Section 1.8: EXPONENTS AND ORDER OF OPERATIONS

When you are done with your homework you should be able to…

- π Evaluate exponential expressions
- π Simplify algebraic expressions with exponents
- π Use the order of operations agreement
- π Evaluate mathematical models

WARM-UP:

1. Determine whether the given number is a solution of the equation.

$$\frac{5m-1}{6}=\frac{3m-2}{4}; \; -4$$

2. Write a numerical expression for each phrase. Then simplify the numerical expression.
   a. 14 added to the product of 4 and -10

   b. The quotient of -18 and the sum of -15 and 12

## DEFINITION OF A NATURAL NUMBER EXPONENT

If $b$ is a real number and $n$ is a natural number,

_________ is read "the ________ __________ of ____" or "____ to the ____ power. The expression _______ is called an _______________ _______________.

Example 1: Evaluate.

1. $(-5)^3$

2. $(-12)^2$

## ORDER OF OPERATIONS

1. Perform all _________________ within ________________ symbols
2. Evaluate all _________________ expressions.
3. Do all _________________________ and ___________________ in the order in which they occur, working from _______________ to _____________.
4. Finally, do all _________________ and __________________ using one of the following procedures:
   - Work from ______________ to ________________ and do additions and subtractions in the ________________ in which they occur.

   or
   - Rewrite subtractions as __________________ of __________________. Combine _______________ and __________________ numbers separately, and then ______________ these results.

Example 2: Simplify.

1. $40 \div 4 \cdot 2$

2. $\dfrac{-5(7-2)-3(4-7)}{-13-(-5)}$

3. $(3 \cdot 5)^2 - 3 \cdot 5^2$

4. $\left[-\dfrac{4}{7}-\left(-\dfrac{2}{5}\right)\right]\left[-\dfrac{3}{8}+\left(-\dfrac{1}{9}\right)\right]$

Example 3: Simplify each algebraic expression.

1. $-6x^2 + 18x^2$

2. $4(7x^3 - 5) - [2(8x^3 - 1) + 1]$

3. $6 - 5[8 - (2y - 4)]$

**APPLICATIONS**

In Palo Alto, CA, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers). The mathematical model $C=\frac{200x}{100-x}$ describes the cost, *C*, in tens of thousands of dollars, for removing *x* percent of the contaminants.

1. Find the cost, in tens of thousands of dollars, for removing 50% of the contaminants.

2. Find the cost, in tens of thousands of dollars, for removing 60% of the contaminants.

3. Describe what is happening to the cost of the cleanup as the percentage of contaminant removed increases.

Section 2.1: THE ADDITION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- $\pi$ Identify linear equations in one variable
- $\pi$ Use the addition property of equality to solve equations
- $\pi$ Solve applied problems using formulas

WARM-UP:

Simplify:

1. $\frac{1}{2}-\frac{2}{3}\div\frac{5}{9}+\frac{3}{10}$

2. $-40\div 5\cdot 2$

**LINEAR EQUATIONS IN ONE VARIABLE**

In Chapter 1, we learned that an ____________________ is a statement that two ____________________ expressions are _________________.We determined whether a given number is an equation's ________________ by substituting that number for each occurrence of the _________________. When the ____________________ resulted in a true statement, that ________________ was a ____________________. When the substituted number resulted in a _______________ statement, that number was ___________ a ________________.

**VOCABULARY**

**Solving an equation:** The ____________ of finding the ______________ (or _________________) that make the equation a __________ statement. These numbers are called the ________________ or ______________ of the equation, and we say that they ________________ the equation.

**DEFINITION OF A LINEAR EQUATION IN ONE VARIABLE**

A _____________ _______________ in ___________ ______________ _____ is an equation that can be written in the form

where ____, ____, and ____ are real numbers, and _____________.

Example 1: Give three examples of a linear equation in one variable.

1.

2.

3.

Example 2: Give two examples of a nonlinear equation in one variable.

1.

2.

**VOCABULARY**

**Equivalent equations:** Equations that have the _______________ solution are _______________ _________________.

**THE ADDITION PROPERTY OF EQUALITY**

The ____________ real number or ________________ expression may be __________ to ___________ sides of an _________________ without changing the equation's _____________________. That is,

Example 3: Solve the following equations. Check your solutions.

1. $y-5=-18$

2. $18+z=14$

3. $x+10.6=-9$

4. $-\frac{1}{8}+x=-\frac{1}{4}$

5. $-3x-5+4x=9$

6. $7x+3=6(x-1)+9$

## ADDING AND SUBTRACTING VARIABLE TERMS ON BOTH SIDES OF AN EQUATION

Our goal is to ________________ all the ______________ terms on one side of the equation. We can use the _______________ _______________ of ________________ to do this.

## APPLICATIONS

1. The cost, *C,* of an item (the price paid by a retailer) plus the markup, *M*, on that item (the retailer's profit) equals the selling price, *S*, of the item. The formula is $C + M = S$.

   The selling price of a television is $650. If the cost to the retailer for the television is $520, find the markup.

2. What is the difference between solving an equation such as $5y + 3 - 4y - 8 = 6 + 9$ and simplifying an algebraic expression such as $5y + 3 - 4y - 8$?

## Section 2.2: THE MULTIPLICATION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- π Use the multiplication property of equality to solve equations
- π Solve equations in the form of $-x = c$
- π Use the addition and multiplication properties to solve equations
- π Solve applied problems using formulas

WARM-UP:

Solve:

1. $5z - 12 = z + 8$

2. $x = -7(2 - x) + 18$

### THE MULTIPLICATION PROPERTY OF EQUALITY

The ____________ ______________ real number or _______________ expression may _______________ __________ sides of an ________________ without changing the ____________________. That is,

Example 1: Solve the following equations. Check your solutions.

1. $-5z = -20$

4. $-\frac{1}{8}x = 6$

2. $-51 = -y$

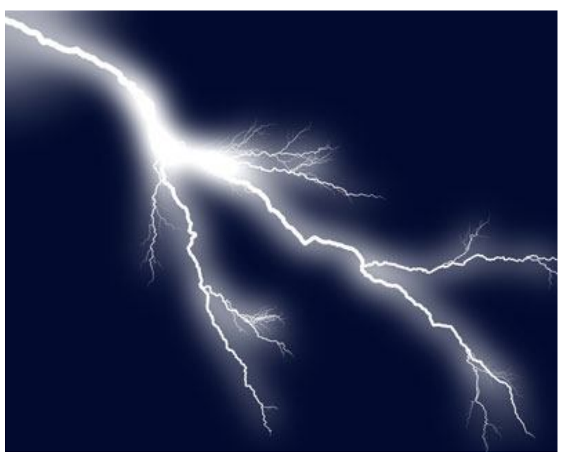
5. $6z - 3 = z + 2$

3. $8x - 3x = -45$

6. $5y + 6 = 3y - 6$

**APPLICATIONS**

The formula $M = \frac{n}{5}$ models your distance, *M,* from a lightning strike in a thunderstorm if it takes *n* seconds to hear thunder after seeing the lightning.



If you are three miles away from the lightning flash, how long will it take the sound of thunder to reach you?

Section 2.3: SOLVING LINEAR EQUATIONS

When you are done with your homework you should be able to…

- π Solve linear equations
- π Solve linear equations containing fractions
- π Identify equations with no solution or infinitely many solutions
- π Solve applied problems using formulas

WARM-UP:

Solve:

1. $-12z = 144$

2. $-x = -7x + 24$

**A STEP-BY-STEP PROCEDURE FOR SOLVING LINEAR EQUATIONS**

1. ____________ the ______________ _______________ on each side.

2. Collect all the ______________ terms on one side and all the

   ______________ terms on the other side.

3. ________________ the _________________ and _____________.

4. ________________ the proposed solution in the _____________ equation.

Example 1: Solve the following equations. Check your solutions.

1. $-z - 34 + 10z = 2 + 10z - 54$

4. $3\left(x+2\right) = x + 30$

2. $20 = 44 - 8(2 - x)$

5. $2(x - 15) + 3x = (6 + 4x) - (9x - 2)$

3. $5x - 4(x + 9) = 2x + 3$

6. $100 = -(x - 1) + 4(x - 6)$

## LINEAR EQUATIONS WITH FRACTIONS

Equations are ____________________ to solve when they do not contain ________________. To remove fractions, we can ____________________ __________ sides of the equation by the ________________ ______________ _________________ of any fractions in the equation. Remember…the ____________ is the _____________________ number that all ____________________ will _______________ into. This is often called "__________________ an equation of _________________".

Example 2: Solve the following equations. Clear the fractions first. Check your solutions.

1. $\frac{x}{2}+13=-22$

3. $\frac{3y}{4}-\frac{2}{3}=\frac{7}{12}$

2. $\frac{z}{5}-\frac{1}{2}=\frac{z}{6}$

4. $\frac{x-2}{3}-4=\frac{x+1}{4}$

## RECOGNIZING INCONSISTENT EQUATIONS AND IDENTITIES

If you attempt to _____________ an equation with ______ _______________ or one that is ______________ for _____________ real number, you will ______________ the ______________.

- π An ________________ equation with ____ __________________ results in a ______________ statement, such as _____________________.
- π An ________________ that is ___________ for _______________ real number results in a _____________ statement, such as ________________.

Example 3: Solve the following equations. Use words or set notation to identify equations that have no solution, or equations that are true for all real numbers. Check your solutions.

1. $2(x-5)=2x+10$

2. $5x-5=3x-7+2(x+1)$

3. $\frac{x}{2}+\frac{2x}{3}+3=x+3$

4. $\frac{x}{4}+3=\frac{x}{4}$

**APPLICATIONS**

The formula $p = 15 + \frac{5d}{11}$ describes the pressure of sea water, *p,* in pounds per square foot, at a depth of *d* feet below the surface.

1. The record depth for breath-held diving, by Francisco Ferreras (Cuba) off Grand Bahama Island, on November 14, 1993, involved pressure of 201 pounds per square foot. To what depth did Francisco descend on this venture? (He was underwater for 2 minutes and 9 seconds!)

2. At what depth is the pressure 20 pounds per square foot?

Section 2.4: FORMULAS AND PERCENTS

When you are done with your homework you should be able to...

- π Solve a formula for a variable
- π Express a percent as a decimal
- π Express a decimal as a percent
- π Use the percent formula
- π Solve applied problems involving percent change

WARM-UP:

Solve:

1. $4 = 0.25B$

2. $1.3 = P \cdot 26$

**SOLVING A FORMULA FOR ONE OF ITS VARIABLES**

Solving a formula for a variable means ___________________ the _____________ so that the ________________ is ____________________ on one side of the equation. To solve a formula for one of its variables, treat that ______________ as if it were the only _______________ in the ________________.

**PERIMETER**

The _______________ of a _________ ____________________ figure is the _________ of the _____________ of its _____________. Perimeter is measured in ____________ units, such as _______________, __________, ____________, or ___________________.

## PERIMETER OF A RECTANGLE

The perimeter, ______, of a rectangle with length ____ and width ____ is given by the formula

## SQUARE UNITS

A _____________ unit is a ___________, each of whose sides is __________ unit in length. The ____________ of a ______ ________________ figure is the number of ______________ _____________ it takes to fill the interior of the figure.

## AREA OF A RECTANGLE

The area, ______, of a rectangle with length ____ and width ____ is given by the formula

Example 1: Solve the following formulas for the specified variable.

1. $d = rt;\ t$
2. $P = C + MC;\ C$

Example 2: Consider a rectangle which has an area of 15 square feet and a width of 3 feet.

1. Find the length.
2. Find the perimeter.

**BASICS OF PERCENTS**

_______________ are the result of _________________ numbers as _______ of ___________. The word ________________ means _______ ____________.

**PERCENT NOTATION**

______________ means _______________.

**STEPS FOR EXPRESSING A PERCENT AS A DECIMAL NUMBER**

1. Move the ______________ point ___________ places to the ___________.
2. Remove the _____________ sign.

Example 3: Express each percent as a decimal.

1. 9.5%
2. 235%

**STEPS FOR EXPRESSING A DECIMAL NUMBER AS A PERCENT**

1. Move the _____________ point __________ places to the ___________.

2. Attach a ____________ sign.

Example 4: Express each decimal as a percent.

1. $1.75$
2. $0.01$

**A FORMULA INVOLVING PERCENT**

________________ are useful in comparing two ____________. To ______________ the number _____ to the number _____ using a percent ____, the following formula is used:

Example 5: Solve.

1. What is 12% of 50?
2. 6 is 30% of what?
3. 200 is what percent of 20?

**PERCENT INCREASE** **PERCENT DECREASE**

**APPLICATIONS**

1. The average, or mean, *A*, of four exam grades, *x, y, z,* and *w,* is given by the formula $A = \frac{x+y+z+w}{4}$.

   a. Solve the formula for *w*.

   b. Use the formula in part (a) to solve this problem: On your first three exams, your grades are 76%, 78%, and 79%: $x = 76,\ y = 78,\ \text{and } z = 79$. What must you get on the fourth exam to have an average of 80%?

2. A charity has raised $225,000, with a goal of raising $500,000. What percent of the goal has been raised?

3. Suppose that the local sales tax rate is 7% and you buy a graphing calculator for $96.
   a. How much tax is due?

   b. What is the calculator's total cost?

Section 2.5: AN INTRODUCTION TO PROBLEM SOLVING

When you are done with your homework you should be able to…

- $\pi$ Translate English phrases into algebraic expressions
- $\pi$ Solve algebraic word problems using linear equations

WARM-UP:

Solve:

A fax machine regularly sells for $380. The sale price is $266. Find the percent decrease in the machine's price.

**STEPS FOR SOLVING WORD PROBLEMS**

1. Analysis: READ the problem. Then, ________________ the problem again!!!

   Draw a _____________________ and/or make a _____________. Identify

   and name all known and unknown ___________________.

2. Translate to Mathese: Write an equation that translates, or ____________,

   the conditions of the problem.

3. Solve: ___________________ the equation. Then ______________ your

   solution.

4. Conclusion: Write your result, in _________________.

Example 1: Solve the following word problems.

1. The sum of a number and 28 is 245. Find the number.

2. Three times the sum of five and a number is 48. Find the number.

3. Eight subtracted from six times a number is 298. Find the number.

4. If the quotient of three times a number and four is decreased by three, the result is nine. Find the number.

5. A car rental agency charges $180 per week plus $0.25 per mile to rent a car. How many miles can you travel in one week for $395?

6. A basketball court is a rectangle with a perimeter of 86 meters. The length is 13 meters more than the width. Find the width and length of the basketball court.

7. This year's salary, $42,074, is a 9% increase over last year's salary. What was last year's salary?

8. A repair bill on a sailboat came to $1603, including $532 for parts and the remainder for labor. If the cost of labor is $35 per hour, how many hours of labor did it take to repair the sailboat?

Section 2.6: PROBLEM SOLVING IN GEOMETRY

When you are done with your homework you should be able to…

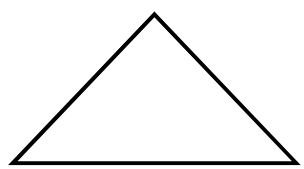
- π Solve problems using formulas for perimeter and area
- π Solve problems using formulas for a circle's area and circumference
- π Solve problems using formulas for volume
- π Solve problems involving the angles of a triangle
- π Solve problems involving complementary and supplementary angles

WARM-UP:

Solve:

After a 30% reduction, you purchase a DVD player for $98. What was the selling price before the reduction?

**COMMON FORMULAS FOR PERIMETER AND AREA**

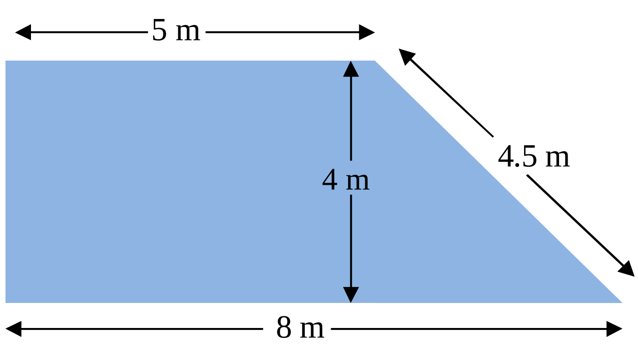


Example 1: Solve.

1. A triangle has a base of 6 feet and an area of 30 square feet. Find the triangle's height.

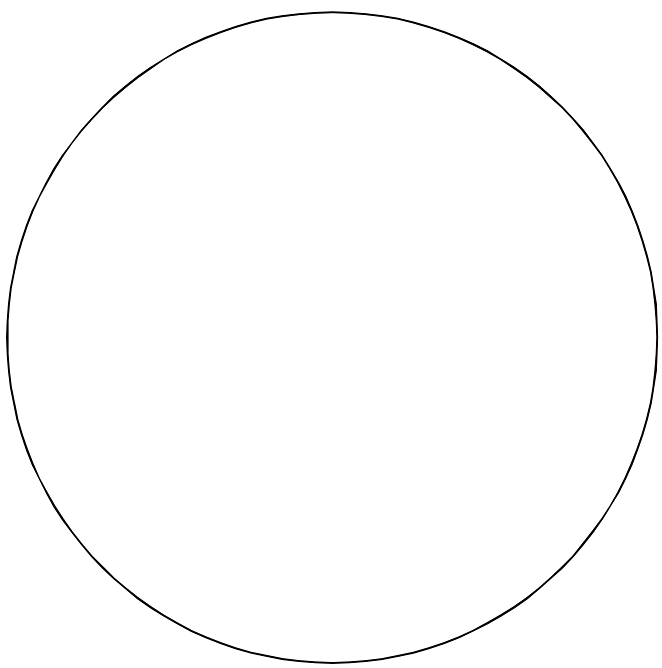
2. A rectangle has a width of 46 cm and a perimeter of 208 cm. What is the rectangle's length?

3. Find the area of the trapezoid.



**GEOMETRIC FORMULAS FOR CIRCUMFERENCE AND AREA OF A CIRCLE**

A ___________________ is the set of all ________________ in the _________ equally distant from a given point, its ________________. A ______________ (plural _______________), _____, is a line _______________ from the _______________ to any point on the ________________. For a given circle, ___________ radii have the same _______________. A _________________, ____, is a _________ segment through the ______________ whose endpoints both lie on the ______________. For a given circle, all ________________ have the _____________ length. In any circle, the length of a _________________ is _______________ the length of a ______________ and the length of a _______________ is ______________ the length of a __________________.



**Area**

**Circumference**

Example 2: Solve.

