

Section 7.3: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH THE SAME DENOMINATOR

When you are done with your homework you should be able to...

- π Add rational expressions with the same denominator
- π Subtract rational expressions with the same denominator
- π Add and subtract rational expressions with opposite denominators

WARM-UP:

Simplify:

a. $\frac{b^2 - a^2}{a^2 - b^2} = \frac{-1(a^2 - b^2)}{a^2 - b^2}$

$$\begin{array}{c} +1 \\ -1 \quad -1 \\ -2 \end{array}$$

b. $\frac{x^2 - 2x + 1}{1 - x} = \frac{x^2 - 1x - 1x + 1}{-1(x-1)}$

$$= \frac{x(x-1) - 1(x-1)}{-1(x-1)}$$

$$= \frac{(x-1)(x-1)}{-1(x-1)}$$

$$= -(x-1) \text{ or } -x+1$$

ADDING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If $\frac{P}{R}$ and $\frac{Q}{R}$ are rational expressions, then

$$\frac{P}{R} + \frac{Q}{R} = \frac{P+Q}{R}$$

$$\frac{5}{6} + \frac{1}{6} = \frac{5+1}{6}$$

added num = $\frac{6}{6}$

simp. = 1

To add rational expressions with the same denominator, add numerators and place the sum over the common denominator. If possible, simplify the result.

SUBTRACTING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If $\frac{P}{R}$ and $\frac{Q}{R}$ are rational expressions, then

$$\frac{P}{R} - \frac{Q}{R} = \frac{P-Q}{R}$$

To subtract rational expressions with the same denominator, subtract numerators and place the difference over the common denominator. If possible, simplify the result.

Example 1: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{x}{15} + \frac{4x}{15} = \frac{(x) + (4x)}{15}$

$$= \frac{\cancel{15}x}{\cancel{15}3}$$

$$= \boxed{\frac{x}{3}}$$

c. $\frac{x}{x-1} - \frac{1}{x-1} = \frac{(x) - (1)}{x-1}$

$$= \frac{\cancel{x-1}}{\cancel{x-1}}$$

$$= \boxed{1}$$

b. $\frac{x+4}{9} + \frac{2x-25}{9} = \frac{(x+4) + (2x-25)}{9}$

$$= \frac{x+4+2x-25}{9}$$

$$= \frac{3x-21}{9} = \boxed{\frac{x-7}{3}}$$

$$= \frac{\cancel{3}(x-7)}{\cancel{9}3}$$

d. $\frac{3x+2}{3x+4} + \frac{3x+6}{3x+4} = \frac{(3x+2) + (3x+6)}{3x+4}$

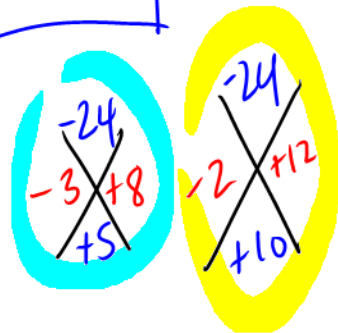
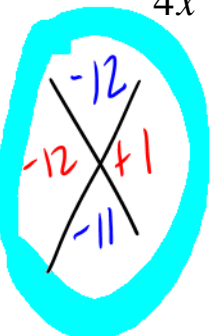
$$= \frac{3x+2+3x+6}{3x+4}$$

$$= \frac{6x+8}{3x+4}$$

$$= \frac{\cancel{2}(3x+4)}{\cancel{3x+4}1} \rightarrow \boxed{2}$$

$$\begin{aligned}
 \text{e. } \frac{x^3-3}{2x^4} - \frac{7x^3-3}{2x^4} &= \frac{(x^3-3) - (7x^3-3)}{2x^4} = -3x^{3-4} \\
 &= \frac{x^3-3-7x^3+3}{2x^4} = -3x^{-1} \\
 &= \frac{-6x^3}{2x^4} = -\frac{3}{x}
 \end{aligned}$$

