Section 6.5: A GENERAL FACTORING STRATEGY

When you are done with your homework you should be able to…

- π Recognize the appropriate method for factoring a polynomial
- π Use a general strategy for factoring polynomials

WARM-UP:

Multiply:

a. $(x+1)(x^2-x+1)$

b. $(2x-3y)(4x^2+6xy+9y^2)$

A STRATEGY FOR FACTORING A POLYNOMIAL

1. If there is a ___________________ factor other than _____, factor the _____________.

2. Determine the ________________ of _____________ in the polynomial and try factoring as follows:

   a. If there are __________ terms, can the _____________ be factored by one of the following special forms?

      _____________________ of ______ ______________:

_____________ of _______ ____________:

_______________ of _________ ___________:

b. If there are _____________ terms, is the _______________ a

________________ ________________ _______________? If so,

factor by one of the following special forms:

________________________ = __________________________

________________________ = __________________________

If the trinomial is ___________ a ________________

_____________, try ________________ by ____________ and

_________________ or ___________________.

c. If there are _______________ or ______________ terms, try

_________________ by ___________________.

3. Check to see if any __________________ with more than one term in the

________________ __________________ can be factored

_____________. If so, _________________ completely.

4. _________________ by ______________________.

Example 1: Factor

a. $5x^4 - 45x^2$

b. $4x^2 - 16x - 48$

c. $4x^5 - 64x$

d. $x^3 - 4x^2 - 9x + 36$

e. $3x^3 - 30x^2 + 75x$

f. $2w^5 + 54w^2$

g. $3x^4y - 48y^5$

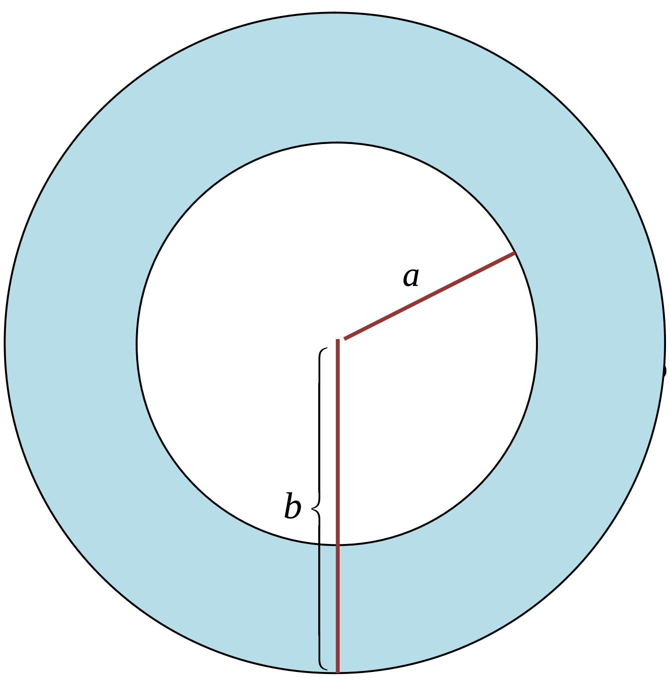
h. $12x^3 + 36x^2y + 27xy^2$

i. $12x^2(x-1)-4x(x-1)-5(x-1)$

j. $x^2+14x+49-16a^2$

APPLICATION

Express the area of the shaded ring shown in the figure in terms of π. Then factor this expression completely.



Section 6.6: SOLVING QUADRATIC EQUATIONS BY FACTORING

When you are done with your homework you should be able to…

- π Use the zero-product principle
- π Solve quadratic equations by factoring
- π Solve problems using quadratic equations

WARM-UP:

a. Factor:

$x^2 - 8x + 7$

b. Solve:

$x - 7 = 0$

DEFINITION OF A QUADRATIC EQUATION

A ________________ ___________________ in ____ is an equation that can

be written in the ________________ ______________

where _____, _____, and _____ are real numbers, with _____________. A

________________ ________________ in _____ is also called a __________-

________________ _________________ equation in ______.

SOLVING QUADRATIC EQUATIONS BY FACTORING

Consider the quadratic equation $x^2 - 8x + 7 = 0$. How is this different from the first warm-up?

We can _______________ the ____________ side of the __________________ equation __________________ to get _____________________. If a quadratic equation has a zero on one side and a __________________ ________________ on the other side, it can be _________________ using the ________________-__________________ principle.

THE ZERO-PRODUCT PRINCIPLE

If the __________________ of two or more ______________ expressions is ________________, then _____ _______________ one of them is __________ to ________________.

Example 1: Solve the following equations:

a. $2x-11=0$

b. $x+1=0$

c. $(2x-11)(x+1)=0$

STEPS FOR SOLVING A QUADRATIC EQUATION BY FACTORING

1. If necessary, ________________ the equation in ________________ form _______________________, moving all _____________ to one side, thereby obtaining ______________ on the other side.
2. ____________________.
3. Apply the _____________-__________________ principle, setting each _______________ equal to ________________.
4. ________________ the equations formed in step 3.
5. ________________ the _________________ in the _________________ equation.

Example 2: Solve:

a. $x(x+9)=0$

b. $8(x-5)(3x+11)=0$

c. $x^2+x-42=0$

d. $x^2=8x$

e. $4x^2 = 12x - 9$

f. $(x+3)(3x+5) = 7$

g. $x^3 - 4x = 0$

h. $(x-3)^2+2(x-3)-8=0$

APPLICATION

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The formula $h=-16t^2+72t$ describes the height of the debris above the ground, *h*, in feet, *t* seconds after the explosion.

a. How long will it take for the debris to hit the ground?

b. When will the debris be 32 feet above the ground?

Section 7.1: RATIONAL EXPRESSIONS AND THEIR SIMPLIFICATION

When you are done with your homework you should be able to…

- π Find numbers for which a rational expression is undefined
- π Simplify rational expressions
- π Solve applied problems involving rational expressions

WARM-UP:

a. Factor:

$x^3 - 8x^2 + 2x - 16$

b. Solve:

$2x^2 - x - 10 = 0$

EXCLUDING NUMBERS FROM RATIONAL EXPRESSIONS

A ________________ expression is the _________________ of two _____________________. Rational expressions indicate ___________________ and division by ________________ is ____________________. This means that we ______________ ____________________ any value or values of the ___________ that make a ____________________ _______________!

Example 1: Find all numbers for which the rational expression is undefined:

a. $\frac{5}{x}$

b. $\frac{x+1}{x-4}$

c. $\frac{8x-40}{x^2+3x-28}$

d. $\frac{x-12}{x^2+4}$

SIMPLIFYING RATIONAL EXPRESSIONS

A ________________ ________________ is ________________ if its ___________________ and _________________ have ______ common _______________ other than ______ or ______.

FUNDAMENTAL PRINCIPLE OF RATIONAL EXPRESSIONS

If _____, _____, and _____ are ___________________ and ______ and _____ are ___________ ______,

STEPS FOR SIMPLIFYING RATIONAL EXPRESSIONS

1. _______________ the _______________ and the __________________ completely.
2. ________________ both the _________________ and the ________________ by any _______________ _______________.

Example 2: Simplify:

a. $\dfrac{4x-64}{16x}$

b. $\dfrac{6y+18}{11y+33}$

c. $\dfrac{x^2-12x+36}{4x-24}$

d. $\dfrac{x^3+4x^2-3x-12}{x+4}$

e. $\dfrac{x+5}{x-5}$

f. $\dfrac{x^3-1}{x^2-1}$

SIMPLIFYING RATIONAL EXPRESIONS WITH OPPOSITE FACTORS IN THE NUMERATOR AND DENOMINATOR

The __________________ of two __________________ that have _____________ signs and are __________________ ____________________ is ________.