1. Find the area and circumference of a circle which has a diameter of 40 feet.

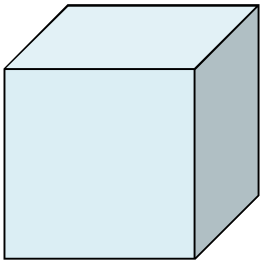
2. Which one of the following is a better buy: a large pizza with a 16-inch diameter for $12 or two small pizzas, each with a 10-inch diameter, for $12?

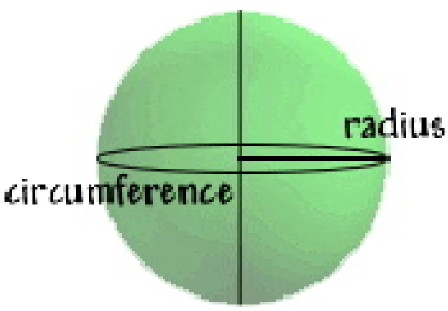
**GEOMETRIC FORMULAS FOR VOLUME**

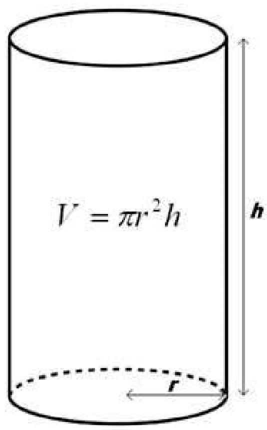
_______________ refers to the amount of _____________ occupied by a

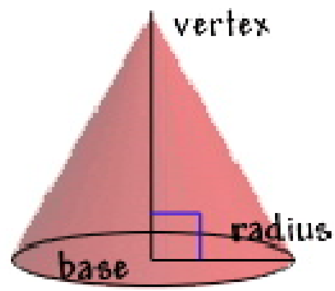
___________-____________________ figure. To measure this space, we use

____________ units.






Example 3: Solve.

1. Solve the formula for the volume of a cone for *h*.

2. A cylinder with radius 2 inches and height 3 inches has its radius quadrupled. How many times greater is the volume of the larger cylinder than the smaller cylinder?

3. Find the volume of a shoebox with dimensions 6 in x 12 in x 5 in.

**THE ANGLES OF TRIANGLES**

An _______________, symbolized by __________, is made up of two __________ that have a common ________________. The common endpoint is called the ________________. The two rays that form the angle are called its __________.

One way to __________________ angles is in _______________, symbolized by a small, raised _____________ ______. There are ____________ in a circle. _____ is _____________ of a complete rotation.

**THE ANGLES OF A TRIANGLE**

The ____________ of the ________________ of the three angles of __________ triangle is ___________.

**COMPLEMENTARY AND SUPPLEMENTARY ANGLES**

Two angles with measures having a _________ of ________ are called _____________________ angles. Two angles with measures having a _________ of ____________ are called ___________________ ________________.

Example 4: Solve.

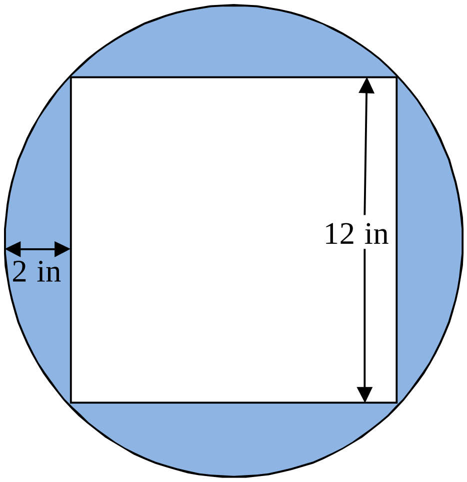
1. One angle of a triangle is three times as large as another. The measure of the third angle is 40º more than that of the smallest angle. Find the measure of each angle.

2. Find the measure of the complement of each angle.
   a. 56º
   b. 89.5º

3. Find the measure of the supplement of each angle.
   a. 177º
   b. 0.2º

4. Find the measure of the angle described.
   The measure of the angle's supplement is 52º more than twice that of its complement.

Example 5: Find the area of the shaded region.



Section 2.7: SOLVING LINEAR INEQUALITIES

When you are done with your homework you should be able to…

- π Graph the solutions of an inequality on a number line
- π Use interval notation
- π Understand properties used to solve linear inequalities
- π Solve linear inequalities
- π Identify inequalities with no solution of infinitely many solutions
- π Solve problems using linear inequalities

WARM-UP:

Solve:

Find the volume of a sphere with diameter 11 meters.

**VOCABULARY**

**<u>Linear inequality in one variable</u>:** An inequality in the form ________________, ____________________, ____________________, or ____________________ is a linear inequality in one variable. _______ means _______ _______, _______ means _______ _______ or _________ _____, _______ means _______ _______, and _______ means _______ _______ or _________ _____.

**Solving an inequality:** The ______________ of finding the ______ of ______________ that will make the inequality a __________ statement. These numbers are called the **solutions** of the ______________, and we say they **satisfy** the __________________. The ___________ of ________ solutions is called the **solution set** of the inequality.

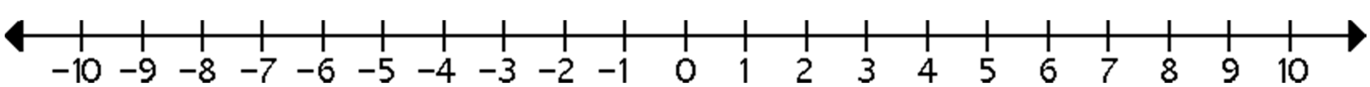
**GRAPHS OF INEQUALITIES**

There are ___________________ _____________ solutions to the inequality $x > 5$. In other words, the solution set for this inequality is all ___________ numbers which are ______________ ____________ ____________. Can we list all these numbers? What does the graph of the solution set look like? Hmmmm...

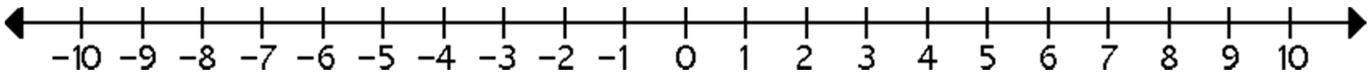
Graphs of __________________ to ______________ __________________ are shown on a _____________ __________ by shading _________ _____________ representing numbers that are __________________. _____________ _____________, ________, indicate _______________ that are _____________ and _________________, _______, indicate ______________ that are __________ __________________.

Example 1: Graph the solutions of each inequality.

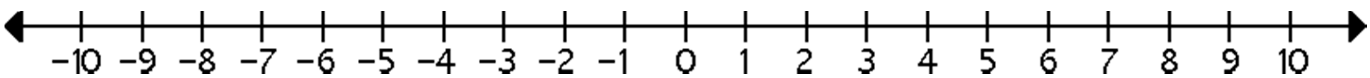
a. $x \leq 6$



b. $x > -\frac{3}{2}$




c. $-\frac{3}{2} < x \le 6$




**SOLUTION SETS OF INEQUALITIES**

| INEQUALITY | INTERVAL NOTATION | SET-BUILDER NOTATION | GRAPH |
|---|---|---|---|
| $x > a$ | | | |
| $x \ge a$ | | | |
| $x < b$ | | | |
| $x \le b$ | | | |
| $a < x < b$ | | | |
| $a \le x \le b$ | | | |
| $a < x \le b$ | | | |
| $a \le x < b$ | | | |

**PARENTHESIS ARE ALWAYS USED WITH _________ OR __________!!!**

## PROPERTIES OF INEQUALITIES

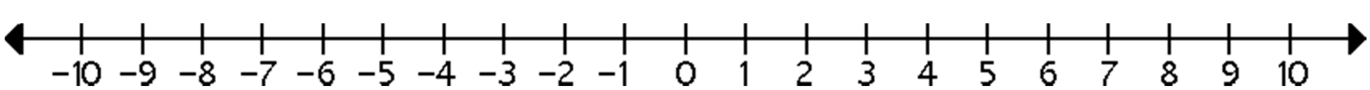
| PROPERTY | THE PROPERTY IN WORDS | EXAMPLE |
|---|---|---|
| THE ADDITION PROPERTY OF INEQUALITY<br>If ___________, then ____________.<br>If ___________, then ____________. | | |
| THE POSITIVE MULTIPLICATION PROPERTY OF INEQUALITY<br>If ___________ and ___ is positive, then ____________.<br>If ___________ and ___ is positive, then ____________. | | |
| THE NEGATIVE PROPERTY OF INEQUALITY<br>If ___________ and ___ is negative, then ____________.<br>If ___________ and ___ is negative, then ____________. | | |

## STEPS FOR SOLVING A LINEAR INEQUALITY

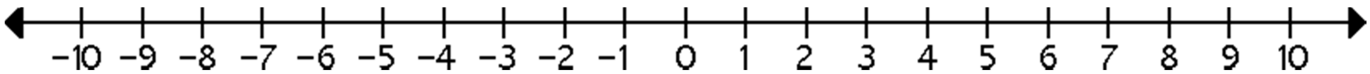
1. Simplify the ______________ ________________ on each side.
2. Use the _______________ property of _________________ to collect all the ________________ terms on one side and all the _______________ terms on the other side.
3. Use the __________________ property of _________________ to ________________ the ________________ and ___________. ________________ the ______________ of the ________________ when ____________________ or _____________________ both sides by a _________________ number.
4. Express the ________________ set in _____________ or _____-_____________ notation, and _____________ the solution set on a _______________ line.

Example 2: Solve each inequality and graph the solution.

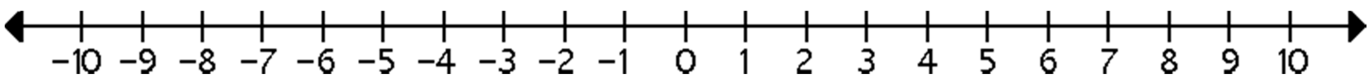
a. $x - 3 \leq 2$



b. $5x+8>2x-7$




c. $4(x+1)\geq 3x+6$




**RECOGNIZING INEQUALITIES WITH NO SOLUTION OR INFINITELY MANY SOLUTIONS**

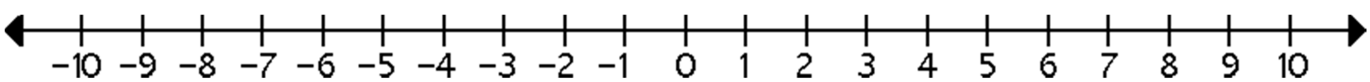
If you attempt to solve an inequality with _____ __________________ or one that is __________ for ________________ __________ number, you will __________________ the ____________________.

- An inequality with ______ ______________________ results in a ______________ statement, such as __________. The solution set is _______ or _______, the ____________ set, and the _______________ is an ________________ number line.

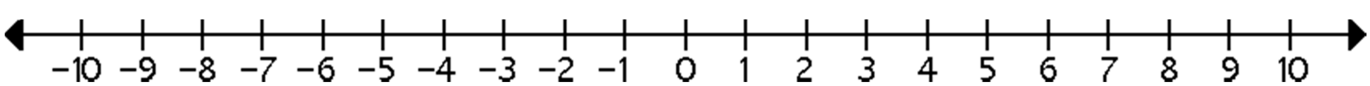
π An inequality that is _______ for ___________ _____________ number results in a ________ statement, such as ___________. The solution set is ___________ or _____________________, and the graph is a ____________ ________________ ___________________line.

Example 3: Solve each inequality and graph the solution.

a. $2\left(x+1\right)-1<2x+1$




b. $5x>2\left(x-7\right)+3x$



**APPLICATION**

On three examinations, you have grades of 88, 78, and 86. There is still a final examination, which counts as one grade.

1. In order to get an A, your average must be at least 90. If you get 100 on the final, compute your average and determine if an A in the course is possible.

2. To earn a B in the course, you must have a final average of at least 80. What must you get on the final to earn a B in the course?

Section 3.1: GRAPHING LINEAR EQUATIONS IN TWO VARIABLES

When you are done with your homework you should be able to…

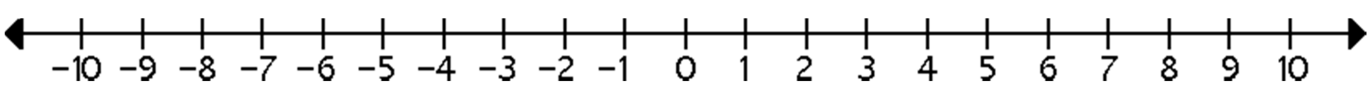
- Plot ordered pairs in the rectangular coordinate system
- Find coordinates of points in the rectangular coordinate system
- Determine whether an ordered pair is a solution of an equation
- Find solutions of an equation in two variables
- Use point plotting to graph linear equations
- Use graphs of linear equations to solve problems

WARM-UP:

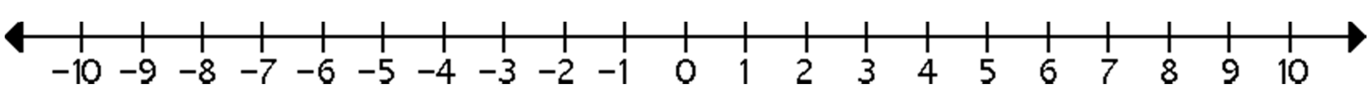
1. Find the volume of a box with dimensions ½ ft by 3 ft by 8 ft.

2. Solve the following inequalities and graph the solution sets.

   a. $x \le 6(3x-5)$

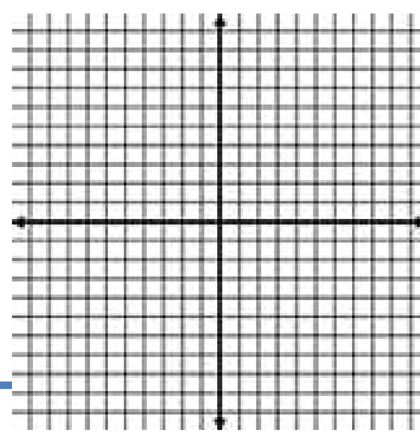



   b. $2x-1 \le 2x$



**POINTS AND ORDERED PAIRS**

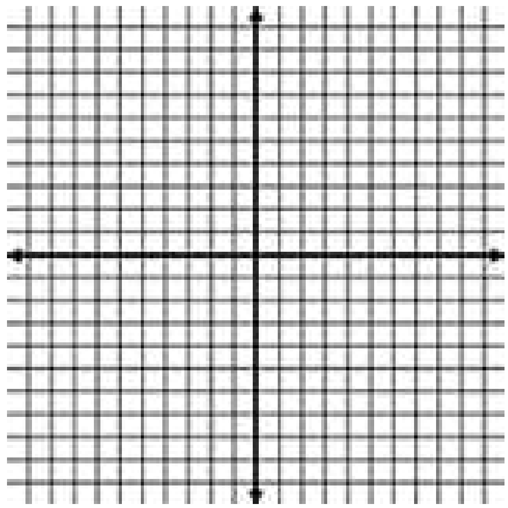
The idea of visualizing equations as geometric figures was developed by the French philosopher and mathematician _____________ ________________. This idea is the _________________ __________________ system or the ______________________ coordinate system. The rectangular coordinate system consists of ________ ______________ lines that _______________ at right ____________ at their ____________ points. The horizontal number line is the ______________ and the vertical number line is the _____________. The point of intersection is a ________________ called the ______________. Positive numbers are to the ______________ and _______________ the origin. Negative numbers are to the _____________ and _____________ the origin. The ________ divide the ___________ into ___________ regions, called _______________. The points located on the ______________ are __________ in any quadrant. Each ______________ in the rectangular coordinate system ________________ to an ______________ ___________ of real numbers, __________. The ________ number in each pair, called the ________________, denotes the _____________ and _______________ from the _____________ along the ______________. The second number, called the ____________________, denotes the ______________ distance along a ________ __________________ to the _______________ or along the ________________ itself.



Example 1: Plot the following ordered pairs.

$(2,5),\ (-3,7),\ (-2,-4)$

| | |
|---|---|
| **(2,5)** | |
| **(-3,7)** | |
| **(-2,-4)** | |



**SOLUTIONS OF EQUATIONS IN TWO VARIABLES**

A ____________________ of an __________________ in __________ variables, _____ and _____, is an ________________ __________ of real numbers with the following property: When the __________________ is substituted for ______ and the __________________ is substituted for ______ in the equation, we obtain a __________ statement.

Example 2: Determine whether each of the given points is a solution of the equation $8x+y=1$.

a. $(0,1)$

b. $(-1,3)$

c. $(2,-15)$

Example 3: Find three solutions of $2y=-x-1$.

**GRAPHING LINEAR EQUATIONS IN THE FORM $y=mx+b$**

The ___________ of the _______________ is the ________ of all _____________

whose _________________ satisfy the equation.

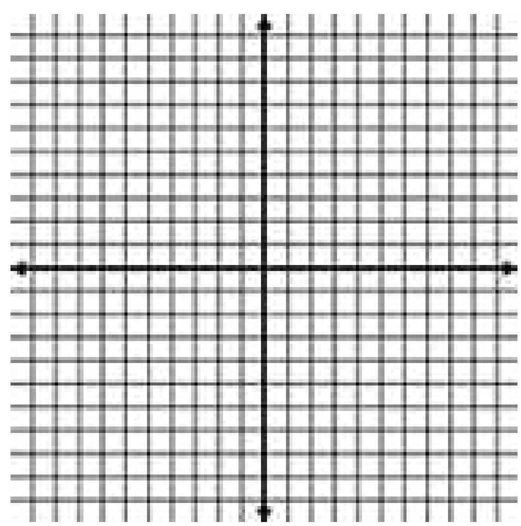
**STEPS FOR USING THE POINT-PLOTTING METHOD FOR GRAPHING AN EQUATION IN TWO VARIABLES**

1. Find several ______________ __________ that are ________________ of the equation.
2. Plot these ordered pairs as _______________ in the ___________________ coordinate system.
3. _________________ the points with a ________________ curve or ________, depending on the type of equation.

Example 3: Graph the following equations by plotting points.