$$\begin{aligned}
 \text{f. } \frac{x^2+9x}{4x^2-11x-3} + \frac{3x-5x^2}{4x^2-11x-3} &= \frac{(x^2+9x) + (3x-5x^2)}{4x^2-11x-3} \\
 &= \frac{x^2+9x+3x-5x^2}{4x^2-11x-3} \\
 &= \frac{-4x^2+12x}{4x^2-11x-3} \\
 &= \frac{-4x(x-3)}{4x^2-11x-3} \\
 &= \frac{-4x(x-3)}{4x(x-3)+1(x-3)} \\
 &= \frac{-4x(x-3)}{(x-3)(4x+1)} \\
 &= \frac{-4x}{4x+1}
 \end{aligned}$$



$$\begin{aligned}
 \text{g. } \frac{3y^2-2}{3y^2+10y-8} - \frac{y+10}{3y^2+10y-8} - \frac{y^2-6y}{3y^2+10y-8} &= \frac{(3y^2-2) - (y+10) - (y^2-6y)}{3y^2+10y-8} \\
 &= \frac{3y^2-2-y-10-y^2+6y}{3y^2+10y-8} \\
 &= \frac{2y^2+5y-12}{3y^2+10y-8} \\
 &= \frac{y(2y-3)+4(2y-3)}{(2y-3)(y+4)} \\
 &= \frac{y(3y-2)+4(3y-2)}{(3y-2)(y+4)} \\
 &= \frac{2y-3}{3y-2}
 \end{aligned}$$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH OPPOSITE DENOMINATORS

When one denominator is the opposite, or additive inverse, of the other, first multiply either rational expression by $\frac{-1}{-1}$ to obtain a common denominator.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{6x+7}{x-6} + \frac{3x}{6-x}$

$\frac{6x+7}{x-6} + \frac{3x \cdot (-1)}{(6-x) \cdot (-1)}$

$= \frac{6x+7}{x-6} + \frac{-3x}{x-6}$

$= \frac{(6x+7) + (-3x)}{x-6}$

$= \frac{6x+7-3x}{x-6}$

$= \frac{3x+7}{x-6}$

b. $\frac{x^2}{x-3} + \frac{9}{3-x}$

$\frac{x^2}{x-3} + \frac{9 \cdot (-1)}{(3-x) \cdot (-1)}$

$= \frac{x^2}{x-3} + \frac{-9}{x-3}$

$= \frac{x^2-9}{x-3}$

$= \frac{(x+3)(x-3)}{x-3}$

$= x+3$

c. $\frac{4-x}{x-9} - \frac{3x-8}{9-x}$

$\frac{(-1)(4-x)}{(-1)(x-9)} - \frac{3x-8}{9-x}$

$= \frac{-4+x}{9-x} - \frac{3x-8}{9-x}$

$= \frac{(-4+x) - (3x-8)}{9-x}$

$= \frac{-4+x-3x+8}{9-x}$

$= \frac{4-2x}{9-x}$

$= \frac{2(2-x)}{9-x}$

d. $\frac{2x+3}{x^2-x-30} + \frac{30+x-x^2}{30+x-x^2}$

$\frac{2x+3}{x^2-x-30} + \frac{-x+2}{x^2-x-30}$

$= \frac{(2x+3) + (-x+2)}{x^2-x-30}$

$= \frac{2x+3-x+2}{x^2-x-30}$

$= \frac{x+5}{x^2-x-30}$

$= \frac{x+5}{x^2-6x+5x-30}$

Diagram: A central tree diagram shows the simplification paths for each problem. It starts with the original expressions and branches into the steps of multiplying by -1, combining numerators, and simplifying. Boxes highlight the final simplified results for each problem.

Warning: A yellow circle contains a crossed-out diagram showing the incorrect simplification of $\frac{x^2-9}{x-3}$ to $\frac{-30}{-1}$.

$$= \frac{x+5}{x(x-6)+5(x-6)}$$

$$= \frac{\cancel{x+5}}{(x-6)\cancel{(x+5)}}$$

$$= \boxed{\frac{1}{x-6}}$$