Example 3: Simplify:

a. $\frac{x-3}{3-x}$

b. $\frac{9x-15}{5-3x}$

c. $\frac{x^2-4}{2-x}$

APPLICATION

A company that manufactures small canoes has costs given by the equation

$$C = \frac{20x + 20000}{x}$$

in which x is the number of canoes manufactured and C is the cost to manufacture each canoe.

a. Find the cost per canoe when manufacturing 100 canoes.

b. Find the cost per canoe when manufacturing 10000 canoes.

c. Does the cost per canoe increase or decrease as more canoes are manufactured?

Section 7.2: MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Multiply rational expressions
- π Divide rational expressions

WARM-UP:

Simplify:

a. $\dfrac{a^2-2ab+b^2}{a^2-b^2}$

b. $\dfrac{x^2-3x+2}{x-1}$

MULTIPLYING RATIONAL EXPRESSIONS

If _____, ______, ______, and _____ are polynomials, where ______ and _______, then

The _________________ of two _______________ _________________ is the _______________ of their _______________, divided by the ________________ of their ________________.

STEPS FOR MULTIPLYING RATIONAL EXPRESSIONS

1. ________________ all ___________________ and _____________________.
2. ________________ ___________________ and ___________________ by common ___________________.
3. ___________________ the remaining factors in the _________________ and ___________________ the remaining factors in the ________________.

Example 1: Multiply.

a. $\frac{x-5}{3}\cdot\frac{18}{x-8}$

c. $\frac{9y+21}{y^2-2y}\cdot\frac{y-2}{3y+7}$

b. $\frac{x}{5}\cdot\frac{30}{x-4}$

d. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$

DIVIDING RATIONAL EXPRESSIONS

If _____, ______, ______, and _____ are polynomials, where ______, ______,

and_______, then

The _________________ of two _______________ _________________ is the _______________ of the ___________ expression and the ___________ of the ______________.

Example 2: Divide.

a. $\frac{x}{3} \div \frac{3}{8}$

b. $\frac{x+5}{7} \div \frac{4x+20}{9}$

c. $\frac{y^2-2y}{15} \div \frac{y-2}{5}$

d. $\frac{x^2-4y^2}{x^2+3xy+2y^2} \div \frac{x^2-4xy+4y^2}{x+y}$

Example 3: Perform the indicated operation or operations.

a. $\frac{5x^2-x}{3x+2}\div\left(\frac{6x^2+x-2}{10x^2+3x-1}\cdot\frac{2x^2-x-1}{2x^2-x}\right)$

b. $\frac{5xy-ay-5xb+ab}{25x^2-a^2}\div\frac{y^3-b^3}{15x+3a}$

Section 7.3: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH THE SAME DENOMINATOR

When you are done with your homework you should be able to…

- π Add rational expressions with the same denominator
- π Subtract rational expressions with the same denominator
- π Add and subtract rational expressions with opposite denominators

WARM-UP:

Simplify:

a. $\frac{b^2-a^2}{a^2-b^2}$

b. $\frac{x^2-2x+1}{1-x}$

ADDING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If _____ and ______ are ___________________ expressions, then

To __________ rational expressions with the _________ ________________,

add ________________ and place the __________ over the _____________

____________________. If possible, __________________ the result.

SUBTRACTING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If _____ and ______ are ___________________ expressions, then

To ______________ rational expressions with the _________ ____________,

subtract ________________ and place the ____________________ over the

_________________ ____________________. If possible, ______________ the result.

Example 1: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{x}{15}+\frac{4x}{15}$

c. $\frac{x}{x-1}-\frac{1}{x-1}$

b. $\frac{x+4}{9}+\frac{2x-25}{9}$

d. $\frac{3x+2}{3x+4}+\frac{3x+6}{3x+4}$

e. $\frac{x^3-3}{2x^4}-\frac{7x^3-3}{2x^4}$

f. $\frac{x^2+9x}{4x^2-11x-3}+\frac{3x-5x^2}{4x^2-11x-3}$

g. $\frac{3y^2-2}{3y^2+10y-8}-\frac{y+10}{3y^2+10y-8}-\frac{y^2-6y}{3y^2+10y-8}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH OPPOSITE DENOMINATORS

When one denominator is the ____________________, or ________________ ________________, of the other, first __________________ either rational expression by __________ to obtain a ______________ _________________.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{6x+7}{x-6}+\frac{3x}{6-x}$

c. $\frac{4-x}{x-9}-\frac{3x-8}{9-x}$

b. $\frac{x^2}{x-3}+\frac{9}{3-x}$

d. $\frac{2x+3}{x^2-x-30}+\frac{x-2}{30+x-x^2}$

Section 7.4: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH DIFFERENT DENOMINATORS

When you are done with your homework you should be able to…

- π Find the least common denominator
- π Add and subtract rational expressions with different denominators

WARM-UP: Perform the indicated operation and simplify.

a. $\frac{-3}{8}+\frac{5}{12}$

b. $\frac{x+2}{x^2+x}+\frac{-1}{x^2+x}$

FINDING THE LEAST COMMON DENOMINATOR (LCD)

The ________________ _________________ denominator of several

________________ ___________________ is a ___________________ consisting

Of the ________________ of all ___________________ ________________ in

the ___________________, with each _______________ raised to the greatest

________________ of its occurrence in any denominator.

FINDING THE LEAST COMMON DENOMINATOR

1. _________________ each _______________ completely.

2. List the factors of the first _________________________.

3. Add to the list in step 2 any __________________ of the second denominator that do not appear in the list. Repeat this step for all denominators.

4. Form the __________________ of the __________________ from the list in step 3. This product is the LCD.

Example 1: Find the LCD of the rational expressions.

a. $\frac{11}{25x^2}$ and $\frac{17}{35x}$

b. $\frac{7}{y^2-49}$ and $\frac{12}{y^2-14y+49}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS THAT HAVE DIFFERENT DENOMINATORS

1. Find the __________ of the ___________________ ____________________.

2. Rewrite each rational expression as an ___________________ expression whose ___________________ is the ____________.

3. Add or subtract __________________, placing the resulting expression over the LCD.

4. If possible, _________________ the resulting rational expression.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{5}{6x}+\frac{7}{8x}$

b. $3+\frac{1}{x}$

c. $\frac{2}{3x}+\frac{x}{x+3}$

d. $\frac{y}{y-5}-\frac{y-5}{y}$

e. $\dfrac{3x+7}{x^2-5x+6}-\dfrac{3}{x-3}$

f. $\dfrac{5}{x^2-36}+\dfrac{3}{(x+6)^2}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WHEN DENOMINATORS CONTAIN OPPOSITE FACTORS

When one denominator contains the ______________ factor of the other, first _________________ either rational expression by ___________. Then apply the _________________ for ________________ or ________________ rational expressions that have _________________ ______________________.

Example 3: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{x+7}{4x+12}+\frac{x}{9-x^2}$

b. $\frac{5x}{x^2-y^2}-\frac{2}{y-x}$

c. $\frac{7y-2}{y^2-y-12}+\frac{2y}{4-y}+\frac{y+1}{y+3}$

Section 7.5: COMPLEX RATIONAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Simplify complex rational expressions by dividing
- π Simplify complex rational expressions by multiplying by the LCD

WARM-UP: Perform the indicated operation. Simplify, if possible.

a. $\frac{x+1}{x}+\frac{3x}{x+1}$

b. $\frac{x^2+x}{x^2-4}\div\frac{12x}{2x-4}$

SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY DIVIDING

1. If necessary, add or subtract to get a _____________ rational expression in the ___________________.
2. If necessary, add or subtract to get a _____________ rational expression in the ___________________.
3. Perform the ___________________ indicated by the main _______________ bar: ________________ the denominator of the complex rational expression and ___________________.
4. If possible, ________________.