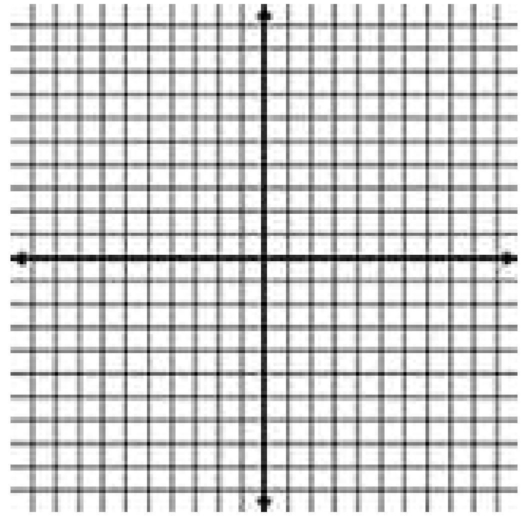
a. $y=2x$

| $x$ | $y=2x$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



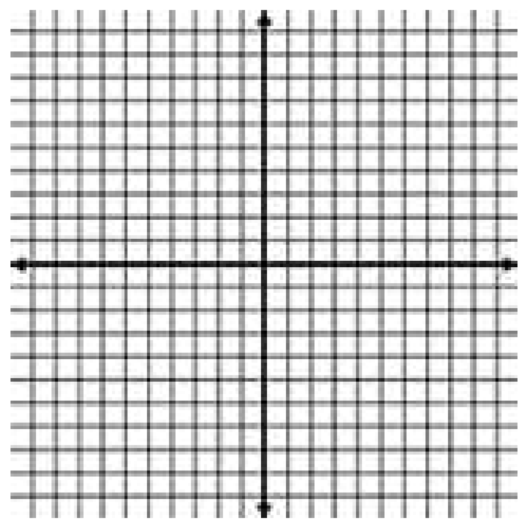
b. $y=-3x+9$

| $x$ | $y=-3x+9$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



c. $y=\frac{2}{5}x+3$

| $x$ | $y=\frac{2}{5}x+3$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



## COMPARING GRAPHS OF LINEAR EQUATIONS

If the value of ______ does not change,

π The graph of ________________ is the graph of __________ shifted

____ units ____ when ____ is a positive number.

π The graph of ________________ is the graph of __________ shifted

____ units ____ when ____ is a negative number.

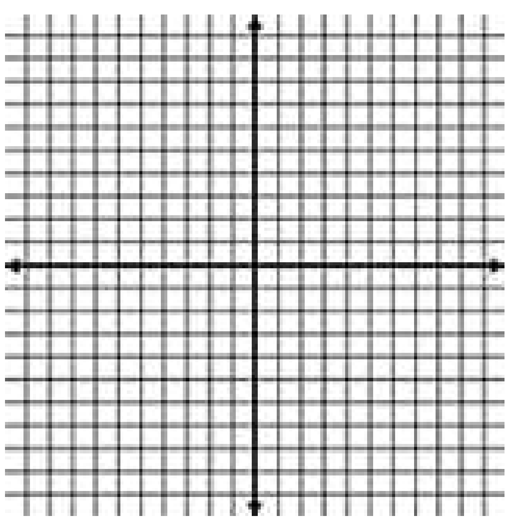
**APPLICATION**

In 1960, per capita fish consumption was 10 pounds. This increased by approximately 0.15 pound per year from 1960 through 2005. These conditions can be described by the mathematical model $F = 0.15n + 10$, where *F* is per capita fish consumption *n* years after 1960.

a. Let *n* = 0, 10, 20, 30, and 40. Make a table of values showing five solutions of the equation.

| $n$ | $F = 0.15n + 10$ | $(n, F)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

b. Graph the formula in a rectangular coordinate system.



c. Use the graph to estimate per capita fish consumption in 2020.

d. Use the formula to project per capita fish consumption in 2020.

## Section 3.2: GRAPHING LINEAR EQUATIONS USING INTERCEPTS

When you are done with your homework you should be able to…

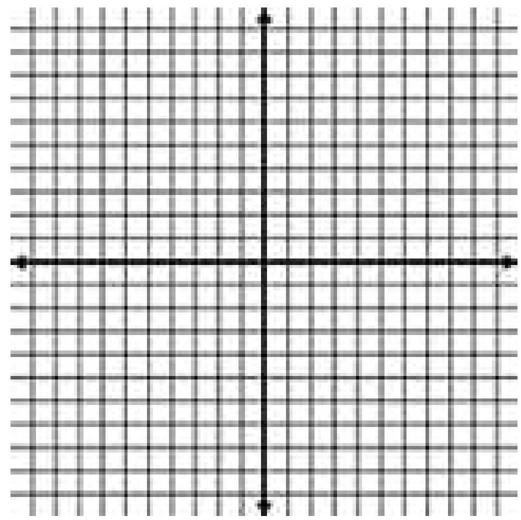
- π Use a graph to identify intercepts
- π Graph a linear equation in two variables using intercepts
- π Graph horizontal or vertical lines

WARM-UP:

Graph the following equations by plotting points.

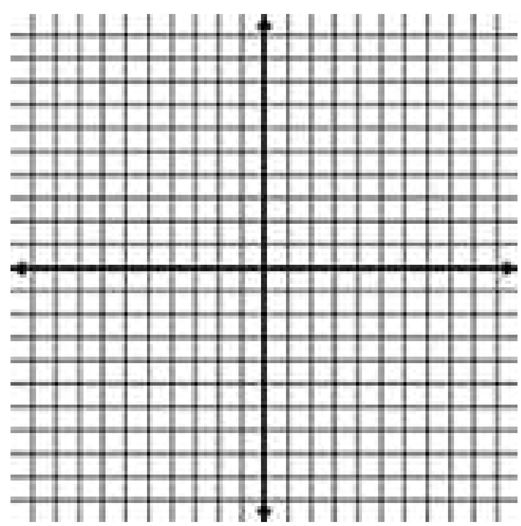
a. $y=-x$

| $x$ | $y=-x$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



b. $y=\frac{2}{3}x-7$

| $x$ | $y=\frac{2}{3}x-7$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



**INTERCEPTS**

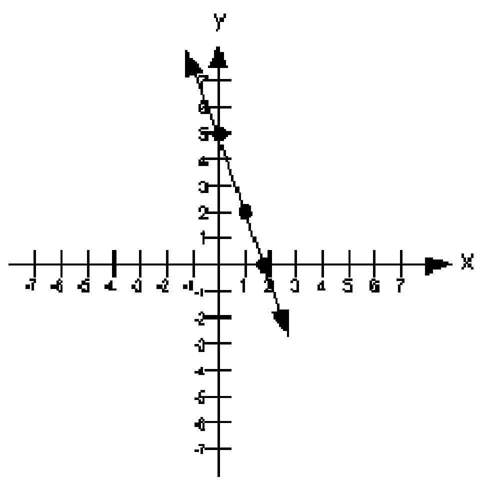
An _________________ of a graph is the __________________ of a point where the graph __________________ the _____________. **The _________________ corresponding to an _________________ is always _________!!!**

A _________________ of a graph is the __________________ of a point where the graph __________________ the _____________. **The _________________ corresponding to a _________________ is always _________!!!**

Example 1: Use the graph to identify the

a. *x*-intercept

b. *y*-intercept




**GRAPHING USING INTERCEPTS**

An equation of the form __________________, where ____, ____, and ____ are integers, is called the _______________ form of a line.

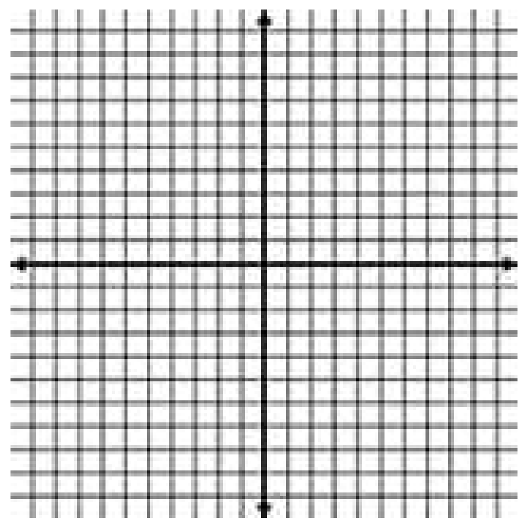
## STEPS FOR USING INTERCEPTS TO GRAPH $Ax+By=C$

1. Find the ___________________. Let _______________ and solve for ____.
2. Find the ___________________. Let _______________ and solve for ____.
3. Find a checkpoint, a ____________ ordered-pair _________________.
4. Graph the equation by drawing a _____________ through the _____ points.

Example 2: Graph using intercepts and a checkpoint.

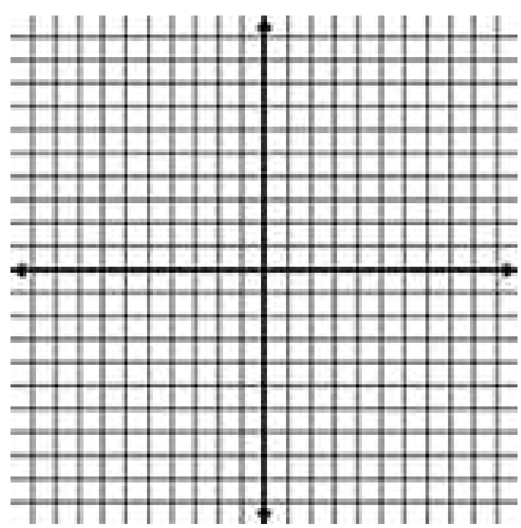
a. $x+y=6$

| | $x+y=6$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |



b. $3x-2y=-7$

| | $3x-2y=-7$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |



## EQUATIONS OF HORIZONTAL AND VERTICAL LINES

We know that the graph of any equation of the form ___________________ is a _____________ as long as ____ and ____ are not both _______. What happens if ____ or ____, but not both, is zero?

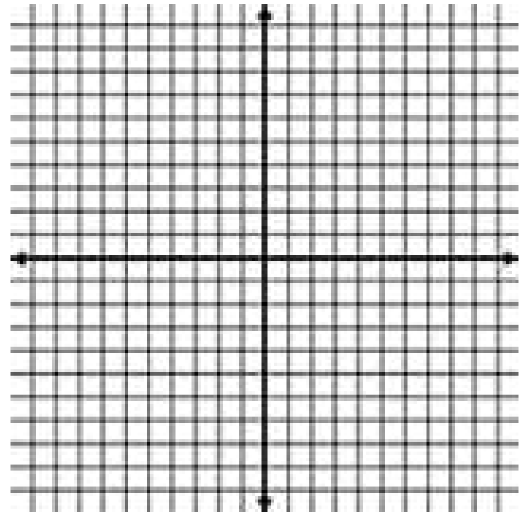
## HORIZONTAL AND VERTICAL LINES

The graph of _____________ is a ________________ line. The _____________ is ______________.

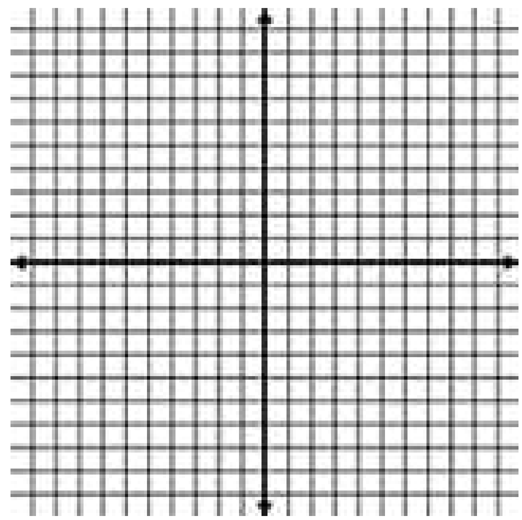
The graph of _____________ is a ________________ line. The _____________ is ______________.

Example 3: Graph.

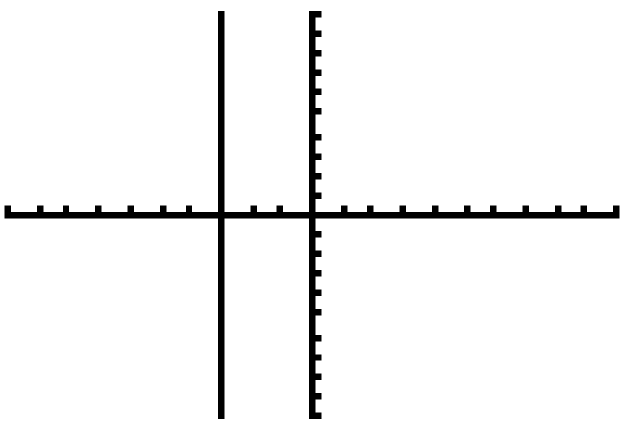
a. $y = 8$



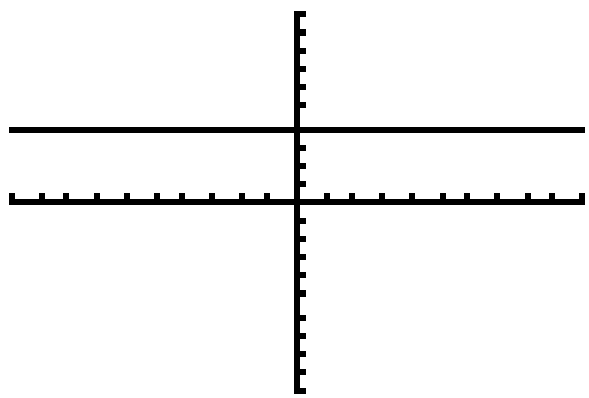
b. $12x = -60$



Example 4: Write an equation for each graph.



a.



**b.**

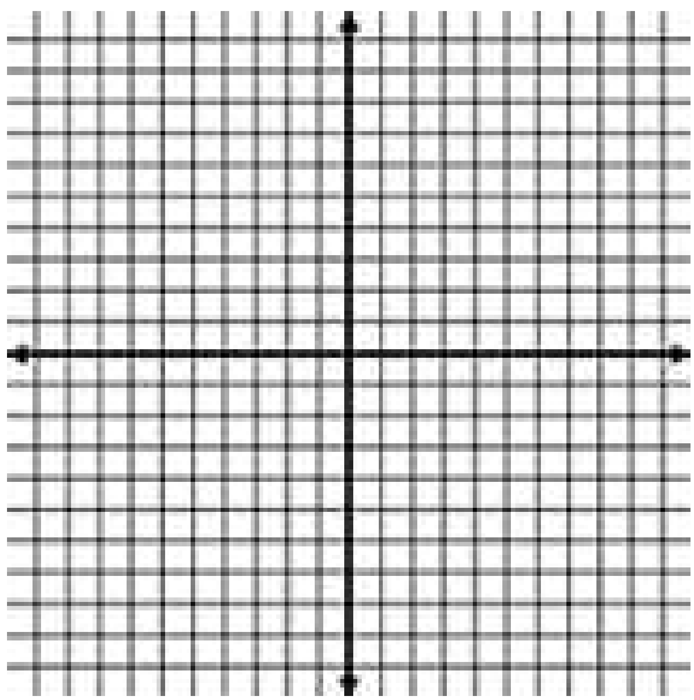
**APPLICATION**

A new car worth $24,000 is depreciating in value by $3000 per year. The mathematical model $y = -3000x + 24000$ describes the car's value, *y*, in dollars, after *x* years.

a. Find the *x*-intercept. Describe what this means in terms of the car's value.

b. Find the *y*-intercept. Describe what this means in terms of the car's value.

c. Use the intercepts to graph the linear equation.



d. Use your graph to estimate the car's value after five years.

Section 3.3: SLOPE

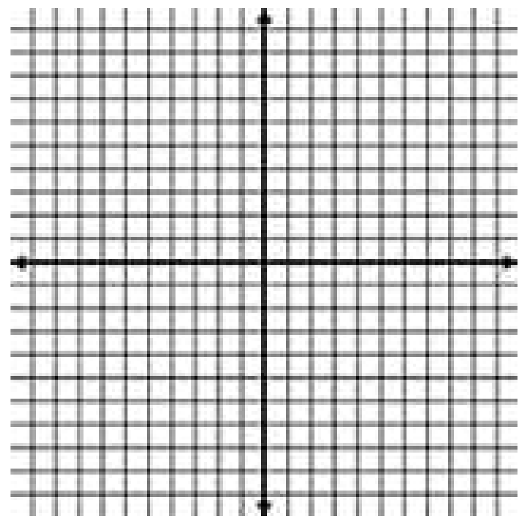
When you are done with your homework you should be able to…

- π Compute a line's slope
- π Use slope to show that lines are parallel
- π Use slope to show that lines are perpendicular
- π Calculate rate of change in applied situations

WARM-UP:

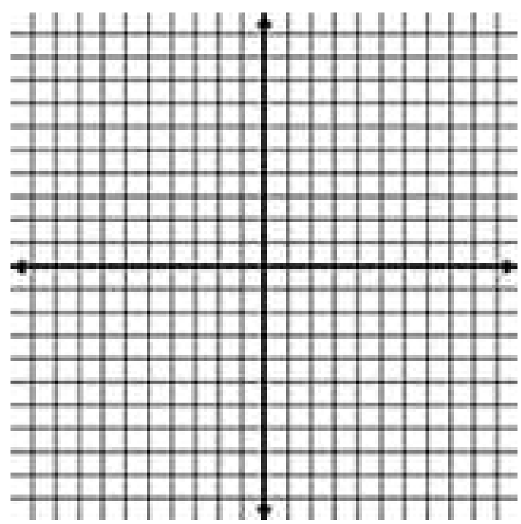
Graph each equation.

a. $y-2=0$



b. $-2x-3y=9$

| | $-2x-3y=9$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |



**THE SLOPE OF A LINE**

Mathematicians have developed a useful _______________ of the

_________________ of a line, called the ______________ of the line. Slope

compares the ___________________ change (the _________) to the

__________________ change (the _______) when moving from one _________

point to another along the line.

**DEFINITION OF SLOPE**

The ______________ of the line through the distinct points ___________ and

____________ is

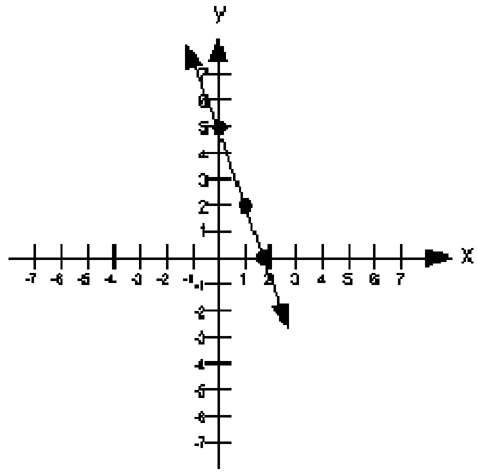
where _________________. It is common to use the letter _____ to represent the slope of a line. This letter is used because it is the first letter of the French verb monter, meaning to rise, or to ascend.

