

Example 4: A hurricane strikes and a rural area is without food or water. Three crews arrive. One can dispense needed supplies in 10 hours, a second in 15 hours, and a third in 20 hours. How long will it take all three crews working together to dispense food and water?

PROBLEMS INVOLVING PROPORTIONS

A **ratio** is the quotient of two numbers or two quantities. The ratio of two numbers *a* and *b* can be written as

$$a \text{ to } b \text{ or}$$
$$a : b \text{ or}$$
$$\frac{a}{b}$$

A **proportion** is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0.$ We call *a, b, c,* and *d* the **terms** of the proportion. The cross-products *ad* and *bc* are equal.

Example 5: According to the authors of *Number Freaking,* in a global village of 200 people, 9 get drunk every day. How many of the world's 6.9 billion people (2010 population) get drunk every day?

Example 6: A person's hair length is proportional to the number of years it has been growing. After 2 years, a person's hair grows 8 inches. The longest moustache on record was grown by Kalyan Sain of India. Sain grew his moustache for 17 years. How long was each side of the moustache?

SIMILAR FIGURES

Two figures are **<u>similar</u>** if their corresponding angle measures are equal and their corresponding sides are proportional.

Example 7: A fifth-grade student is conducting an experiment to find the height of a tree in the schoolyard. The student measures the length of the tree's shadow and then immediately measures the length of the shadow that a yardstick forms. The tree's shadow measures 30 feet and the yardstick's shadow measures 6 feet. Find the height of the tree.