Let’s simplify the problem below using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let’s replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 1: Simplify each complex rational expression.

a. $\dfrac{\frac{4}{5}-x}{\frac{4}{5}+x}$

b. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$

c. $\dfrac{\dfrac{8}{x^2}-\dfrac{2}{x}}{\dfrac{10}{x}-\dfrac{6}{x^2}}$

d. $\dfrac{\dfrac{1}{x-2}}{1-\dfrac{1}{x-2}}$

SIMPLIFYING A COMPLEX RATIONAL EXPRESSION BY MULTIPLYING BY THE LCD

1. Find the LCD of ALL ______________ expressions within the ______________ rational expression.
2. ___________________ both the ________________ and ________________ by this LCD.
3. Use the ___________________ property and multiply each ___________ in the numerator and denominator by this ____________. ______________ each term. No ________________ expressions should remain.
4. If possible, _________________ and _________________.

Let's simplify the earlier problem using this method:

$$\frac{\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}$$

Now let's replace the constants with variables and simplify using the same method.

$$\frac{\frac{1}{x}+\frac{2}{x+1}}{4-\frac{2}{x+1}}$$

Example 2: Simplify each complex rational expression.

a. $\dfrac{4-\frac{7}{y}}{3-\frac{2}{y}}$

b. $$\frac{\frac{3}{x}+\frac{x}{3}}{\frac{x}{3}-\frac{3}{x}}$$

c. $$\frac{\frac{2}{x^3y}+\frac{5}{xy^4}}{\frac{5}{x^3y}-\frac{3}{xy}}$$

d. $\dfrac{\dfrac{1}{x-2}}{1-\dfrac{1}{x-2}}$

Example 3: Simplify each complex rational expression using the method of your choice.

a. $\dfrac{\dfrac{3}{x+2}-\dfrac{3}{x-2}}{\dfrac{5}{x^2-4}}$

b. $\frac{y^{-1}-(y+2)^{-1}}{2}$

Application:

The average rate on a round-trip commute having a one-way distance d is given by the complex rational expression $\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}$ in which r_1 and r_2 are the average rates on the outgoing and return trips, respectively.

a. Simplify the expression.

b. Find your average rate if you drive to the campus averaging 40 mph and return home on the same route averaging 30 mph.

Section 7.6: SOLVING RATIONAL EQUATIONS

When you are done with your homework you should be able to…

- π Solve rational equations
- π Solve problems involving formulas with rational expressions
- π Solve a formula with a rational expression for a variable

WARM-UP:

Solve.

$3x^2 - 2x - 8 = 0$

SOLVING RATIONAL EQUATIONS

1. List ______________ on the variable. (Remember—no _________ in the denominator!)

2. Clear the equation of fractions by multiplying __________ sides of the equation by the LCD of _________ rational expressions in the equation.

3. ___________ the resulting equation.

4. Reject any proposed solution that is in the list of _________________ on the variable. ________________ other proposed solutions in the ________________ equation.

Example 1: Solve each rational equation.

a. $\frac{7}{2x} = \frac{5}{3x} + \frac{22}{3}$

b. $\frac{10}{y+2} = 3 - \frac{5y}{y+2}$

c. $\frac{x-1}{2x+3}=\frac{6}{x-2}$

d. $\frac{2t}{t^2+2t+1}+\frac{t-1}{t^2+t}=\frac{6t+8}{t^3+2t^2+t}$

e. $3y^{-2} + 1 = 4y^{-1}$

SOLVING A FORMULA FOR A VARIABLE

Formulas and ____________ models frequently contain rational expressions. The goal is to get the ________________ variable ___________ on one side of the equation. It is sometimes necessary to _____________ out the variable you are solving for.

Example 2: Solve each formula for the specified variable.

a. $\frac{V_1}{V_2} = \frac{P_2}{P_1}$ for V_2

b. $z = \frac{x - \overline{x}}{s}$ for x

c. $f = \frac{f_1 f_2}{f_1 + f_2}$ for f_2

Section 7.7: APPLICATIONS USING RATIONAL EQUATIONS AND PROPORTIONS

When you are done with your homework you should be able to…

- π Solve problems involving motion
- π Solve problems involving work
- π Solve problems involving proportions
- π Solve problems involving similar triangles

WARM-UP:

A motorboat traveled 36 miles downstream, with the current, in 1.5 hours. The return trip upstream, against the current, covered the same distance, but took 2 hours. Find the boat's rate in still water and the rate of the current.

PROBLEMS INVOLVING MOTION

Recall that ____________. Rational expressions appear in _____________ problems when the conditions of the problem involve the ___________ traveled.

When we isolate time in the formula above, we get

Example 1: As part of an exercise regimen, you walk 2 miles on an indoor track. Then you jog at twice your walking speed for another 2 miles. If the total time spent walking and jogging is 1 hour, find the walking and jogging rates.

Example 2: The water's current is 2 mph. A canoe can travel 6 miles downstream, with the current, in the same amount of time that it travels 2 miles upstream, against the current. What is the canoe's average rate in still water?

PROBLEMS INVOLVING WORK

In ___________ problems, the number ____ represents one ________ job ________________. Equations in work problems are based on the following condition:

Example 3: Shannon can clean the house in 4 hours. When she worked with Rory, it took 3 hours. How long would it take Rory to clean the house if he worked alone?

Example 4: A hurricane strikes and a rural area is without food or water. Three crews arrive. One can dispense needed supplies in 10 hours, a second in 15 hours, and a third in 20 hours. How long will it take all three crews working together to dispense food and water?

A **ratio** is the quotient of two numbers or two quantities. The ratio of two numbers *a* and *b* can be written as

$$a \text{ to } b \text{ or}$$
$$a : b \text{ or}$$
$$\frac{a}{b}$$

A **proportion** is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$. We call *a, b, c,* and *d* the **terms** of the proportion. The cross-products *ad* and *bc* are equal.

Example 5: According to the authors of *Number Freaking,* in a global village of 200 people, 9 get drunk every day. How many of the world's 6.9 billion people (2010 population) get drunk every day?

Example 6: A person's hair length is proportional to the number of years it has been growing. After 2 years, a person's hair grows 8 inches. The longest moustache on record was grown by Kalyan Sain of India. Sain grew his moustache for 17 years. How long was each side of the moustache?

SIMILAR FIGURES

Two figures are **similar** if their corresponding angle measures are equal and their corresponding sides are proportional.

Example 7: A fifth-grade student is conducting an experiment to find the height of a tree in the schoolyard. The student measures the length of the tree's shadow and then immediately measures the length of the shadow that a yardstick forms. The tree's shadow measures 30 feet and the yardstick's shadow measures 6 feet. Find the height of the tree.

Section 8.1: INTRODUCTION TO FUNCTIONS

When you are done with your homework you should be able to…

- π Find the domain and range of a relation
- π Determine whether a relation is a function
- π Evaluate a function

WARM-UP:

Evaluate $y = -x^2 - 22x + 5$ at $x = -3$.

DEFINITION OF A RELATION

A ________________ is any _______ of ordered pairs. The set of all ________ components of the ____________ pairs is called the ____________ of the relation and the set of all second components is called the ____________ of the ____________.

Example 1: Find the domain and range of the relation.