Example 1: Find the slope of the line passing through each pair of points:

a. $(-1,4)$ and $(3,-6)$

*b.* $\left(8,\frac{3}{2}\right)$ and $\left(-\frac{5}{2},7\right)$

Example 2: Use the graph to find the slope of the line




**POSSIBILITIES FOR A LINE'S SLOPE**

| POSITIVE SLOPE | NEGATIVE SLOPE | ZERO SLOPE | UNDEFINED SLOPE |
|---|---|---|---|
| | | | |

## SLOPE AND PARALLEL LINES

Two _______________________ lines that lie in the same plane are _______________. If two lines do not ________________, the ____________ of the ________________ change to the __________________ change is the _____________ for each __________. Because two parallel lines have the same _________________, they must have the same ________________.

1. If two nonvertical lines are ________________, then they have the same ______________.

2. If two distinct nonvertical lines have the same _____________, then they are ___________________.

3. Two distinct vertical lines, each with ________________ slope, are _________________.

## SLOPE AND PERPENDICULAR LINES

Two lines that _______________________ at a ___________ _____________ (_____) are said to be _______________________.

1. If two nonvertical lines are ________________, then the ______________ of their ______________ is __________.

2. If the _________________ of the ______________ of two lines is _____, then the lines are ______________________.

3. A ____________________ line having __________ slope is

______________________ to a vertical line having ______________ slope.

Example 3: Determine whether the lines through each pair of points are parallel, perpendicular, or neither.

a. $(-2,-15)$ and $(0,-3)$; $(-12,6)$ and $(6,3)$

b. $(-2,-7)$ and $(3,13)$; $(-1,-9)$ and $(5,15)$

c. $(-1,-11)$ and $(0,-5)$; $(0,-8)$ and $(12,-6)$

**APPLICATION**

Construction laws are very specific when it comes to access ramps for the disabled. Every vertical rise of 1 foot requires a horizontal run of 12 feet. What is the grade of such a ramp? Round to the nearest tenth of a percent.

## Section 3.4: THE SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

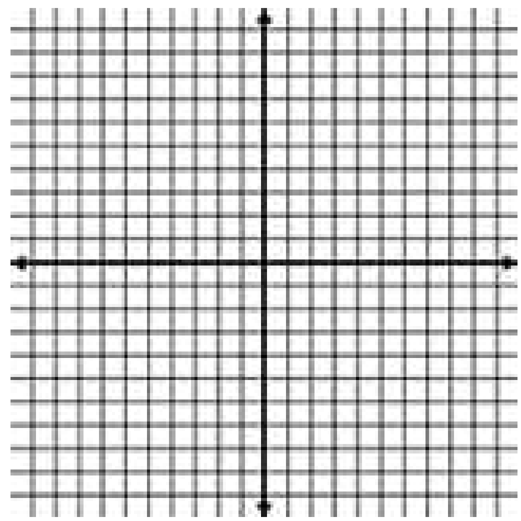
- Find a line's slope and $y$-intercept from its equation
- Graph lines in slope-intercept form
- Use slope and $y$-intercept to graph $Ax+By=C$
- Use slope and $y$-intercept to model data

WARM-UP:

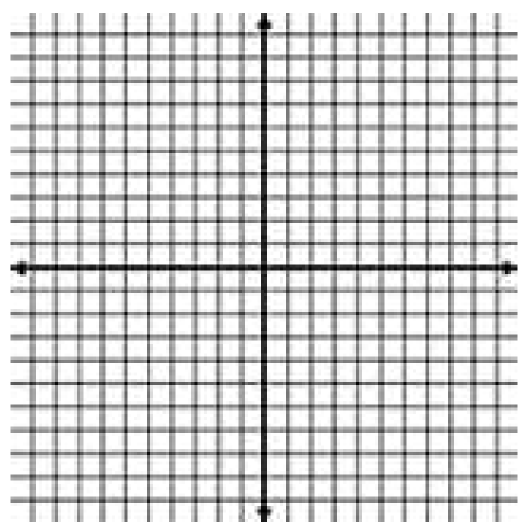
Graph each equation.

a. $4x-8y-2=0$

| | $4x-8y-2=0$ | $(x,y)$ |
|---|---|---|
| | | |
| | | |
| | | |



b. The line which passes through the points $(-1,2)$ and $(3,0)$.



## SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

The _______________ - ___________________ form of the ____________________ of a nonvertical line with slope ________ and _________________ ______ is

Example 1: Find the slope and the $y$-intercept of the line with the given equation:

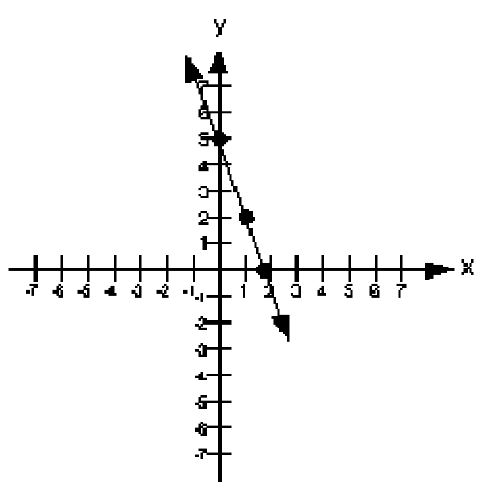
a. $y=-4x-1$

b. $6x-y=-1$

c. $y=\frac{5}{7}x+2$

d. $y=-\frac{x}{3}+\frac{2}{3}$

Example 2: Use the graph to find the equation of the line in slope-intercept form.



## GRAPHING BY USING $y = mx + b$ SLOPE AND *Y*-INTERCEPT

1. Plot the point containing the ____________________ on the _____ axis.

   This is the point ____________.

2. Obtain a second ______________ using the _____________, _____. Write

   _____ as a _______________, and use ___________ over _________,

   starting at the ________________.

3. Use a ___________________ to draw a ______________ through the two

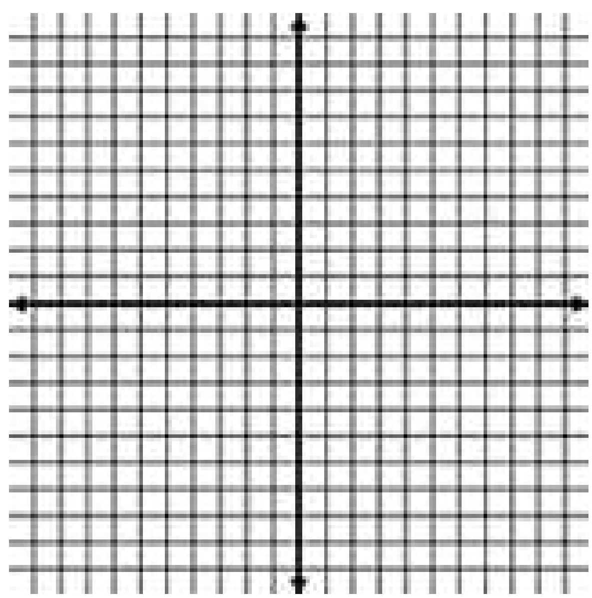
   _____________. Draw _________________________ at the ___________

   of the line to show that the line continues ___________________ in both

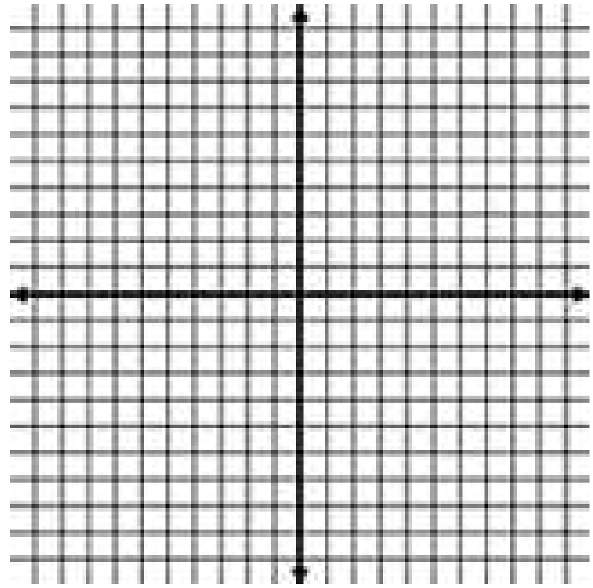
   directions.

Example 3: Graph using the slope and *y*-intercept.

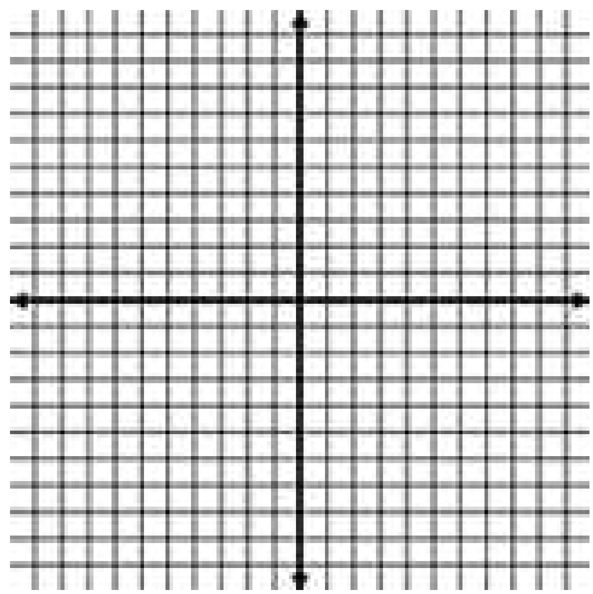
a. $y = -5x + 3$



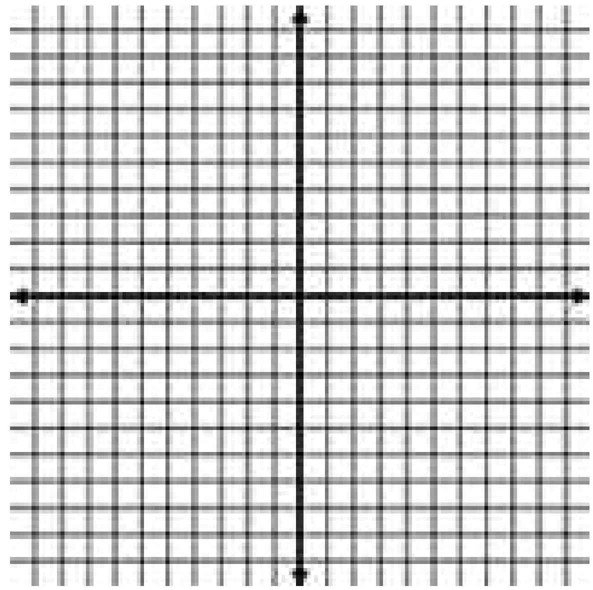
b. $10x - 5y = 25$



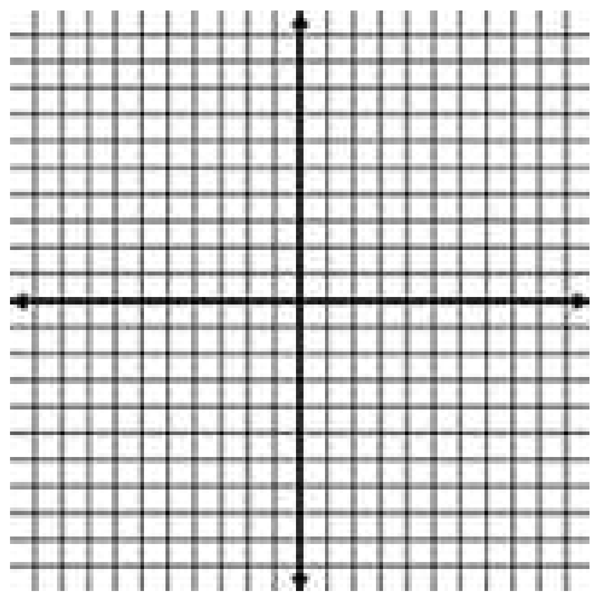
c. $x = 2y - 3$



d. $-y = x - 1$



e. $y = -\frac{6}{7}x + 4$



**APPLICATION**

Write an equation in the form of $y = mx + b$ of the line that is described.

1. The $y$-intercept is -4 and the line is parallel to the line whose equation is $2x + y = 8$.

2. The line falls from left to right. It passes through the origin and a second point with opposite $x$- and $y$-coordinates.

## Section 3.5: THE POINT-SLOPE FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

- Use the point-slope form to write equations of a line
- Find slopes and equations of parallel and perpendicular lines
- Write linear equations that model data and make predictions

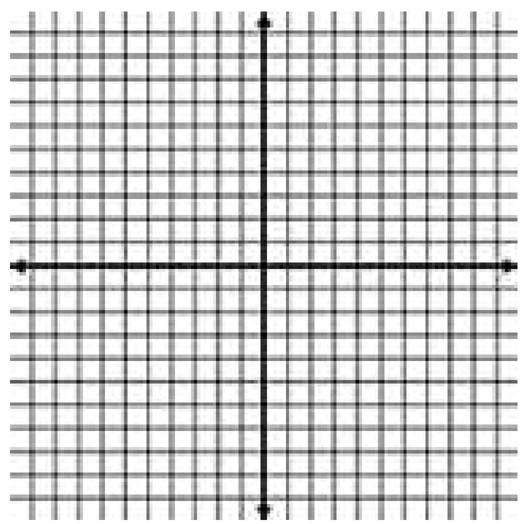
WARM-UP:

1. Simplify.

$$2-5\left[2-\left(7x+2\right)\right]$$

2. Graph the equation using the slope and *y*-intercept.

$$-\frac{x}{3}-\frac{y}{4}=1$$



## POINT-SLOPE FORM

We can use the ________________ of a line to obtain another useful form of the line's equation. Consider a nonvertical line that has slope _____ and contains the point ______________. Now let _________ represent any other ____________ on the ______________. Keep in mind that the point _____________ is _________________ and is ______________ in _____________ _____________ position. The point ________________ is ________________.

## POINT-SLOPE FORM OF THE EQUATION OF A LINE

The ______________ - _________________ form of the __________________ of a nonvertical line with slope _______ that passes through the point _________ is

Example 1: Write the point-slope form of the equation of the line with the given slope that passes through the given point.

a. $m = -2;\ (5, -11)$

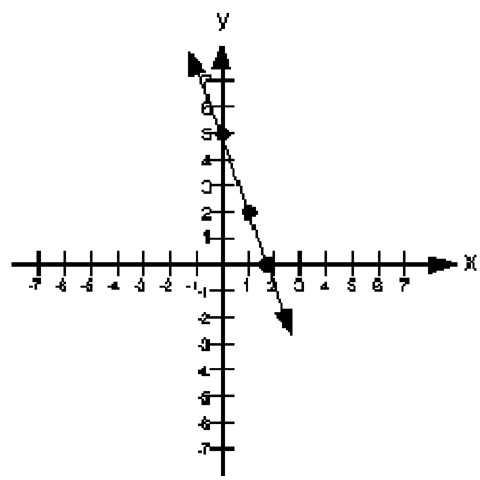
b. $m = \frac{5}{8};\ \left(\frac{1}{4}, 7\right)$

c. $m=0;\ (-21,5)$

d. $m=\text{undefined};\ (0,0)$

Example 2: Use the graph to find two equations of the line in point-slope form.

1.




2.

Now write the slope-intercept form:

1.

2.

## EQUATIONS OF LINES

| FORM | WHAT YOU SHOULD KNOW |
|---|---|
| Standard Form | Graph equations in this form using __________________ and a ______________________. |
| $y = b$ | Graph equations in this form as __________________ lines with _____ as the _____________________. |
| $x = a$ | Graph equations in this form as __________________ lines with _____ as the _____________________. |
| Slope-Intercept Form | Graph equations in this form using the__________________, _____ and the slope, ______.<br>*Start with this form when writing a ______________ equation if you know a line's _______________ and ____________________. |
| Point-Slope Form | Start with this form when writing a linear equation if you know the ______________ of the line and a _________ on the ____________ NOT containing the __________________<br>OR<br>____________ points on the line, _____________ of which contains the ___________________.<br>Calculate the _____________ using |

## PARALLEL AND PERPENDICULAR LINES

Recall that parallel lines have the _____________ ________________ and

perpendicular lines have ________________ which are _________________

________________.

Example 3: Use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

a. Passing through $(-2,-7)$ and parallel to the line whose equation is $y=-5x+4$.

b. Passing through $(-4,2)$ and perpendicular to the line whose equation is $y=-\frac{1}{3}x+7$.

c. Passing through $(5,-9)$ and parallel to the line whose equation is $x+7y=12$.

## Section 4.1: SOLVING SYSTEMS OF LINEAR EQUATIONS BY GRAPHING

When you are done with your homework you should be able to…

- $\pi$ Decide whether an ordered pair is a solution of a linear system
- $\pi$ Solve systems of linear equations by graphing
- $\pi$ Use graphing to identify systems with no solution or infinitely many solutions
- $\pi$ Use graphs of linear systems to solve problems

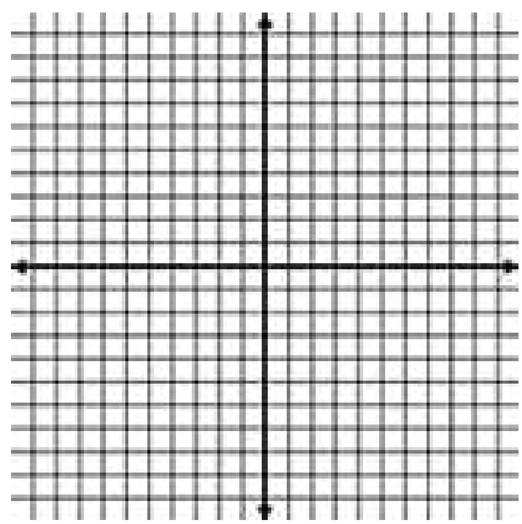
WARM-UP:

1. Determine if the given number or ordered pair is a solution to the given equation.

a. $5x+3=21;\ \frac{18}{5}$

b. $-x+2y=0;\ (4,1)$

2. Graph the line which passes through the points (0,1) and (-5,3).



## SYSTEMS OF LINEAR EQUATIONS AND THEIR SOLUTIONS

We have seen that all __________________ in the form ________________ are straight _____________ when graphed. ___________ such equations are called a _________________ of ________________ __________________ or a _________________ __________________. A __________________ to a system of two ________________ equations in two _______________ is an _________________ _____________ that _______________ _______________ equations in the ________________.

Example 1: Determine whether the given ordered pair is a solution of the system.

a.

$(-2,-5)$

$6x-2y=-2$

$3x+\ \ y=-11$

b.

$(10,7)$

$6x-\ \ 5y=25$

$4x+15y=13$

## SOLVING LINEAR SYSTEMS BY GRAPHING

The ______________ of a __________________ of two linear equations in _______ variables can be found by ________________ ______________ of the __________________ in the ________________ rectangular __________________ system. For a system with ____________ solution, the ___________________ of the point of ____________________ give the _____________________ solution.