VEHICLE	NUMBER OF WHEELS
CAR	4
MOTORCYCLE	2
BOAT	0

DEFINITION OF A FUNCTION

A ________________ is a __________________ from a first set, called the

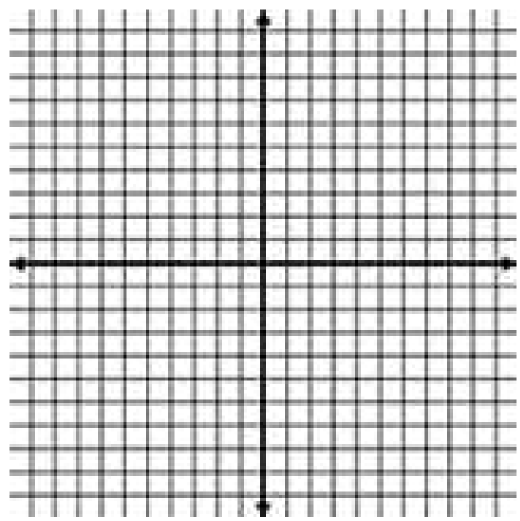
_______________, to a second set, called the _______________, such that each

_______________ in the ______________ corresponds to ___________ _______

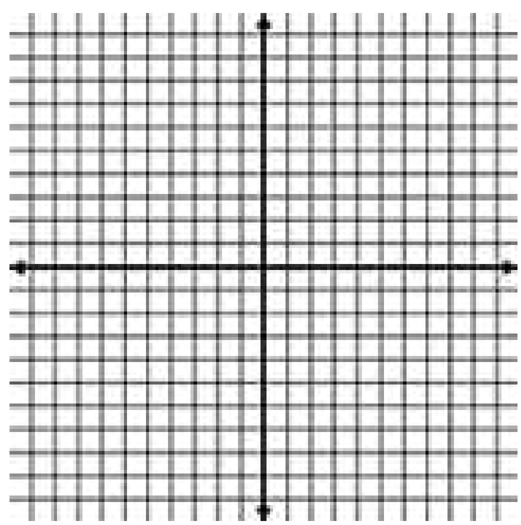
element in the ______________.

Example 2: Determine whether each relation represents a function. Then identify the domain and range.

a. $\{(-6,1),\ (-1,1),\ (0,1),\ (1,1),\ (2,1)\}$



b. $\{(3,3),\ (-2,0),\ (4,0),\ (-2,-5)\}$



FUNCTIONS AS EQUATIONS AND FUNCTION NOTATION

Functions are often given in terms of _______________ rather than as _______

of ________________ ______________. Consider the equation below, which describes the position of an object, in feet, dropped from a height of 500 feet after x seconds.

$$y = -16x^2 + 500$$

The variable ___ is a ______________ of the variable ____. For each value of x,

there is one and only one value of ____. The variable x is called the

_______________ variable because it can be _______________ any value from

the _______________. The variable y is called the _______________ variable

because its value ______________ on x. When an ________________

represents a ________________, the function is often named by a letter such as

$f,\ g,\ h,\ F,\ G,$ or H. Any letter can be used to name a function. The domain is

the _____ of the function's ______________ and the range is the _____ of the

function's _______________. If we name our function ____, the input is

represented by ____, and the output is represented by _____. The notation

_____ is read " ___ of ___" or "___ at ___. So we may rewrite $y = -16x^2 + 500$

as _____________________. Now let's evaluate our function after 10 seconds:

Example 3: Find the indicated function values for $f(x)=(-x)^3-x^2-x+10$.

a. $f(0)$

b. $f(2)$

c. $f(-2)$

d. $f(1)+f(-1)$

Example 3: Find the indicated function and domain values using the table below.

a. $h(-2)$

b. $h(1)$

c. For what values of x is $h(x)=1$?

x	$h(x)$
-2	2
-1	1
0	0
1	1
2	2

Section 8.2: GRAPHS OF FUNCTIONS

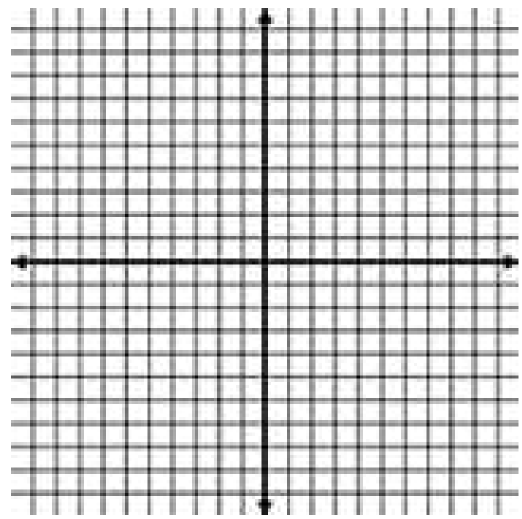
When you are done with your homework you should be able to…

- π Use the vertical line test to identify functions
- π Obtain information about a function from its graph
- π Review interval notation
- π Identify the domain and range of a function from its graph

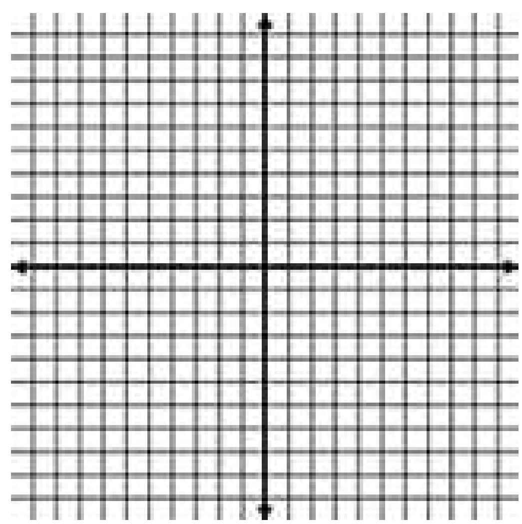
WARM-UP:

Graph the following equations by plotting points.

a. $y = x^2$



b. $y = 3x - 1$

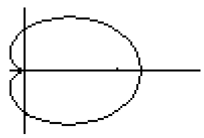


THE VERTICAL LINE TEST FOR FUNCTIONS

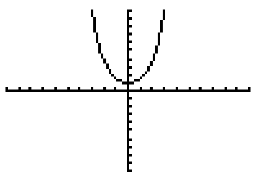
If any vertical line _________________ a graph in more than __________ point, the graph _________ __________ define ____ as a function of ____.

Example 1: Determine whether the graph is that of a function.

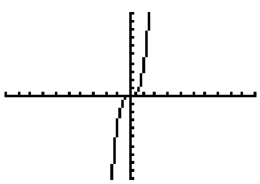
a.



b.



c.



OBTAINING INFORMATION FROM GRAPHS

You can obtain information about a function from its graph. At the right or left of a graph, you will often find ___________ dots, ___________ dots, or __________.

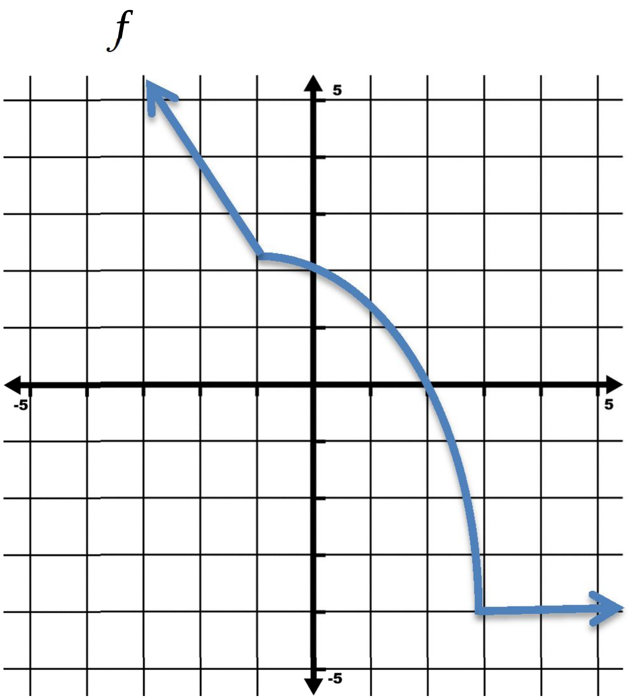
- A closed dot indicates that the graph does not ___________ beyond this point and the ____________ belongs to the ______________
- An open dot indicates that the graph does not ___________ beyond this point and the ____________ DOES NOT belong to the ______________

π An arrow indicates that the graph extends ________________ in the direction in which the arrow ________________

REVIEWING INTERVAL NOTATION

INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
(a,b)		$\longleftrightarrow x$
$[a,b]$		$\longleftrightarrow x$
$[a,b)$		$\longleftrightarrow x$
$(a,b]$		$\longleftrightarrow x$
(a,∞)		$\longleftrightarrow x$
$[a,\infty)$		$\longleftrightarrow x$
$(-\infty,b)$		$\longleftrightarrow x$
$(-\infty,b]$		$\longleftrightarrow x$
$(-\infty,\infty)$		$\longleftrightarrow x$