**STEPS FOR SOLVING SYSTEMS OF TWO LINEAR EQUATIONS IN TWO VARIABLES, *x* AND *y*, BY GRAPHING**

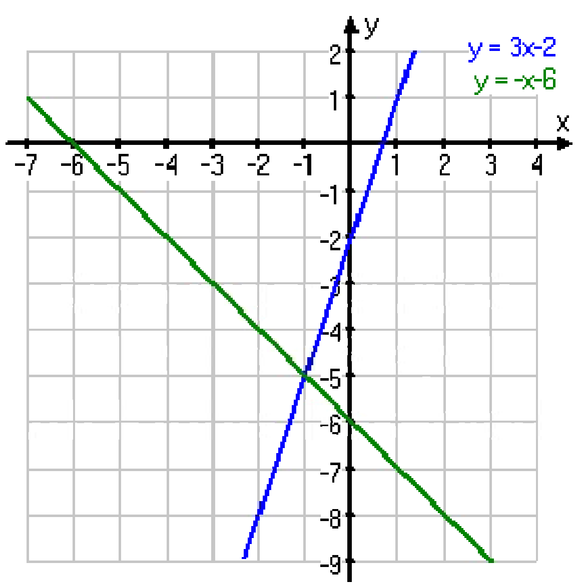
1. Graph the first ______________________.

2. ________________ the second equation on the _______________ set of ____________.

3. If the _______________ representing the __________ graphs ______________ at a _______________, determine the __________________ of this point of intersection. The ______________ ___________ is the __________________ of the ________________.

4. __________________ the __________________ in ______________ equations.

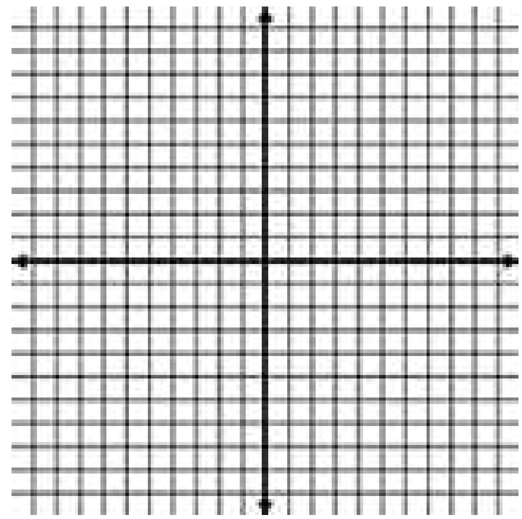
Example 2: Use the graph below to find the solution of the system of linear equations.



Example 3: Solve each system by graphing. Use set notation to express solution sets.

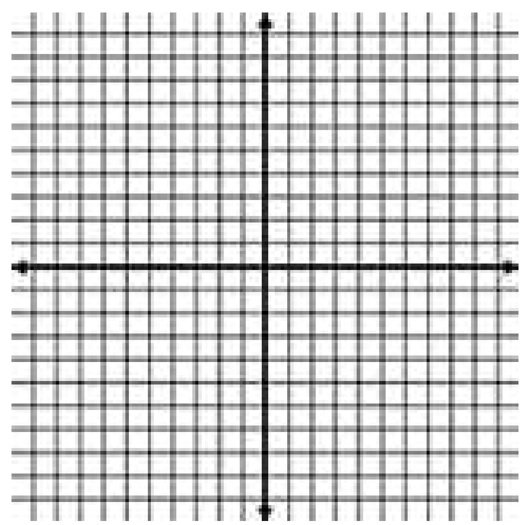
a.

$$x + y = 2$$
$$x - y = 4$$



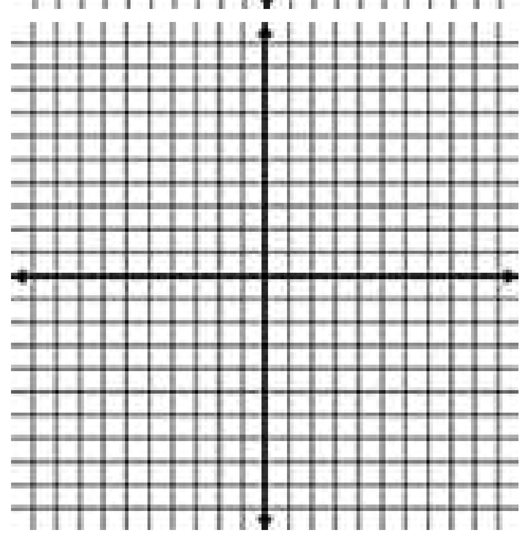
b.

$$y = 3x - 4$$
$$y = -2x + 1$$



c.

$$x + y = 6$$
$$y = -3$$



## LINEAR SYSTEMS HAVING NO SOLUTION OR INFINITELY MANY SOLUTIONS

We have seen that a ____________________ of linear equations in __________ variables represents a _______________ of _______________. The lines either __________________ at ___________ point, are _______________, or are ______________________. Thus, there are ________________ possibilities for the _______________ of solutions to a system of two linear equations.

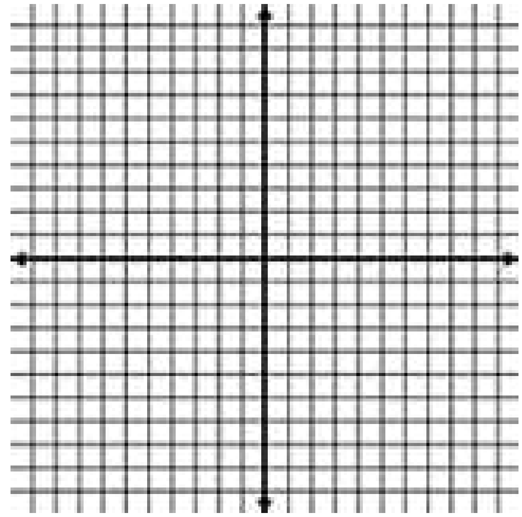
## THE NUMBER OF SOLUTIONS TO A SYSTEM OF TWO LINEAR EQUATIONS

| NUMBER OF SOLUTIONS | WHAT THIS MEANS GRAPHICALLY |
|---|---|
| Exactly ______________ ordered pair solution. | The two lines _________________ at _________ point. This is a __________________ system. |
| ________ Solution | The two lines are __________________. This is an __________________ system. |
| ___________________ many solutions | The two lines are __________________. This is a system with ________________ equations. |

Example 4: Solve each system by graphing. If there is no solution or infinitely many solutions, so state. Use set notation to express solution sets.

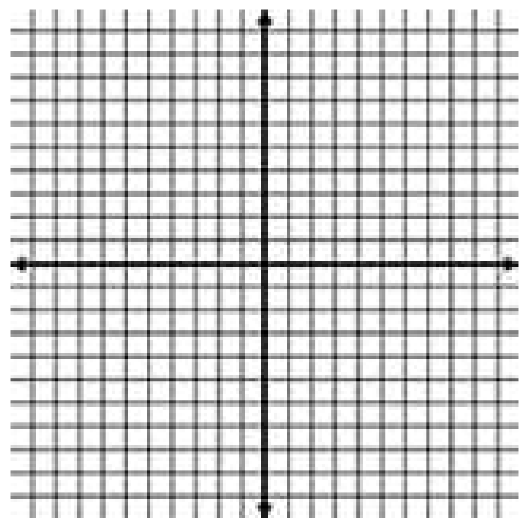
a.

$$\begin{aligned} x + y &= 4 \\ 2x + 2y &= 8 \end{aligned}$$



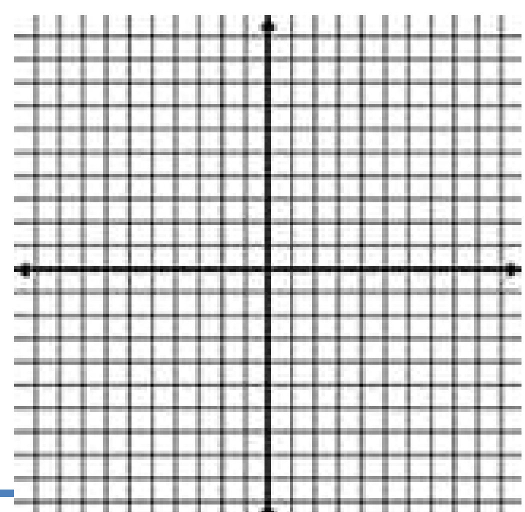
b.

$$\begin{aligned} y &= 3x - 1 \\ y &= 3x + 2 \end{aligned}$$



c.

$$\begin{aligned} 2x - y &= 0 \\ y &= 2x \end{aligned}$$

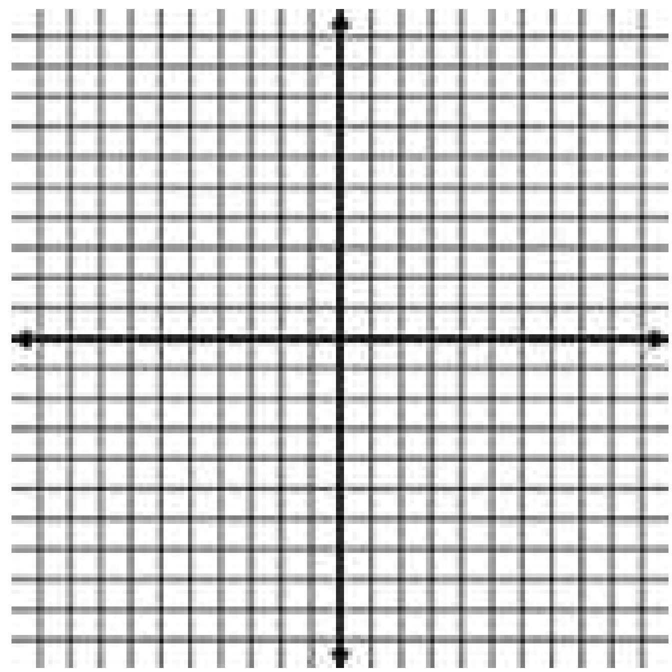


**APPLICATION**

A band plans to record a demo. Studio A rents for \$100 plus \$50 per hour. Studio B rents for \$50 plus \$75 per hour. The total cost, *y,* in dollars, of renting the studios for *x* hours can be modeled by the linear system

$$y = 50x + 100$$
$$y = 75x + 50$$

a. Use graphing to solve the system. Extend the *x*-axis from 0 to 4 and let each tick mark represent 1 unit (one hour in a recording studio). Extend the *y*-axis from 0 to 400 and let each tick mark represent 100 units (a rental cost of \$100).



b. Interpret the coordinates of the solution in practical terms.

When you are done with your 4.2 homework you should be able to…

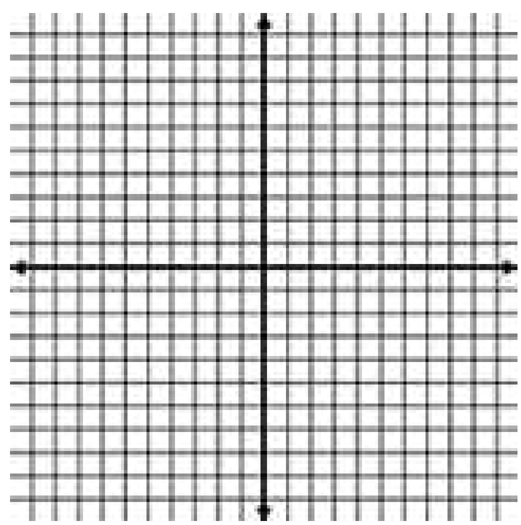
- π Solve linear systems by the substitution method
- π Use the substitution method to identify systems with no solution or infinitely many solutions
- π Solve problems using the substitution method

WARM-UP:

1. Solve.
   $-5x+3(2x-7)=x-21$

2. Solve the following system of linear equations by graphing. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

$$y=-4x+6$$
$$y=-2x$$



**Steps for Solving a System of Two Linear Equations Containing Two Variables by Substitution**

1. Solve one of the equations for one of the unknowns.
2. Substitute the expression solved for in Step 1 into the **<u>other</u>** equation. The

   result will be a _________________ equation in _______ variable.

3. ______________ the linear equation in one variable found in Step 2.

4. _____________________ the value of the variable found in Step 3 into one of

   the **<u>original</u>** equations to find the _________________ of the other

   _________________.
5. Check your answer by ___________________________ the ________________

   ______________ into ________________ of the original equations.

Example 1: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

a.
$$5x+2y=-5$$
$$3x-\ y=-14$$

b.

$$y = 5x - 3$$
$$y = 2x - \frac{21}{5}$$

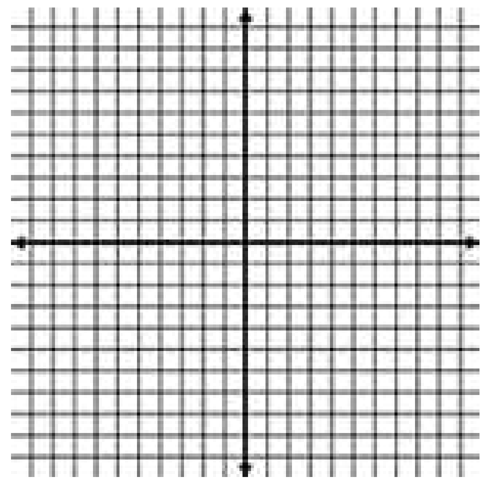
π Suppose you are solving a system of equations and you end up with 5 = 0. This is a ______________________ and yields a result of ________ or ________. This system consists of two __________________ lines which never ___________________.

π Suppose you are solving a system of equations and you end up with 5 = 5 or x = x. This is an ________________ and yields a result of all _____________ ____________ which are on the ______________. In other words, the system would have _____________________ ________________ solutions. This system consists of two lines which are ______________________.

Example 2: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Graph the system.

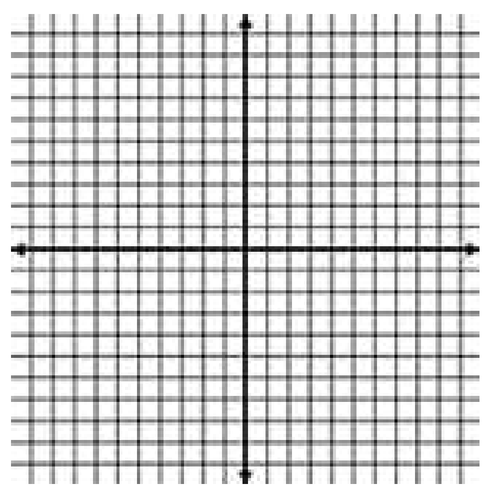
a.

$$-x+3y=4$$
$$2x-6y=-8$$



b.

$$x - 5y=3$$
$$-2x+10y=8$$



Example 3: Write a system of equations that has infinitely many solutions.

*APPLICATIONS*

1. Christa is a waitress and collects her tips at the table. At the end of the shift she has 68 bills in her tip wallet, all ones and fives. If the total value of her tips is $172, how many of each bill does she have?

2. Melody wishes to enclose a rectangular garden with fencing, using the side of her garage as one side of the rectangle. A neighbor gave her 30 feet of fencing, and Melody wants the length of the garden along the garage to be 3 feet more than the width. What are the dimensions of the garden?

When you are done with your 4.3 homework you should be able to…

- π Solve linear systems by the addition method
- π Use the addition method to identify systems with no solution or infinitely many solutions
- π Determine the most efficient method for solving a linear system

WARM-UP:

1. Solve the following system of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

$$y=\frac{7}{2}x-3$$
$$y=-4x+2$$

**ELIMINATING A VARIABLE USING THE ADDITION METHOD**

The ____________________ method is most useful if one of the equations has an _______________ variable. A third method for solving a linear system is the ___________________ method. The addition method __________________ a variable by _______________ the equations. When we use the addition method, we want to obtain two equations whose ____________ is an equation containing only __________ variable. The key step is to obtain, for one of the variables, __________________ that differ only in _____________.

**Steps for Solving a System of Two Linear Equations Containing Two Variables by Addition**

1. If necessary, _______________ both equations in the form ______________.
2. If necessary, _______________ either equation or both equations by appropriate nonzero numbers so that the ___________ of the $x$-coefficients or $y$-coefficients is __________.
3. __________ the equations in step 2. The __________ is an _______________ in ___________ variable.
4. ______________ the equation in one variable.
5. _____________-_______________ the value obtained in step 4 into either of the _____________ equations and _____________ for the other variable.
6. _____________ the solution in __________ of the original equations.

Example 1: Solve the following systems of linear equations by the addition method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x + y = 6$$
$$x - y = -2$$

b.

$$3x - y = 11$$
$$2x + 5y = 13$$

**COMPARING SOLUTION METHODS**

| **METHOD** | **ADVANTAGES** | **DISADVANTAGES** |
|---|---|---|

| | | |
|---|---|---|
| GRAPHING | You can _________ the _________________. | If the solutions do not involve _____________ or are too ___________ or ____________ to be ____________ on the graph, it's impossible to tell exactly what the __________________ are. |
| SUBSTITUTION | Gives _____________ solutions. Easy to use if a _________________ is on _________ side by itself. | Solutions cannot be ___________. Can introduce extensive work with ______________ when no variable has a coefficient of ______ or _______. |
| ADDITION | Gives ____________ solutions. Easy to use! | Solutions cannot be _____________. |

Example 2: Solve the following systems of linear equations by any method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$\begin{aligned} 2x+5y &= -6 \\ 7x-2y &= 11 \end{aligned}$$

b.

$$\begin{aligned} 4x - y &= 1 \\ y &= 7x - 15 \end{aligned}$$

c.

$$\begin{aligned} 4x - 2y &= 2 \\ 2x - y &= 1 \end{aligned}$$

d.

$$3x = 4y + 1$$
$$4x + 3y = 1$$

e.

$$2x + 4y = 5$$
$$3x + 6y = 6$$

## Section 4.4: PROBLEM USING SOLVING SYSTEMS OF EQUATIONS

When you are done with your homework you should be able to…

- $\pi$ Solve problems using linear systems
- $\pi$ Solve simple interest problems
- $\pi$ Solve mixture problems
- $\pi$ Solve motion problems

WARM-UP:

1. Solve the system of linear equations using the substitution or the addition method. Determine if the system is consistent or inconsistent, and if the equations are dependent or independent. Give your result in set notation.

a.

$$2x-3y=4$$
$$3x+4y=0$$

b.

$$x-y=3$$
$$2x=4+2y$$

## A STRATEGY FOR SOLVING WORD PROBLEMS USING SYSTEMS OF EQUATIONS

When we solved problems in chapter 2, we let *x* represent a _________________ that was ____________________. Problems in this section involve ___________ unknown ____________________. We will let _______ and _______ represent the _______________ quantities and _________________ the English words into a _________________ of _______________ equations.