Example 2: Use the graph of f to determine each of the following.



a. $f(0)$

b. $f(-2)$

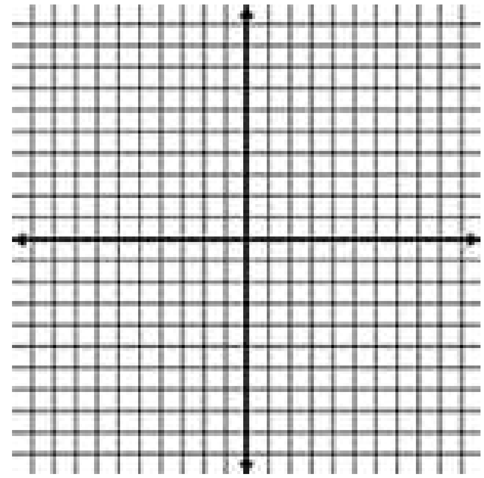
c. For what value of x is $f(x)=3$?

d. The domain of f

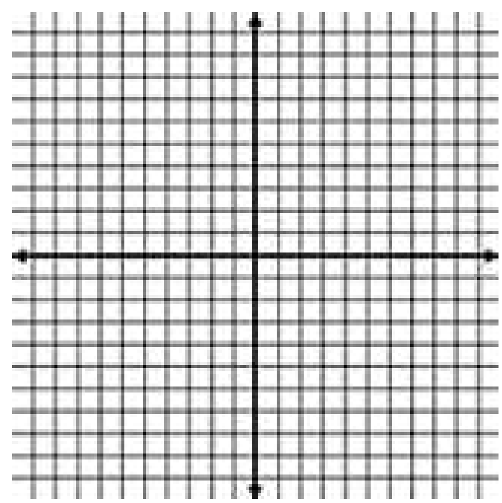
e. The range of f

Example 3: Graph the following functions by plotting points and identify the domain and range.

a. $f(x) = -x - 2$



b. $H(x) = x^2 + 1$



Section 8.3: THE ALGEBRA OF FUNCTIONS

When you are done with your homework you should be able to...

- π Find the domain of a function
- π Use the algebra of functions to combine functions and determine domains

WARM-UP:

Find the following function values for $f(x)=\sqrt{x}$

a. $f(4)$

b. $f(0)$

c. $f(196)$

FINDING A FUNCTION'S DOMAIN

If a function f does not model data or verbal conditions, its domain is the _______________ set of __________ numbers for which the value of $f(x)$ is a real number. _________________ from a function's _______________ real numbers that cause _________________ by _______________ and real numbers that result in a _______________ root of a ____________________ number.

Example 1: Find the domain of each of the following functions.

a. $f(x)=\sqrt{x-1}$

b. $g(x)=\dfrac{4-x}{1-x^2}$

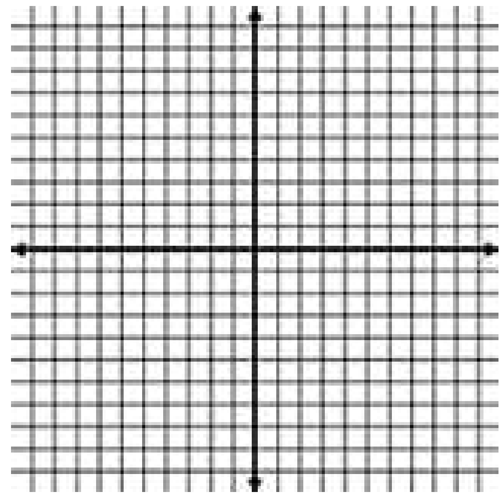
c. $h(t)=3t+5$

THE ALGEBRA OF FUNCTIONS

Consider the following two functions:

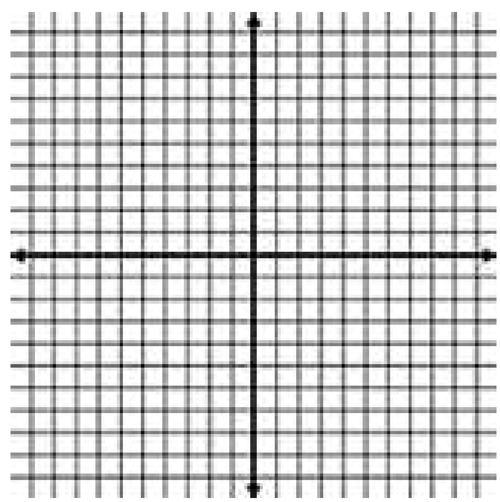
$f(x) = -x$ and $g(x) = 3x - 5$

Let's graph these two functions on the same coordinate plane.



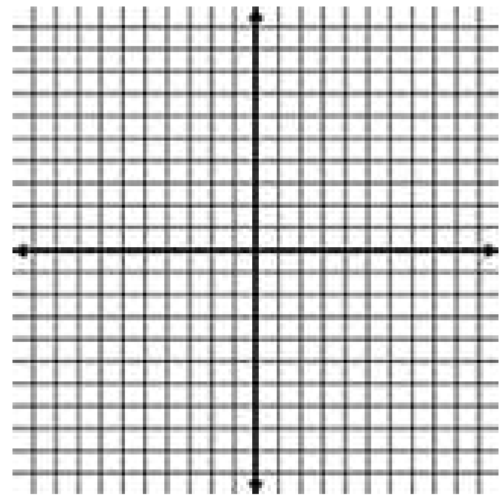
Now find and graph the sum of f and g.

$(f+g)(x) =$



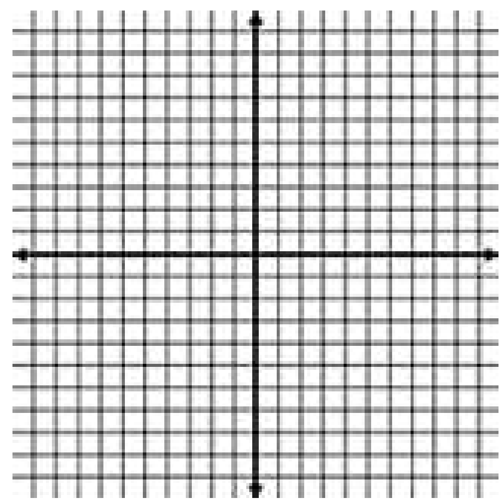
Now find and graph the difference of f and g.

$(f-g)(x)=$



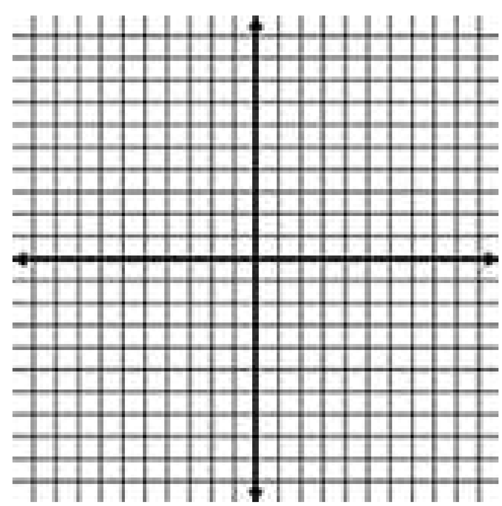
Now find and graph the product of f and g.

$(fg)(x)=$



Now find and graph the quotient of f and g.

$\left(\frac{f}{g}\right)(x)=$



THE ALGEBRA OF FUNCTIONS: SUM, DIFFERENCE, PRODUCT, AND QUOTIENT OF FUNCTIONS

Let f and g be two functions. The ______ $f+g$, the ______________ $f-g$, the _____________ fg, and the _____________ $\frac{f}{g}$ are _____________ whose domains are the set of all real numbers ________________ to the domains of f and g, defined as follows:

1. Sum: ________________________________
2. Difference: ___________________________
3. Product: ______________________________
4. Quotient: _____________________________, provided ________________

Example 2: Let $f(x)=x^2+4x$ and $g(x)=2-x$. Find the following:

a. $(f+g)(x)$

b. $(f+g)(4)$

c. $f(-3)+g(-3)$

d. $(fg)(x)$

e. $(fg)(3)$

f. The domain of $\left(\frac{f}{g}\right)(x)$

Section 8.4: COMPOSITE AND INVERSE FUNCTIONS

When you are done with your homework you should be able to…

- π Form composite functions
- π Verify inverse functions
- π Find the inverse of a function
- π Use the horizontal line test to determine if a function has an inverse function
- π Use the graph of a one-to-one function to graph its inverse function

WARM-UP:

Find the domain and range of the function $\{(-1,0),(0,1),(1,2),(2,3)\}$:

THE COMPOSITION OF FUNCTIONS

The composition of the function ____ with ____ is denoted by __________ and is defined by the equation

The domain of the ______________ function __________ is the set of all ____ such that

1. ____ is in the domain of ____ and
2. ______ is in the domain of _____.

Example 1: Given $f(x) = -x^2 + 8$ and $g(x) = 6x - 1$, find each of the following composite functions.

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

DEFINITION OF THE INVERSE OF A FUNCTION

Let f and g be two functions such that

__________________ for every ______ in the domain of _____

and

__________________ for every ______ in the domain of _____.

The function ______ is the ______________ of the function ______ and is denoted

by _______ (read " f -inverse"). Thus ________________ and _______________.

The ________________ of _____ is equal to the _____________ of _______ and

vice versa.

Example 2: Show that each function is the inverse of the other.

$f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$

FINDING THE INVERSE OF A FUNCTION

The equation of the inverse of a function f can be found as follows:

1. Replace ________ with ______ in the equation for ________.
2. Interchange _____ and _____.
3. Solve for _____. If this equation does not define ___ as a function of ___, the function _____ doe not have an _____________ function and this procedure ends. If this equation does define ___ as a function of ___, the function ___ has an inverse function.
4. If ___ has an inverse function, replace ___ in step 3 with ________. We can verify our result by showing that __________________ and ________________.

Example 3: Find an equation for $f^{-1}(x)$, the inverse function.

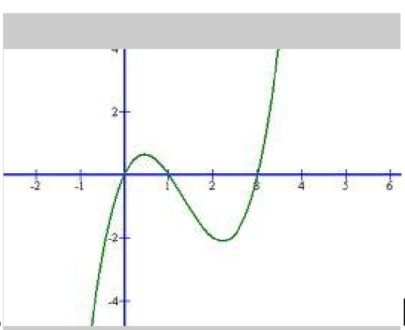
a. $f(x) = 4x$

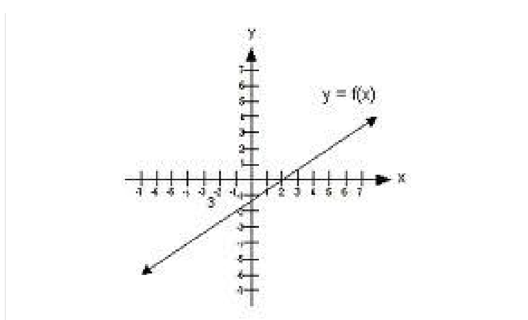
b. $f(x) = \dfrac{2x-3}{x+1}$

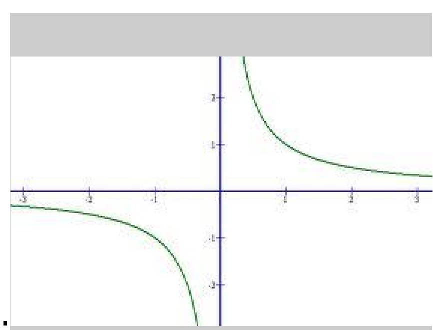
THE HORIZONTAL LINE TEST FOR INVERSE FUNCTIONS

A function f has an inverse that is a function ________, if there is no __________________ line that intersects the graph of the function ______ at more than __________ point.

Example 4: Which of the following graphs represent functions that have inverse functions?

a. 

b. 

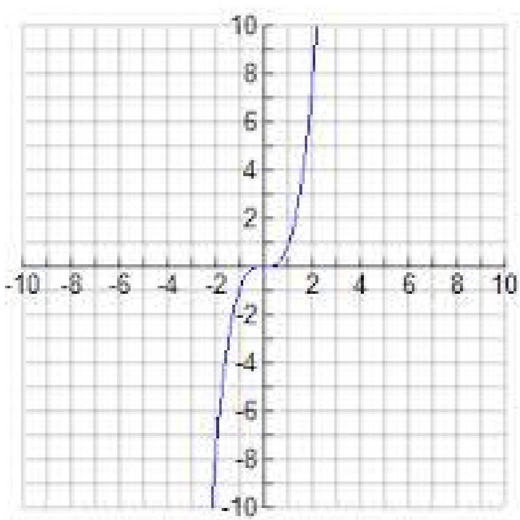
c. 

GRAPHS OF A FUNCTION AND ITS INVERSE FUNCTION

There is a ____________________ between the graph of a one-to-one function ____ and its inverse _______. Because inverse functions have ordered pairs with the coordinates ________________, if the point __________ is on the graph of _____, the point ____________ is on the graph of _________. The points ______ and ______ are ________________ with respect to the line _______.

Therefore, the graph of _______ is a ___________________ of the graph of _____ about the line __________.

Example 5: Use the graph of f below to draw the graph of its inverse function.



Section 9.3: EQUATIONS AND INEQUALITIES INVOLVING ABSOLUTE VALUE

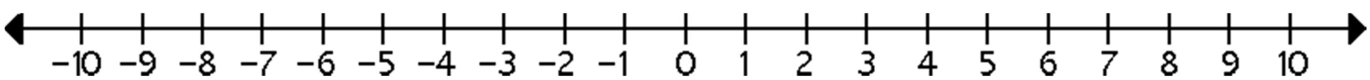
When you are done with your homework you should be able to…

- π Solve absolute value equations
- π Solve absolute value inequalities in the form $|u| < c$
- π Solve absolute value inequalities in the form $|u| > c$
- π Recognize absolute value inequalities with no solution or all real numbers as solutions
- π Solve problems using absolute value inequalities

WARM-UP:

Graph the solutions of the inequality.

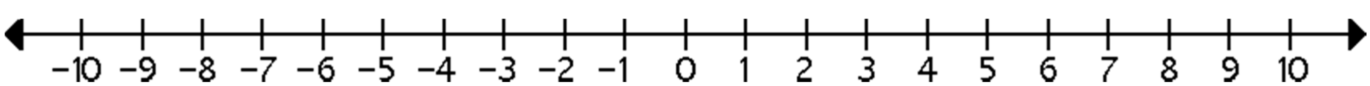
a. $-6 < x < 6$



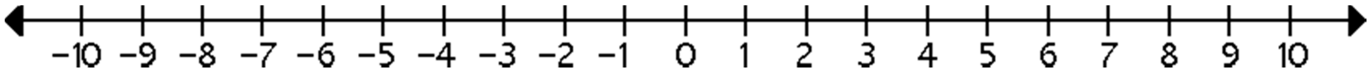
REWRITING AN ABSOLUTE VALUE EQUATION WITHOUT ABSOLUTE VALUE BARS

If ____ is a positive real number and ____ represents any ________________ expression, then ________________ is equivalent to ____________ or __________.

Consider $|x| = 6$.



Now consider $|x-3|=6$.