Example 1: The sum of two numbers is five. If one number is subtracted from the other, their difference is thirteen. Find the numbers.

Example 2: Each day, the sum of the average times spent on grooming for 15- to 19-year-old women and men is 96 minutes. The difference between grooming times for 15- to 19-year-old women and men is 22 minutes. How many minutes per day do 15- to 19-year-old women and men spend on grooming?

Example 3: A rectangular lot whose perimeter is 1600 feet is fenced along three sides. An expensive fencing along the lot's length costs \$20 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. The total cost of the fencing along the three sides comes to \$13000. What are the lot's dimensions?

Example 4: On a special day, tickets for a minor league baseball game cost \$5 for adults and \$1 for students. The attendance that day was 1281 and \$3425 was collected. Find the number of each type of ticket sold.

Example 5: You invested $11000 in stocks and bonds, paying 5% and 8% annual interest. If the total interest earned for the year was $730, how much was invested in stocks and how much was invested in bonds?

Example 6: A jeweler needs to mix an alloy with a 16% gold content and an alloy with a 28% gold content to obtain 32 ounces of a new alloy with a 25% gold content. How many ounces of each of the original alloys must be used?

## A FORMULA FOR MOTION

Distance equals _______________ times ______________.

Example 7: When a plane flies with the wind, it can travel 4200 miles in 6 hours. When the plane flies in the opposite direction, against the wind, it takes 7 hours to fly the same distance. Find the rate of the plane in still air and the rate of the wind.

Example 8: With the current, you can row 24 miles in 3 hours. Against the same current, you can row only 2/3 of this distance in 4 hours. Find your rowing rate in still water and the rate of the current.

Section 5.1: ADDING AND SUBTRACTING POLYNOMIALS

When you are done with your homework you should be able to…

- $\pi$ Understand the vocabulary used to describe polynomials
- $\pi$ Add polynomials
- $\pi$ Subtract polynomials
- $\pi$ Graph equations defined by polynomials of degree 2

WARM-UP:

Simplify:

$-6x+5y-2x^2-2y+x^2$

**DESCRIBING POLYNOMIALS**

A ____________________ is a ___________ term or the ____________ of two or more ________________ containing _________________ with ____________ number ________________. It is customary to write the ______________ in the order of _________________ powers of the __________________. This is the _________________ form of a _____________________. We begin this chapter by limiting discussion to polynomials containing __________ variable. Each term of such a _____________________ in _____ is of the form _______________. The ______________ of ______________ is ______.

## THE DEGREE OF $ax^n$

If ____________ and ______ is a _____________ number, the _____________ of ____________ is _________. The _______________ of a nonzero constant term is _________. The constant zero has no defined degree.

Example 1: Identify the terms of the polynomial and the degree of each term.

a. $-4x^5-13x^3+5$

b. $-x^2+3x-7$

A polynomial is _________________ when it contains no _____________ symbols and no ____________ _________________. A simplified polynomial that has exactly ___________ term is called a ___________________. A simplified polynomial that has _____________ terms is called a ______________ and a simplified polynomial with _____________ terms is called a ________________. Simplified polynomials with _____________ or more ___________ have no special names. The ________________ of a __________________ is the ________________ degree of ____________ the _____________ of a __________________.

Example 2: Find the degree of the polynomial.

a. $5x^2-x^8+16x^4$

b. $-2$

**ADDING POLYNOMIALS**

Recall that _________ ____________ are terms containing __________ the same _____________ to the __________ powers. _____________ are added by ______________ ____________ ____________.

Example 3: Add the polynomials.

a. $(8x-5)+(-13x+9)$

b. $(7y^3+5y-1)+(2y^2-6y+3)$

c. $\left(\frac{2}{5}x^4+\frac{2}{3}x^3+\frac{5}{8}x^2+7\right)+\left(-\frac{4}{5}x^4+\frac{1}{3}x^3-\frac{1}{4}x^2-7\right)$

d.

$$\begin{array}{r} 7x^2-5x-6 \\ \underline{-9x^2+4x+6} \end{array}$$

## SUBTRACTING POLYNOMIALS

We ______________ real numbers by _____________ the _______________ of the number being ________________. Subtraction of polynomials also involves ____________________. If the sum of two polynomials is __________, the polynomials are ___________________ of each other.

Example 4: Find the opposite of the polynomial.

a. $x+8$

b. $-12x^3-x+1$

## SUBTRACTING POLYNOMIALS

To ______________ two polynomials, ________ the first polynomial and the _______________ of the second polynomial

Example 5: Subtract the polynomials.

a. $(x-2)-(7x+9)$

b. $(3x^2-2x)-(5x^2-6x)$

c. $\left(\frac{3}{8}x^2-\frac{1}{3}x-\frac{1}{4}\right)-\left(-\frac{1}{8}x^2+\frac{1}{2}x-\frac{1}{4}\right)$

d.

$$\begin{array}{r} 3x^5-5x^3+6 \\ \underline{-\left(7x^5+4x^3-2\right)} \end{array}$$

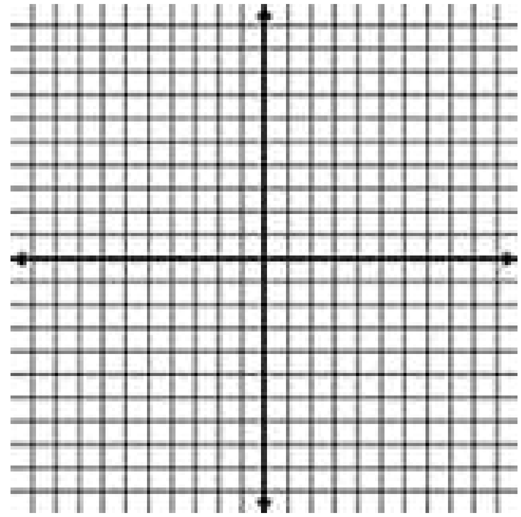
**GRAPHING EQUATIONS DEFINED BY POLYNOMIALS**

Graphs of equations defined by ___________________ of degree ______ have a ___________________ quality. We can obtain their graphs, shaped like _____________ or _______________ bowls, using the _____________-___________________ method for graphing an equation in two variables.

Example 6: Graph the following equations by plotting points.

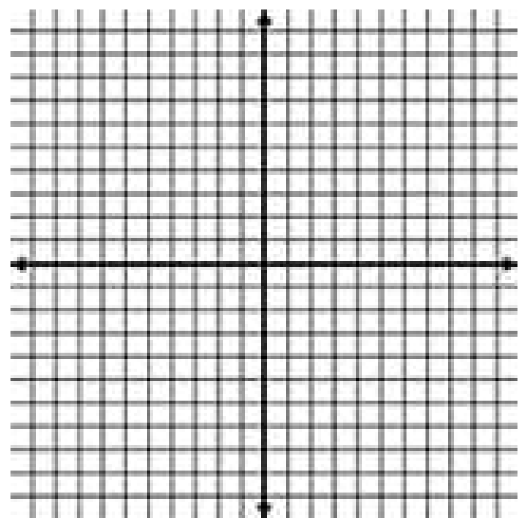
a. $y = x^2 - 1$

| $x$ | $y = x^2 - 1$ | $(x, y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



b. $y = 9 - x^2$

| $x$ | $y = 9 - x^2$ | $(x, y)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |



Section 5.2: MULTIPLYING POLYNOMIALS

When you are done with your homework you should be able to…

- $\pi$ Use the product rule for exponents
- $\pi$ Use the power rule for exponents
- $\pi$ Use the products-to-power rule
- $\pi$ Multiply monomials
- $\pi$ Multiply a monomial and a polynomial
- $\pi$ Multiply polynomials when neither is a monomial

WARM-UP:

Add or subtract the following polynomials:

a. $\left(-22r^7+6r^3-r^2\right)-\left(2r^7+r^2-1\right)$

b. $\left(8x^4-x^3-x^2\right)+\left(-8x^4+x^3\right)$

**THE PRODUCT RULE FOR EXPONENTS**

We have seen that __________________ are used to indicate ________________ multiplication. Recall that $3^4$ = __________________. Now consider $3^4 \cdot 3^2$:

**THE PRODUCT RULE**

When multiplying _________________ expressions with the ___________ base, ________ the _________________. Use this ________ as the _______________ of the _________________ base.

Example 1: Simplify each expression.

a. $2^5 \cdot 2^3$

b. $x^2 \cdot x \cdot x^4$

**THE POWER RULE (POWERS TO POWERS)**

When an ___________________ _________________ is ___________ to a

________________, _______________ the _________________. Place the

_______________ of the __________________on the __________ and

________________ the ____________________.

Example 2: Simplify each expression.

a. $\left(4^2\right)^3$

b. $\left(x^{12}\right)^5$

## THE PRODUCTS-TO-POWERS RULE FOR EXPONENTS

When a ________________ is _______________ to a __________, __________ each _____________ to the ________________.

Example 3: Simplify each expression.

a. $(-2y)^5$

b. $(10x^3)^2$

## MULTIPLYING MONOMIALS

To _________________ _________________ with the _____________ _________________ base, _______________ the ____________________ and then multiply the _________________. Use the _____________ rule for _______________ to multiply the __________________.

Example 4: Multiply.

d. $(8x)(-11x^4)$

e. $(7y^3)(2y^2)$

f. $\left(\frac{2}{5}x^4\right)\left(-\frac{5}{6}x^7\right)$

## MULTIPLYING A MONOMIAL AND A POLYNOMIAL THAT IS NOT A MONOMIAL

To __________________ a ___________________ and a __________________, use the ________________ property to _________________ each ____________ of the __________________ by the ______________________.

Example 5: Multiply.

a. $3x^2(2x-5)$

b. $-x(x^2+6x-5)$

## MULTIPLYING POLYNOMIALS WHEN NEITHER IS A MONOMIAL

Multiply each __________________ of one __________________ by each __________ of the other polynomial. Then _____________ _____________ terms.

Example 6: Multiply.

a. $(x+2)(x+5)$

b. $(2x+5)(x+3)$

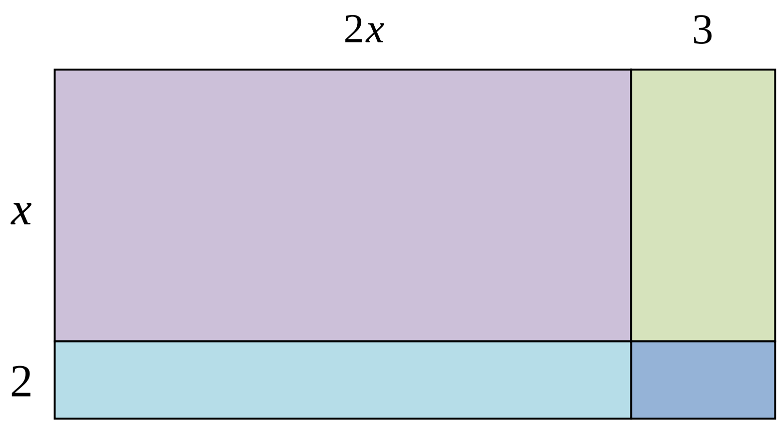
c. $\left(x^2-7x+9\right)\left(x+4\right)$

Example 7: Simplify.

a. $3x^2\left(6x^3+2x-3\right)-4x^3\left(x^2-5\right)$

b. $\left(y+6\right)^2-\left(y-2\right)^2$

**APPLICATION**




a. Express the area of the large rectangle as the product of two binomials.

b. Find the sum of the areas of the four smaller rectangles.

c. Use polynomial multiplication to show that your expressions for area in parts (a) and (b) are equal.

Section 5.3: SPECIAL PRODUCTS

When you are done with your homework you should be able to…

- π Use FOIL in polynomial multiplication
- π Multiply the sum and difference of two terms
- π Find the square of a binomial sum
- π Find the square of a binomial difference

WARM-UP:

Multiply the following polynomials:

a. $(x-1)^2$

b. $(x-5)(x+5)$

**THE PRODUCT OF TWO BINOMIALS: FOIL**

**F** represents the _______________ of the ______________ terms in each _______________, **O** represents the _________________ of the ___________ terms, **I** represents the ________________ of the ________________ terms, and **L** represents the ________________ of the ________________ terms.

**USING THE FOIL METHOD TO MULTIPLY BINOMIALS**

$(ax+b)(cx+d) =$ ______________________________________________

Example 1: Multiply using FOIL.

a. $(5x+3)(3x+8)$

b. $(x-10)(x+9)$

**THE PRODUCT OF THE SUM AND DIFFERENCE OF TWO TERMS**

$(A+B)(A-B)=$ ________________________________________

The ________________ of the ________ and the __________________ of the ___________ two terms is the ____________ of the _____________ ________________ the _________________ of the second.

Example 2: Multiply.

a. $(x+4)(x-4)$

b. $(3x-7y)(3x+7y)$

**THE SQUARE OF A BINOMIAL SUM**

$(A+B)^2=$ ________________________________________

The ________________ of a _______________ _________ is the ____________ term _____________ ________ ___ times the _____________ of the terms _____________ the last term _______________.

Example 3: Multiply.

a. $\left(x+6\right)^2$

b. $\left(x^2+9\right)^2$

**THE SQUARE OF A BINOMIAL DIFFERENCE**

$$\left(A-B\right)^2 = \rule{10cm}{0.4pt}$$

The ________________ of a _______________ _________ is the _____________ term ______________ ________ ___ times the ______________ of the terms ______________ the last term _______________.

Example 4: Multiply.

a. $\left(5x-y\right)^2$

b. $\left(x^3-11\right)^2$

Section 5.4: POLYNOMIALS IN SEVERAL VARIABLES

When you are done with your homework you should be able to…

- π Evaluate polynomials in several variables
- π Understand the vocabulary of polynomials in two variables
- π Add and subtract polynomials in several variables
- π Multiply polynomials in several variables

WARM-UP:

Evaluate the polynomial:

$x^3y + 2xy^2 + 5x - 2;\ x = -2 \text{ and } y = 3$

**EVALUATING A POLYNOMIAL IN SEVERAL VARIABLES**

1. ___________________ the given value for each ______________________.

2. Perform the resulting _______________________ using the ____________ of _______________________.

**DESCRIBING POLYNOMIALS IN TWO VARIABLES**

In general, a ____________________ in ___________ ___________________, ___ and ___, contains the ____________ of one or more ____________________ in the form ______________. The constant, ______, is the __________________. The ____________________, _____ and _____, represent _______________ numbers. The ___________________ of the ____________________ __________ is ______________.

Example 1: Determine the coefficient of each term, the degree of each term, and the degree of the polynomial.

$8xy^4 - 17x^5y^3 + 4x^2y - 9y^3 + 7$

**ADDING AND SUBTRACTING POLYNOMIALS IN SEVERAL VARIABLES**

________________________ in __________________ variables are added by

___________________ _______________ _________________.

Example 2: Add or subtract.

a. $\left(x^3 - y^3\right) - \left(-4x^3 - x^2y + xy^2 + 3y^3\right)$

b. $\left(7x^2y + 5xy + 13\right) + \left(-3x^2y + 6xy + 4\right)$

## MULTIPLYING POLYNOMIALS IN SEVERAL VARIABLES

The _______________ of _____________ ________ the basis of ___________ ___________________. _____________________ can be done _____________ by ___________________ ___________________ and _______________ __________________ on ____________________ with the ________________ _______________.

Example 3: Multiply.

a. $\left(5xy^3\right)\left(-10x^2y^4\right)$

c. $\left(x-2y^4\right)\left(x+2y^4\right)$

b. $-x^7y^2\left(x^2+7xy-4\right)$

d. $\left(x^2-y\right)^2$

Section 5.5: DIVIDING POLYNOMIALS

When you are done with your homework you should be able to…

- π Use the quotient rule for exponents
- π Use the zero-exponent rule for exponents
- π Use the quotients-to-power rule
- π Divide monomials
- π Check polynomial division
- π Divide a polynomial by a monomial

WARM-UP:

1. Find the missing exponent, designated by the question mark, in the final step:

$$\frac{x^8}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^?$$

2. Simplify:

$$\frac{\left(2a^3\right)^5}{\left(b^4\right)^5}$$

**THE QUOTIENT RULE FOR EXPONENTS**

When dividing _________________ expressions with the ___________ nonzero base, ________________ the exponent in the _________________ from the ____________________ in the __________________. Use this ______________________ as the _______________ of the _________________ base.

Example 1: Simplify each expression.

a. $\frac{2^5}{2^3}$

b. $\frac{x^{10}}{x^8}$

**THE ZERO-EXPONENT RULE**

If ______ is any _____________ number other than ______,

Example 2: Simplify each expression.

a. $\left(4^2\right)^0$

b. $-7x^0$

## THE QUOTIENTS-TO-POWERS RULE FOR EXPONENTS

If _____ and _____ are real numbers and ____ is nonzero, then

When a ________________ is _______________ to a __________, __________ the _____________ to the ________________ and _______________ by the ________________ raised to the ____________________.

Example 3: Simplify each expression.

a. $\left(\frac{x}{3}\right)^5$

b. $\left(\frac{4x^3}{5y}\right)^2$

## DIVIDING MONOMIALS

To _________________ _________________, ___________________the ______________________ and then divide the ______________________.

Use the _____________ rule for _______________ to divide the ____________.

Example 4: Divide.

a. $\dfrac{16x^4}{2x^4}$

b. $\dfrac{6x^2y^5}{21xy^3}$

c. $\dfrac{35r^8}{14r^7}$

**DIVIDING A POLYNOMIAL THAT IS NOT A MONOMIAL BY A MONOMIAL**

To ________________ by a _________________, ________________ each

__________________ of the ____________________ by the ______________.

Example 5: Find the quotient.

a. $\left(24x^6 - 12x^4 + 8x^3\right) \div \left(4x^3\right)$

b. $\dfrac{459x^{10}y^9 + 18x^5y^3 - 9x^4y}{-9x^3y}$

## Section 5.6: LONG DIVISION OF POLYNOMIALS AND SYNTHETIC DIVISION

When you are done with your homework you should be able to…

- π Use long division to divide by a polynomial containing more than one term
- π Divide polynomials using synthetic division

WARM-UP:

a. Divide using long division:

$$56\overline{)1234567}$$

b. Simplify:

$$\frac{5x^5-8x^3+x^2}{2x^2}$$

## STEPS FOR DIVIDING A POLYNOMIAL BY A BINOMIAL

1. ________________ the terms of ___________ the ________________ and the ________________ in ___________________ powers of the variable.
2. _________________ the ____________ term in the _______________ by the ______________ term in the _________________. The result is the _______________ term of the __________________.
3. _______________ every term in the ___________________ by the ______________ term in the ________________. Write the resulting ________________ beneath the ________________ with __________ terms lined up.
4. _____________ the _______________ from the ____________________.
5. _________________ down the next term in the ________________ dividend and write it next to the __________________ to form a new __________________.
6. Use this new expression as the ____________________ and repeat the process until the ______________________ can no longer be __________________. This will occur when the ______________ of the __________________ is ______________ than the _______________ of the ________________.

Example 1: Divide.

a. $\dfrac{x^2+7x+10}{x+5}$

b. $\dfrac{2y^2-13y+21}{y-3}$

c. $\dfrac{x^3+2x^2-3}{x-2}$

d. $\left(8y^3+y^4+16+32y+24y^2\right)\div\left(y+2\right)$

## DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

We can use ___________________ division to divide _________________ if the _______________ is of the form _______________. This method provides a _______________ more quickly than _____________ division.

## STEPS FOR SYNTHETIC DIVISION

1. Arrange the ___________________ in __________________ powers, with a _______ coefficient for any _______________ term.
2. Write _____ for the ______________, ___________. To the _________, write the _________________ of the _________________.
3. Write the _______________ ___________________ of the _______________ on the _______________ row.
4. __________________ _____ times the ____________ just written on the _______________ row. Write the _______________ in the next ______________ in the ________________ row.
5. __________ the values in this new column, writing the __________ in the ______________ row.
6. Repeat this series of _______________________ and ________________ until all _________________ are filled in.

7. Use the numbers in the last row to write the ________________ plus the

___________________ _________________ the _________________. The

_________________ of the ________________ term of the quotient will be

___________ less than the __________________ of the first term of the

_____________________. The final value in this row is the ______________.

Example 2: Divide using synthetic division.

a. $\left(x^2+x-2\right)\div\left(x-1\right)$

b. $\left(x^2-6x-6x^3+x^4\right)\div\left(6+x\right)$

c. $\dfrac{x^7-128}{x-2}$

d. $\left(y^5-2y^4-y^3+3y^2-y+1\right)\div\left(y-2\right)$

**APPLICATION**

You just signed a contract for a new job. The salary for the first year is \$30,000 and there is to be a percent increase in your salary each year. The algebraic expression

$$\frac{30000x^n - 30000}{x-1}$$

describes your total salary over *n* years, where *x* is the sum of 1 and the yearly percent increase, expressed as a decimal.

a. Use the given expression and write a quotient of polynomials that describes your total salary over four years.

b. Simplify the expression in part (a) by performing the division.

c. Suppose you are to receive an increase of 8% per year. Thus, *x* is the sum of 1 and 0.08, or 1.08. Substitute 1.08 for *x* in the expression in part (a) as well as the simplified expression in part (b). Evaluate each expression. What is your total salary over the four-year period?

## Section 5.7: NEGATIVE EXPONENTS AND SCIENTIFIC NOTATION

When you are done with your homework you should be able to…

- π Use the negative exponent rule
- π Simplify exponential expressions
- π Convert from scientific notation to decimal notation
- π Convert from decimal notation to scientific notation
- π Compute with scientific notation
- π Solve applied problems using scientific notation

WARM-UP:

1. Divide:

$$\left(7x^4 - 8x\right) \div \left(x + 3\right)$$

2. Simplify:

$$\frac{1}{\left(6x^3\right)^2}$$

## NEGATIVE INTEGERS AS EXPONENTS

A nonzero base can be raised to a _________________ power. The _______________ rule can be used to help determine what a ______________ __________________ as an __________________ should mean.

## THE NEGATIVE EXPONENT RULE

If _____ is any real number other than ____ and ____ is a natural number, then

## NEGATIVE EXPONENTS IN NUMERATORS AND DENOMINATORS

If _____ is any real number other than ____ and ____ is a natural number, then

When a _______________ number appears as an ________________, ______________ the position of the ___________ (from ________________ to _________________ or from _________________ to ____________________) and make the ___________________ _________________. The sign of the _____________ does ______________ change.

Example 1: Write each expression with positive exponents only. Then simplify, if possible.

a. $-7^{-2}$

c. $3^{-1}-6^{-1}$

b. $(-7)^{-2}$

d. $\dfrac{x^{-12}}{y^{-1}}$

## SIMPLIFYING EXPONENTIAL EXPRESSIONS

Properties of ___________________ are used to _________________ exponential expressions. An exponential ___________________ is _________________ when

- Each _______________ occurs only _________________
- No _________________________ appear
- No ____________________ are raised to ____________________
- No _________________ or _______________ exponents appear

## STEPS FOR SIMPLIFYING EXPONENTIAL EXPRESSIONS

1. If necessary, be sure that each _____________ appears only __________, using ___________________ or __________________.

2. If necessary, ________________ parentheses using _________________ or _______________________.

3. If necessary, simplify ________________ to _________________ using ______________________.

4. If necessary, _________________ exponential expressions with _______ powers as _____ (_____________). Furthermore, write the answer with _________________ exponents using ___________________________.

Example 2: Simplify. Assume that variables represent nonzero real numbers.

a. $\dfrac{45z^4}{15z^{12}}$

b. $\dfrac{(3y^4)^3 y^{-7}}{y^7}$

c. $\dfrac{(5x^3)^2}{x^7}$

d. $\left(\dfrac{x^3}{y^2}\right)^{-4}$

## SCIENTIFIC NOTATION

A _________________ number is written in _________________ notation when it is expressed in the form

where ____ is a number ______________ than or equal to ____ and __________ than _____ (_____________________) and ____ is an _____________________.

It is customary to use the _____________________ symbol, ____, rather than a dot, when writing a number in ___________________ _________________. We can use ____, the exponent on the ______ in _______________, to change a number in scientific notation to ________________ notation. If _____ is _________________, move the decimal point in _____ to the _____________ ____ places. If _____ is __________________, move the decimal point in ____ to the ______________ _________ places.

Example 3: Write each number in decimal notation.

a. $7.85\times10^{8}$

c. $1.001\times10^{2}$

b. $9\times10^{-5}$

d. $9.999\times10^{-1}$

## CONVERTING FROM DECIMAL TO SCIENTIFIC NOTATION

Write the number in the form ________________.

- Determine _____, the numerical _______________. Move the ___________ point in the ____________ number to obtain a number _______________ than or equal to _____ and _________ than ______.
- Determine _____, the _______________ on _______. The ____________ ________________ of _____ is the ________________ of places the decimal point was _____________. The exponent _____ is _____________ if the given number is ________________ than ______ and ____________ if the given number is _________________ _____ and _____.

Example 4: Write each number in scientific notation.

a. $0.00000006589$

c. $0.234$

b. $6,789,000,000,000$

d. $1,000,234,000$

## COMPUTATIONS WITH NUMBERS IN SCIENTIFIC NOTATION

**MULTIPLICATION**

**DIVISION**

**EXPONENTIATION**

After the computation is ____________________, the _________________ may require an additional ___________________ before it is expressed in ______________________ notation.

Example 5: Perform the indicated operations, writing the answers in scientific notation.

a. $\left(3\times10^{4}\right)\left(4\times10^{2}\right)$

b. $\left(2\times10^{-3}\right)^{5}$

c. $\frac{180\times10^{8}}{2\times10^{4}}$

d. $\left(5\times10^{4}\right)^{-1}$

**APPLICATIONS**

1. A human brain contains $3\times10^{10}$ neurons and a gorilla brain contains $7.5\times10^{9}$ neurons. How many times as many neurons are in the brain of a human as in the brain of a gorilla?

2. If the sun is approximately $9.14\times10^{7}$ miles from the earth, how many seconds, to the nearest tenth of a second does it take sunlight to reach Earth? Use the motion formula, $d=rt$, and the fact that light travels at the rate of $1.86\times10^{5}$ miles per second.

Section 6.1: THE GREATEST COMMON FACTOR AND FACTORING BY GROUPING

When you are done with your homework you should be able to…

- π Find the greatest common factor (GCF)
- π Factor out the GCF of a polynomial
- π Factor by grouping

WARM-UP:

1. Multiply:

$x^2\left(7x^4-8\right)$

2. Divide:

$$\frac{16x^4-8x^2}{4x^2}$$

**FACTORING A ______________________ CONTAINING THE SUM OF _________________ MEANS FINDING AN ________________ EXPRESSION THAT IS A ____________________.**

## FACTORING OUT THE GREATEST COMMON FACTOR (GCF)

We use the ____________________ property to ________________ a monomial and a ____________________ of ____________ or more ____________.

When we ________________, we _________________ this process, expressing the __________________ as a ____________________.

**MULTIPLICATION** **FACTORING**

In any ________________________ problem, the first step is to look for the ____________________ ___________________ __________________. The ____________ is an __________________ of the _________________ degree that _________________ each _____________ of the __________________.

**The ________________ part of the ______________ always contains the ________________ _________________ of a ________________ that appears in __________ terms of the ____________________.**

Example 1: Find the greatest common factor of each list of monomials:

a. $5$ and $15x$

b. $-3x^4$ and $6x^3$

c. $x^2y,\ 7x^3$y and $14x^2$

## STEPS FOR FACTORING A MONOMIAL FROM A POLYNOMIAL

1. Determine the ________________ _______________ factor of _________ terms in the __________________________.
2. Express each _________________ as the __________________ of the ________ and its other ___________________.
3. Use the _________________ _________________ to factor out the ______.

Example 2: Factor each polynomial using the GCF:

a. $9x+9$

b. $32x-24$

c. $18x^3\mathrm{y}^2-12x^3y-24x^2y$

d. $7(x+1)+21x(x+1)$

## FACTORING BY GROUPING

1. ________________ terms that have a ________________ _____________ factor. There will usually be _______ groups. Sometimes the terms must be ____________________.

2. _______________ out the ________________ monomial ____________ from each _______________.

3. ________________ out the remaining common ________________ factor (if one exists).

Example 3: Factor by grouping:

a. $x^2+3x+5x+15$

c. $xy-6x+2y-12$

b. $x^3-3x^2+4x-12$

d. $10x^2-12xy+35xy-42y^2$

Example 4: Factor each polynomial:

a. $x^3 - 5 + 2x^3y - 10y$

c. $8x^5(x+2) - 10x^3(x+2) - 2x^2(x+2)$

b. $7x^5 - 7x^4 + x^3 - x^2 + 3x - 3$

d. $12x^2 - 25$

**APPLICATION**

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The polynomial $72x - 16x^2$ describes the height of the debris above the ground, in feet, after $x$ seconds.

a. Find the height of the debris after 4 seconds.

b. Factor the polynomial.

c. Use the factored form of the polynomial in part (b) to find the height of the debris after 4 seconds. Do you get the same answer as you did in part (a)? If so, does this prove that your factorization is correct?

## Section 6.2: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS 1

When you are done with your homework you should be able to…

- π Factor trinomials of the form $x^2+bx+c$

WARM-UP:

Multiply:

a. $(x+1)(x+8)$

c. $(x+1)(x-8)$

b. $(x-1)(x-8)$

d. $(x-1)(x+8)$

### A STRATEGY FOR FACTORING $ax^2+bx+c$ : USING GROUPING

1. Multiply the leading coefficient (in this case 1) and the constant, ______.
2. Find the __________________ of _________ whose _________ is ______.
3. Rewrite the ____________ term, _____, as a _________ or a

   _______________ using the factors from step 2.
4. ______________ by ________________.

Example 1: Factor each trinomial

a. $x^2+9x+8$

b. $x^2+7x+10$

c. $x^2-13x+40$

d. $x^2+3x-28$

e. $x^2-4x-5$

f. $w^2+12w-64$

g. $y^2-15y+5$

h. $x^2-9xy+14y^2$

Some ___________________ can be _________________ using more than one ____________________. **Always begin by looking for the _______________ __________________ _________________ and, if there is one, _________ it out!** A polynomial is ______________ ________________ when it is written as the _________________ of ______________ ______________________.

Example 4: Factor completely

a. $3x^2+21x+36$

b. $20x^2y-5xy-120y$

c. $y^4 - 12y^3 + 35y^2$

d. $(a+b)x^2 - 13(a+b)x + 36(a+b)$

**APPLICATION**

You dive directly upward from a board that is 48 feet high. After *t* seconds, your height above the water is described by the polynomial $-16t^2 + 32t + 48$.

a. Factor the polynomial completely.

b. Evaluate both the original polynomial and its factored form for *t* = 3.

c. Do you get the same answer? Describe what this answer means?

## Section 6.3: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS NOT 1

When you are done with your homework you should be able to…

- π Factor trinomials by trial and error
- π Factor trinomials by grouping

WARM-UP:

Factor:

a. $x^2y - xy^2$

c. $2x^3 - 6x^2 + 4x$

b. $x^2 - 14x - 51$

d. $z^2 + z - 72$

## A STRATEGY FOR FACTORING $ax^2+bx+c$ : USING TRIAL AND ERROR

Assume, for the moment, that there is no _______________ ______________ factor other than _____.

1. ____________ two First ______________ whose ___________ is ________.

2. ____________ two Last ______________ whose ___________ is _____.

3. By _____________ and _______________, perform steps 1 and 2 until the __________ of the Outside ________________ and the Inside ______________ is ___________.

If ___________ such ___________________ exist, the polynomial is __________.

Example 1: Factor using trial and error.

a. $5x^2-14x+8$

b. $6x^2+19x-7$

c. $3x^2 - 13xy + 4y^2$

d. $9z^2 + 3z + 2$

## A STRATEGY FOR FACTORING $ax^2 + bx + c$ : USING GROUPING

1. Multiply the leading coefficient and the constant, _______.
2. Find the __________________ of _________ whose _________ is _______.
3. Rewrite the _____________ term, ______, as a _________ or a

   ________________ using the factors from step 2.
4. _______________ by _________________.

Example 1: Factor using grouping.

a. $3x^2 - x - 10$

b. $8x^2 - 10x + 3$

c. $9y^2+5y-4$

d. $12x^2+7xy-12y^2$

Example 4: Factor completely

a. $4x^2-18x-10$

c. $24y^4+10y^3-4y^2$

b. $3x^3+14x^2+8x$

d. $6(y+1)x^2+33(y+1)x+15(y+1)$

Section 6.4: FACTORING SPECIAL FORMS

When you are done with your homework you should be able to…

- $\pi$ Factor the difference of two squares
- $\pi$ Factor perfect square trinomials
- $\pi$ Factor the sum of two cubes
- $\pi$ Factor the difference of two cubes

WARM-UP:

Factor:

a. $3a^2 - ab - 14b^2$

c. $80z^3 + 80z^2 - 60z$

b. $12x^2 - 33x + 21$

d. $-10x^2y^4 + 14xy^4 + 12y^4$

## THE DIFFERENCE OF TWO SQUARES

If _____ and _____ are real numbers, or ________________ expressions, then

The __________________ of the _____________ of ___________ ___________ factors as the _______________ of a ____________ and a _________________ of those terms.

## 16 PERFECT SQUARES

| | | | |
|---|---|---|---|
| $1 =$ ________ | $25 =$ ________ | $81 =$ ________ | $169 =$ ________ |
| $4 =$ ________ | $36 =$ ________ | $100 =$ ________ | $196 =$ ________ |
| $9 =$ ________ | $49 =$ ________ | $121 =$ ________ | $225 =$ ________ |
| $16 =$ _______ | $64 =$ ________ | $144 =$ ________ | $256 =$ ________ |

Example 1: Factor.

a. $x^2 - 144$

c. $25 - 4x^{10}$

b. $16x^2 - 196y^2$

d. $18x^3 - 2x$

## FACTORING PERFECT SQUARE TRINOMIALS

Let ______ and ______ be real numbers, _____________________, or _________________ expressions.

1. $A^2 + 2AB + B^2 =$ ____________________________

2. $A^2 - 2AB + B^2 =$ ____________________________

- The ________________ and ________________ terms are ______________ of__________________ or ____________________.
- The _____________________ term is __________________ the _______________ of the _____________________ being ________________ in the ___________ and ________________ terms.

Example 2: Factor.

a. $9x^2 + 6x + 1$

c. $x^2 - 18xy + 81y^2$

b. $x^2 + 4x + 4$

d. $2y^2 - 40y + 200$

## FACTORING THE SUM OR DIFFERENCE OF TWO CUBES

Let _____ and _____ be real numbers, _________________, or _______________ expressions.

1. $A^3 + B^3 =$ ______________________________

2. $A^3 - B^3 =$ ______________________________

Example 3: Factor.

a. $x^3 + 64$

c. $128 - 250y^3$

b. $8y^3 - 1$

d. $125x^3 + y^3$

Example 4: Factor completely

a. $25x^2 - \frac{4}{49}$

c. $(y+6)^2 - (y-2)^2$

b. $20x^3 - 5x$

d. $0.064 - x^3$

## Section 6.5: A GENERAL FACTORING STRATEGY

When you are done with your homework you should be able to…

- π Recognize the appropriate method for factoring a polynomial
- π Use a general strategy for factoring polynomials

WARM-UP:

Multiply:

a. $(x+1)(x^2-x+1)$

b. $(2x-3y)(4x^2+6xy+9y^2)$

### A STRATEGY FOR FACTORING A POLYNOMIAL

1. If there is a ___________________ factor other than _____, factor the _____________.

2. Determine the _______________ of _____________ in the polynomial and try factoring as follows:

   a. If there are __________ terms, can the _____________ be factored by one of the following special forms?

      ____________________ of ______ ______________:

_____________ of _______ _____________:

_______________ of _________ ____________:

b. If there are _____________ terms, is the _______________ a

________________ ________________ _______________? If so,

factor by one of the following special forms:

________________________ = ___________________________

________________________ = ___________________________

If the trinomial is ___________ a ________________

_____________, try ________________ by ____________ and

_________________ or ____________________.

c. If there are _______________ or ______________ terms, try

_________________ by ___________________.

3. Check to see if any ___________________ with more than one term in the

________________ ___________________ can be factored

_____________. If so, _________________ completely.

4. _________________ by ______________________.