Example 1: Solve.

a. $|5x+7|=12$

b. $7|-x+11|=21$

c. $|x-4|-8=9$

d. $|x|+5=4$

REWRITING AN ABSOLUTE VALUE EQUATION WITH TWO ABSOLUTE VALUES WITHOUT ABSOLUTE VALUE BARS

If ______________, then ____________ or ______________.

Example 2: Solve.

$$|2x-7|=|x-12|$$

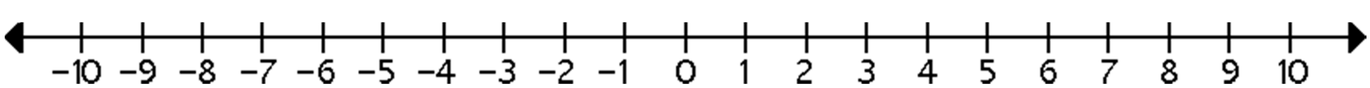
SOLVING ABSOLUTE VALUE INEQUALITIES OF THE FORM $|u|<c$

If ____ is a positive real number and ____ represents any ________________ expression, then

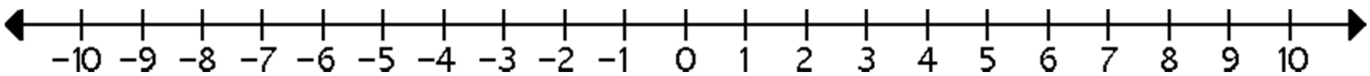
This rule is valid if __________ is replaced by _________.

Example 3: Solve and graph the solution set on a number line:

a. $|x|<6$



b. $-3|2x+7|+8 \geq -1$



SOLVING ABSOLUTE VALUE INEQUALITIES OF THE FORM $|u| > c$

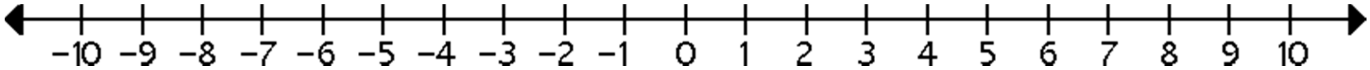
If _____ is a positive real number and _____ represents any ________________

expression, then

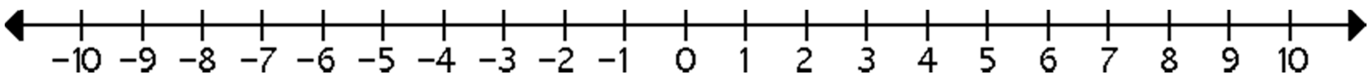
This rule is valid if __________ is replaced by __________.

Example 4: Solve and graph the solution set on a number line:

a. $|x| > 6$



b. $5|12x-1|-10 \geq 2$



ABSOLUTE VALUE INEQUALITIES WITH UNUSUAL SOLUTION SETS

If _____ algebraic expression and ______ is a __________________ number,

1. The inequality _________________ has ______ solution.

2. The inequality _________________ is _________ for all real numbers for which

   ______ is defined.

APPLICATION

The inequality $|T-50| \leq 22$ describes the range of monthly average temperature T, in degrees Fahrenheit, for Albany, New York. Solve the inequality and interpret the solution.

Section 10.1: RADICAL EXPRESSIONS AND FUNCTIONS

When you are done with your homework you should be able to…

- π Evaluate square roots
- π Evaluate square root functions
- π Find the domain of square root functions
- π Use models that are square root functions
- π Simplify expressions of the form $\sqrt{a^2}$
- π Evaluate cube root functions
- π Simplify expressions of the form $\sqrt[3]{a^3}$
- π Find even and odd roots
- π Simplify expressions of the form $\sqrt[n]{a^n}$

WARM-UP:

1. Fill in the blank.

   a. $5 \cdot \_\_ = 5^2$

   b. $x^3 \cdot \_\_ = x^6$

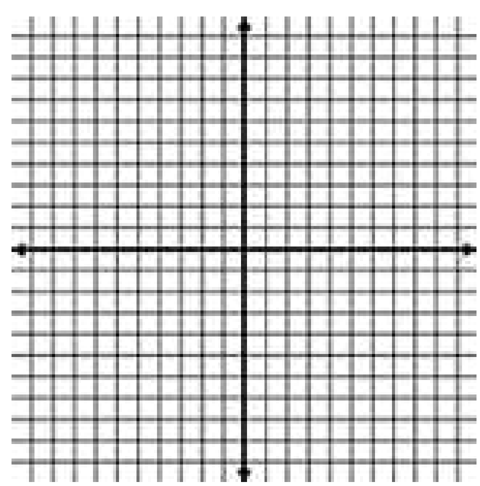
   c. $\left(y^2\right)^{\_\_} = y^{16}$

   d. $(-16)^2 = \_\_\_\_$

   e. $-(16)^2 = \_\_\_\_\_$

2. Solve $|x| = 3$.

3. Graph $f(x) = \sqrt{x}$



DEFINITION OF THE PRINCIPAL SQUARE ROOT

If _____ is a nonnegative real number, the __________________ number ____

such that ________________, denoted by ________________, is the

________________ ____________ ______________ of _____.

Example 1: Evaluate.

a. $\sqrt{169}$

b. $\sqrt{0.04}$

c. $\sqrt{\frac{49}{64}}$

d. $\sqrt{36+64}$

e. $\sqrt{36}+\sqrt{64}$

SQUARE ROOT FUNCTIONS

Because each ____________________ number, ____, has precisely one principal

square root, _______, there is a square root function defined by

The domain of this function is ___________. We can graph ____________ by

selecting nonnegative real numbers for _____. It is easiest to pick perfect

______________.

How is this different than the graph we sketched in the warm-up?

Example 2: Find the indicated function value.

a. $f(x)=\sqrt{6x+10};\ f(1)$

b. $g(x)=-\sqrt{50-2x};\ f(5)$

Example 3: Find the domain of $f(x)=\sqrt{10x-7}$

SIMPLIFYING $\sqrt{a^2}$

For any real number *a*,

In words, the principal square root of ______ is the _____________ _________ of _____.

Example 4: Simplify each expression.

a. $\sqrt{(-9)^2}$

b. $\sqrt{(x-23)^2}$

c. $\sqrt{100x^{10}}$

d. $\sqrt{x^2-14x+49}$

DEFINITION OF THE CUBE ROOT OF A NUMBER

The cube root of a real number *a* is written ____________.

____________________ means that _________________.

CUBE ROOT FUNCTIONS

Unlike square roots, the cube root of a negative number is a ____________

number. All real numbers have cube roots. Because every ____________________

number, ____, has precisely one cube root, ________, there is a cube root

function defined by

The domain of this function is ____________. We can graph _____________ by

selecting real numbers for _____. It is easiest to pick perfect ____________.

SIMPLIFYING $\sqrt[3]{a^3}$

For any real number *a*,

In words, the cube root of any expression ___________ is that expression.

Example 5: Find the indicated function value.

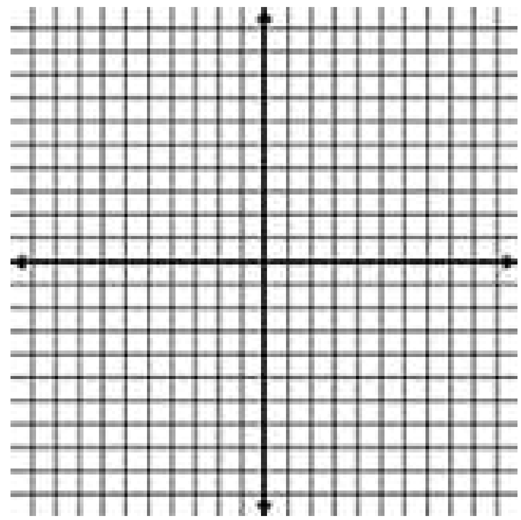
a. $f(x)=\sqrt[3]{x-20};\ f(12)$

b. $g(x)=\sqrt[3]{2x};\ g(32)$

Example 6: Graph the following functions by plotting points.