Example 1: Factor

a. $5x^4 - 45x^2$

b. $4x^2 - 16x - 48$

c. $4x^5 - 64x$

d. $x^3 - 4x^2 - 9x + 36$

e. $3x^3 - 30x^2 + 75x$

f. $2w^5 + 54w^2$

g. $3x^4 y - 48y^5$

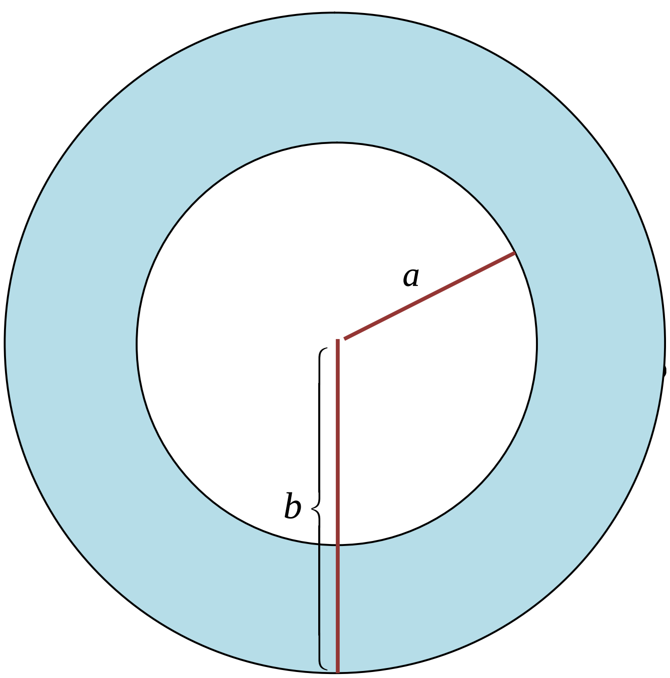
h. $12x^3 + 36x^2 y + 27xy^2$

i. $12x^2(x-1)-4x(x-1)-5(x-1)$

j. $x^2+14x+49-16a^2$

**APPLICATION**

Express the area of the shaded ring shown in the figure in terms of $\pi$. Then factor this expression completely.



Section 6.6: SOLVING QUADRATIC EQUATIONS BY FACTORING

When you are done with your homework you should be able to…

- π Use the zero-product principle
- π Solve quadratic equations by factoring
- π Solve problems using quadratic equations

WARM-UP:

a. Factor:

$x^2 - 8x + 7$

b. Solve:

$x - 7 = 0$

**DEFINITION OF A QUADRATIC EQUATION**

A _________________ ___________________ in ____ is an equation that can

be written in the _________________ ______________

where _____, _____, and _____ are real numbers, with _____________. A

_________________ ________________ in _____ is also called a ___________-

________________ _________________ equation in ______.

## SOLVING QUADRATIC EQUATIONS BY FACTORING

Consider the quadratic equation $x^2 - 8x + 7 = 0$. How is this different from the first warm-up?

We can _______________ the ____________ side of the __________________ equation __________________ to get _____________________. If a quadratic equation has a zero on one side and a __________________ ________________ on the other side, it can be _________________ using the ________________-__________________ principle.

**THE ZERO-PRODUCT PRINCIPLE**

If the __________________ of two or more ______________ expressions is ________________, then _____ _______________ one of them is __________ to ________________.

Example 1: Solve the following equations:

a. $2x-11=0$

b. $x+1=0$

c. $(2x-11)(x+1)=0$

**STEPS FOR SOLVING A QUADRATIC EQUATION BY FACTORING**

1. If necessary, ________________ the equation in ________________ form ________________________, moving all _____________ to one side, thereby obtaining ______________ on the other side.
2. ____________________.
3. Apply the _____________-__________________ principle, setting each _______________ equal to _________________.
4. ________________ the equations formed in step 3.
5. ________________ the _________________ in the _________________ equation.

Example 2: Solve:

a. $x(x+9)=0$

b. $8(x-5)(3x+11)=0$

c. $x^2+x-42=0$

d. $x^2=8x$

e. $4x^2 = 12x - 9$

f. $(x+3)(3x+5) = 7$

g. $x^3 - 4x = 0$

h. $(x-3)^2+2(x-3)-8=0$

**APPLICATION**

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The formula $h=-16t^2+72t$ describes the height of the debris above the ground, *h*, in feet, *t* seconds after the explosion.

a. How long will it take for the debris to hit the ground?

b. When will the debris be 32 feet above the ground?

Section 7.1: RATIONAL EXPRESSIONS AND THEIR SIMPLIFICATION

When you are done with your homework you should be able to…

- π Find numbers for which a rational expression is undefined
- π Simplify rational expressions
- π Solve applied problems involving rational expressions

WARM-UP:

a. Factor:

$$x^3 - 8x^2 + 2x - 16$$

b. Solve:

$$2x^2 - x - 10 = 0$$

**EXCLUDING NUMBERS FROM RATIONAL EXPRESSIONS**

A ________________ expression is the _________________ of two ______________________. Rational expressions indicate ___________________ and division by ________________ is ____________________. This means that we _______________ ____________________ any value or values of the ___________ that make a ____________________ _______________!

Example 1: Find all numbers for which the rational expression is undefined:

a. $\frac{5}{x}$

b. $\frac{x+1}{x-4}$

c. $\frac{8x-40}{x^2+3x-28}$

d. $\frac{x-12}{x^2+4}$

**SIMPLIFYING RATIONAL EXPRESSIONS**

A ________________ ________________ is ________________ if its

__________________ and _________________ have ______ common

_______________ other than ______ or ______.

**FUNDAMENTAL PRINCIPLE OF RATIONAL EXPRESSIONS**

If _____, _____, and _____ are __________________ and ______ and _____ are __________ ______,

## STEPS FOR SIMPLIFYING RATIONAL EXPRESSIONS

1. _______________ the _______________ and the ___________________ completely.

2. ________________ both the _________________ and the ________________ by any _______________ _______________.

Example 2: Simplify:

a. $\frac{4x-64}{16x}$

b. $\frac{6y+18}{11y+33}$

c. $\frac{x^2-12x+36}{4x-24}$

d. $\dfrac{x^3+4x^2-3x-12}{x+4}$

e. $\dfrac{x+5}{x-5}$

f. $\dfrac{x^3-1}{x^2-1}$

## SIMPLIFYING RATIONAL EXPRESIONS WITH OPPOSITE FACTORS IN THE NUMERATOR AND DENOMINATOR

The _________________ of two _________________ that have _____________ signs and are _________________ ___________________ is _______.

Example 3: Simplify:

a. $\dfrac{x-3}{3-x}$

b. $\dfrac{9x-15}{5-3x}$

c. $\dfrac{x^2-4}{2-x}$

**APPLICATION**

A company that manufactures small canoes has costs given by the equation

$$C = \frac{20x + 20000}{x}$$

in which $x$ is the number of canoes manufactured and $C$ is the cost to manufacture each canoe.

a. Find the cost per canoe when manufacturing 100 canoes.

b. Find the cost per canoe when manufacturing 10000 canoes.

c. Does the cost per canoe increase or decrease as more canoes are manufactured?

Section 7.2: MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Multiply rational expressions
- π Divide rational expressions

WARM-UP:

Simplify:

a. $\dfrac{a^2-2ab+b^2}{a^2-b^2}$

b. $\dfrac{x^2-3x+2}{x-1}$

**MULTIPLYING RATIONAL EXPRESSIONS**

If _____, ______, ______, and _____ are polynomials, where ______ and _______, then

The _________________ of two _______________ _________________ is the _______________ of their _______________, divided by the ________________ of their ________________.

## STEPS FOR MULTIPLYING RATIONAL EXPRESSIONS

1. ________________ all ___________________ and ____________________.

2. ________________ ___________________ and ___________________ by

   common ___________________.

3. ___________________ the remaining factors in the _________________

   and ___________________ the remaining factors in the ________________.

Example 1: Multiply.

a. $\frac{x-5}{3}\cdot\frac{18}{x-8}$

c. $\frac{9y+21}{y^2-2y}\cdot\frac{y-2}{3y+7}$

b. $\frac{x}{5}\cdot\frac{30}{x-4}$

d. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$

## DIVIDING RATIONAL EXPRESSIONS

If _____, ______, ______, and _____ are polynomials, where ______, ______,

and_______, then

The _________________ of two _______________ _________________ is the _______________ of the ___________ expression and the ___________ of the ______________.

Example 2: Divide.

a. $\frac{x}{3} \div \frac{3}{8}$

c. $\frac{y^2 - 2y}{15} \div \frac{y-2}{5}$

b. $\frac{x+5}{7} \div \frac{4x+20}{9}$

d. $\frac{x^2 - 4y^2}{x^2 + 3xy + 2y^2} \div \frac{x^2 - 4xy + 4y^2}{x+y}$

Example 3: Perform the indicated operation or operations.

e. $\dfrac{5x^2-x}{3x+2} \div \left(\dfrac{6x^2+x-2}{10x^2+3x-1} \cdot \dfrac{2x^2-x-1}{2x^2-x}\right)$

f. $\dfrac{5xy-ay-5xb+ab}{25x^2-a^2} \div \dfrac{y^3-b^3}{15x+3a}$

## Section 7.4: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH DIFFERENT DENOMINATORS

When you are done with your homework you should be able to…

- π Find the least common denominator
- π Add and subtract rational expressions with different denominators

WARM-UP: Perform the indicated operation and simplify.

1. $\frac{-3}{8}+\frac{5}{12}$

b. $\frac{x+2}{x^2+x}+\frac{-1}{x^2+x}$

### FINDING THE LEAST COMMON DENOMINATOR (LCD)

The _________________ __________________ denominator of several

_________________ ___________________ is a ___________________ consisting

Of the _________________ of all ___________________ _________________ in

the ___________________, with each _______________ raised to the greatest

_________________ of its occurrence in any denominator.

## FINDING THE LEAST COMMON DENOMINATOR

1. _________________ each _______________ completely.

2. List the factors of the first _________________________.

3. Add to the list in step 2 any __________________ of the second denominator that do not appear in the list. Repeat this step for all denominators.

4. Form the __________________ of the __________________ from the list in step 3. This product is the LCD.

Example 1: Find the LCD of the rational expressions.

a. $\frac{11}{25x^2}$ and $\frac{17}{35x}$

b. $\frac{7}{y^2-49}$ and $\frac{12}{y^2-14y+49}$

## ADDING AND SUBTRACTING RATIONAL EXPRESSIONS THAT HAVE DIFFERENT DENOMINATORS

1. Find the __________ of the ___________________ ____________________.

2. Rewrite each rational expression as an ___________________ expression

   whose ___________________ is the ____________.

3. Add or subtract __________________, placing the resulting expression over the LCD.

4. If possible, _________________ the resulting rational expression.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\dfrac{5}{6x}+\dfrac{7}{8x}$

b. $3+\dfrac{1}{x}$

c. $\dfrac{2}{3x}+\dfrac{x}{x+3}$

d. $\dfrac{y}{y-5}-\dfrac{y-5}{y}$

e. $\dfrac{3x+7}{x^2-5x+6}-\dfrac{3}{x-3}$

f. $\dfrac{5}{x^2-36}+\dfrac{3}{(x+6)^2}$

**ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WHEN DENOMINATORS CONTAIN OPPOSITE FACTORS**

When one denominator contains the _______________ factor of the other, first __________________ either rational expression by ___________. Then apply the __________________ for _________________ or _________________ rational expressions that have __________________ _______________________.

Example 3: Add or subtract as indicated. Simplify the result, if possible.

a. $\dfrac{x+7}{4x+12}+\dfrac{x}{9-x^2}$

b. $\dfrac{5x}{x^2-y^2}-\dfrac{2}{y-x}$

c. $\dfrac{7y-2}{y^2-y-12}+\dfrac{2y}{4-y}+\dfrac{y+1}{y+3}$

Section 7.5: COMPLEX RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Simplify complex rational expressions by dividing
- π Simplify complex rational expressions by multiplying by the LCD

WARM-UP: Perform the indicated operation. Simplify, if possible.

1. $\frac{x+1}{x}+\frac{3x}{x+1}$

2. $\frac{x^2+x}{x^2-4}\div\frac{12x}{2x-4}$

**SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY DIVIDING**

1. If necessary, add or subtract to get a ______________ rational expression in the ____________________.
2. If necessary, add or subtract to get a ______________ rational expression in the ____________________.
3. Perform the ____________________ indicated by the main ________________ bar: __________________ the denominator of the complex rational expression and ____________________.
4. If possible, __________________.

Let's simplify the problem below using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let's replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 1: Simplify each complex rational expression.

a. $$\frac{\frac{4}{5}-x}{\frac{4}{5}+x}$$

b. $$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$$

c. $$\frac{\frac{8}{x^2}-\frac{2}{x}}{\frac{10}{x}-\frac{6}{x^2}}$$

d. $$\frac{\frac{1}{x-2}}{1-\frac{1}{x-2}}$$

## SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY MULTIPLYING BY THE LCD

1. Find the LCD of ALL ______________ expressions within the ______________ rational expression.
2. ___________________ both the ________________ and ________________ by this LCD.
3. Use the ___________________ property and multiply each ___________ in the numerator and denominator by this ____________. _______________ each term. No ________________ expressions should remain.
4. If possible, _________________ and _________________.

Let's simplify the earlier problem using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let's replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 2: Simplify each complex rational expression.

a. $\dfrac{4-\frac{7}{y}}{3-\frac{2}{y}}$

b. $$\frac{\frac{3}{x}+\frac{x}{3}}{\frac{x}{3}-\frac{3}{x}}$$

c. $$\frac{\frac{2}{x^3y}+\frac{5}{xy^4}}{\frac{5}{x^3y}-\frac{3}{xy}}$$

d. $\dfrac{\dfrac{1}{x-2}}{1-\dfrac{1}{x-2}}$

Example 3: Simplify each complex rational expression using the method of your choice.

a. $\dfrac{\dfrac{3}{x+2}-\dfrac{3}{x-2}}{\dfrac{5}{x^2-4}}$

b. $\dfrac{y^{-1}-(y+2)^{-1}}{2}$

Application:

The average rate on a round-trip commute having a one-way distance $d$ is given by the complex rational expression $\dfrac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}}$ in which $r_1$ and $r_2$ are the average rates on the outgoing and return trips, respectively.

a. Simplify the expression.

b. Find your average rate if you drive to the campus averaging 40 mph and return home on the same route averaging 30 mph.

Section 7.6: SOLVING RATIONAL EQUATIONS

When you are done with your homework you should be able to…

- $\pi$ Solve rational equations
- $\pi$ Solve problems involving formulas with rational expressions
- $\pi$ Solve a formula with a rational expression for a variable

WARM-UP:

Solve.

$$3x^2 - 2x - 8 = 0$$

SOLVING RATIONAL EQUATIONS

1. List _____________ on the variable. (Remember—no _________ in the denominator!)

2. Clear the equation of fractions by multiplying __________ sides of the equation by the LCD of _________ rational expressions in the equation.

3. ___________ the resulting equation.

4. Reject any proposed solution that is in the list of _________________ on the variable. ________________ other proposed solutions in the ________________ equation.

Example 1: Solve each rational equation.

a. $\dfrac{7}{2x}=\dfrac{5}{3x}+\dfrac{22}{3}$

b. $\dfrac{10}{y+2}=3-\dfrac{5y}{y+2}$

c. $\dfrac{x-1}{2x+3}=\dfrac{6}{x-2}$

d. $\dfrac{2t}{t^2+2t+1}+\dfrac{t-1}{t^2+t}=\dfrac{6t+8}{t^3+2t^2+t}$

e. $3y^{-2}+1=4y^{-1}$

## SOLVING A FORMULA FOR A VARIABLE

Formulas and ____________ models frequently contain rational expressions. The goal is to get the ________________ variable ___________ on one side of the equation. It is sometimes necessary to _____________ out the variable you are solving for.

Example 2: Solve each formula for the specified variable.

a. $\frac{V_1}{V_2}=\frac{P_2}{P_1}$ for $V_2$

b. $z=\frac{x-\overline{x}}{s}$ for $x$

c. $f=\frac{f_1 f_2}{f_1+f_2}$ for $f_2$

## Section 7.7: APPLICATIONS USING RATIONAL EQUATIONS AND PROPORTIONS

When you are done with your homework you should be able to…

- π Solve problems involving motion
- π Solve problems involving work
- π Solve problems involving proportions
- π Solve problems involving similar triangles

WARM-UP:

A motorboat traveled 36 miles downstream, with the current, in 1.5 hours. The return trip upstream, against the current, covered the same distance, but took 2 hours. Find the boat's rate in still water and the rate of the current.

PROBLEMS INVOLVING MOTION

Recall that ____________. Rational expressions appear in _____________

problems when the conditions of the problem involve the ___________ traveled.

When we isolate time in the formula above, we get

Example 1: As part of an exercise regimen, you walk 2 miles on an indoor track. Then you jog at twice your walking speed for another 2 miles. If the total time spent walking and jogging is 1 hour, find the walking and jogging rates.

Example 2: The water's current is 2 mph. A canoe can travel 6 miles downstream, with the current, in the same amount of time that it travels 2 miles upstream, against the current. What is the canoe's average rate in still water?

PROBLEMS INVOLVING WORK

In ___________ problems, the number ____ represents one _________ job _______________. Equations in work problems are based on the following condition:

Example 3: Shannon can clean the house in 4 hours. When she worked with Rory, it took 3 hours. How long would it take Rory to clean the house if he worked alone?

Example 4: A hurricane strikes and a rural area is without food or water. Three crews arrive. One can dispense needed supplies in 10 hours, a second in 15 hours, and a third in 20 hours. How long will it take all three crews working together to dispense food and water?

## PROBLEMS INVOLVING PROPORTIONS

A **ratio** is the quotient of two numbers or two quantities. The ratio of two numbers *a* and *b* can be written as

$$a \text{ to } b \text{ or}$$
$$a : b \text{ or}$$
$$\frac{a}{b}$$

A **proportion** is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0 \text{ and } d \neq 0.$ We call *a, b, c,* and *d* the **terms** of the proportion. The cross-products *ad* and *bc* are equal.

Example 5: According to the authors of *Number Freaking,* in a global village of 200 people, 9 get drunk every day. How many of the world's 6.9 billion people (2010 population) get drunk every day?

Example 6: A person's hair length is proportional to the number of years it has been growing. After 2 years, a person's hair grows 8 inches. The longest moustache on record was grown by Kalyan Sain of India. Sain grew his moustache for 17 years. How long was each side of the moustache?

SIMILAR FIGURES

Two figures are **similar** if their corresponding angle measures are equal and their corresponding sides are proportional.

Example 7: A fifth-grade student is conducting an experiment to find the height of a tree in the schoolyard. The student measures the length of the tree's shadow and then immediately measures the length of the shadow that a yardstick forms. The tree's shadow measures 30 feet and the yardstick's shadow measures 6 feet. Find the height of the tree.