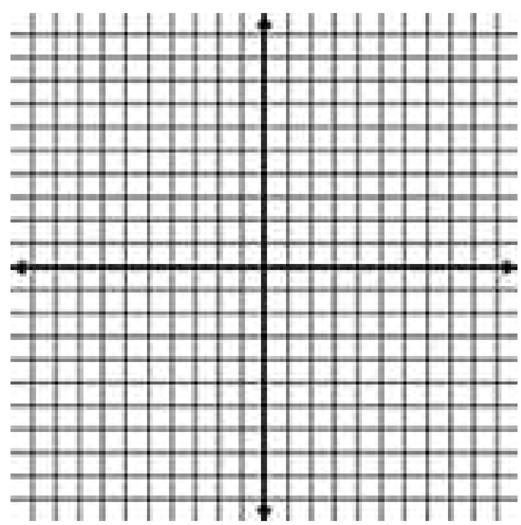
a. $f(x)=\sqrt{x+1}$

x	$f(x)=\sqrt{x+1}$	(x,y)



b. $g(x)=\sqrt[3]{x}$

x	$g(x)=\sqrt[3]{x}$	(x,y)



SIMPLIFYING $\sqrt[n]{a^n}$

For any real number *a*,

1. If *n* is even, _______________.

2. If *n* is odd, _______________.

Example 7: Simplify.

a. $\sqrt[6]{x^6}$

b. $\sqrt[5]{(2x-1)^5}$

c. $\sqrt[8]{(-2)^8}$

APPLICATION

Police use the function $f(x)=\sqrt{20x}$ to estimate the speed of a car, $f(x)$, in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road. A motorist is involved in an accident. A police officer measures the car's skid marks to be 45 feet long. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her?

Section 10.2: RATIONAL EXPONENTS

When you are done with your homework you should be able to…

- π Use the definition of $a^{\frac{1}{n}}$
- π Use the definition of $a^{\frac{m}{n}}$
- π Use the definition of $a^{-\frac{m}{n}}$
- π Simplify expressions with rational exponents
- π Simplify radical expressions using rational exponents

WARM-UP:

1. $\frac{1}{2}-\frac{3}{8}$

2. Simplify $\frac{x^2y^5}{\left(2x^3\right)^{-3}}$

THE DEFINITION OF $a^{\frac{1}{n}}$

If _____________ represents a real number and __________ is an integer, then

If *n* is even, *a* must be __________________. If *n* is odd, *a* can be any real number.

Example 1: Use radical notation to rewrite each expression. Simplify, if possible.

a. $400^{\frac{1}{2}}$

b. $\left(7xy^2\right)^{\frac{1}{3}}$

c. $\left(-32\right)^{\frac{1}{5}}$

Example 2: Rewrite with rational exponents.

a. $\sqrt[4]{12st}$

b. $\sqrt[3]{\frac{3z^2}{10}}$

c. $\sqrt{5xyz}$

THE DEFINITION OF $a^{\frac{m}{n}}$

If ____________ represents a real number, _______ is a positive rational number reduced to lowest terms, and _________ is an integer, then

and

Example 3: Use radical notation to rewrite each expression. Simplify, if possible.

a. $16^{\frac{3}{4}}$

b. $(-243)^{\frac{2}{3}}$

c. $(9)^{\frac{5}{2}}$

Example 4: Rewrite with rational exponents.

a. $\sqrt[3]{12^4}$

b. $\sqrt[5]{\left(\frac{x}{y}\right)^4}$

c. $\sqrt{(11t)^3}$

THE DEFINITION OF $a^{-\frac{m}{n}}$

If _____________ is a nonzero real number, then

Example 5: Rewrite each expression with a positive exponent. Simplify, if possible.

a. $144^{-\frac{1}{2}}$

b. $(-8)^{-\frac{2}{3}}$

c. $(32)^{-\frac{3}{5}}$

PROPERTIES OF RATIONAL EXPONENTS

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m b^n =$ ____________________.

2. $\dfrac{b^m}{b^n} =$ ____________________.

3. $\left(b^m\right)^n =$ ____________________.

4. $(ab)^n =$ ____________________.

5. $\left(\dfrac{a}{b}\right)^n =$ ____________________.

Example 6: Use properties of rational exponents to simplify each expression. Assume that all variables represent positive numbers.

a. $5^{\frac{2}{3}} \cdot 5^{\frac{1}{3}}$

b. $\left(125x^9y^6\right)^{\frac{1}{3}}$

c. $\dfrac{\left(2y^{\frac{1}{5}}\right)^4}{y^{\frac{3}{10}}}$

SIMPLIFYING RADICAL EXPRESSIONS USING RATIONAL EXPONENTS

1. Rewrite each radical expression as an ______________________ expression with a __________________ __________________.
2. Simplify using ________________ of rational exponents.
3. _______________ in radical notation if rational exponents still appear.

Example 7: Use rational exponents to simplify. If rational exponents appear after simplifying, write the answer in radical notation. Assume that all variables represent positive numbers.

a. $\left(\sqrt[3]{xy}\right)^{21}$

b. $\sqrt{3}\cdot\sqrt[3]{3}$

c. $\dfrac{\sqrt[4]{a^3b^3}}{\sqrt{ab}}$

Section 10.3: MULTIPLYING AND SIMPLIFYING RADICAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Use the product rule to multiply radicals
- π Use factoring and the product rule to simplify radicals
- π Multiply radicals and then simplify

WARM-UP:

1. Use properties of rational exponents to simplify each expression. Assume that all variables represent positive numbers.

a. $\dfrac{4^{\frac{2}{3}}}{4^{\frac{1}{3}}}$

b. $\left(196x^{10}y^{22}\right)^{\frac{1}{2}}$

2. Factor out the greatest common factor.

$8x^{\frac{1}{4}}+16x$

3. Multiply

$\left(x^{\frac{1}{2}}+3\right)\left(x^{\frac{3}{2}}-10\right)$

THE PRODUCT RULE FOR RADICALS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

The _________________ of two ___________ ______________ is the ________ root of the ________________ of the radicals.

Example 1: Multiply.

a. $\sqrt{2}\cdot\sqrt{11}$

b. $\sqrt[3]{4x}\cdot\sqrt[3]{12x}$

c. $\sqrt{x-1}\cdot\sqrt{x+1}$

SIMPLIFYING RADICAL EXPRESSIONS BY FACTORING

A radical expression whose index is *n* is __________________ when its radicand has no ________________ that are perfect ________ powers. To simplify, use the following procedure:

1. Write the radicand as the ________________ of two factors, one of which is the _______________ perfect ___________ power.
2. Use the _______________ rule to take the _______ root of each factor.
3. Find the ________ root of the perfect *n*th power.

Example 2: Simplify by factoring. Assume that all variables represent positive numbers.

a. $\sqrt{12}$

b. $\sqrt[3]{81x^5}$

c. $\sqrt{288x^{11}y^{14}z^3}$

For the remainder of this chapter, in situations that do not involve functions, we will **assume that no radicands involve negative quantities raised to even powers. Based upon this assumption, absolute vale bars are not necessary when taking even roots.

SIMPLIFYING WHEN VARIABLES TO EVEN POWERS IN A RADICAND ARE NONNEGATIVE QUANTITIES

For any ________________________ real number *a*,

Example 3: Simplify.

a. $\sqrt{108x^4y^3}$

b. $\sqrt[5]{64x^8y^{10}z^5}$

c. $\sqrt[4]{32x^{12}y^{15}}$

Example 4: Multiply and simplify.

a. $\sqrt{15xy}\cdot\sqrt{3xy}$

b. $\sqrt[3]{10x^2y}\cdot\sqrt[3]{200x^2y^2}$

Example 5: Simplify.

a. $\sqrt{5xy}\cdot\sqrt{10xy^2}$

b. $\sqrt[5]{8x^4y^3z^3}\cdot\sqrt[5]{8xy^9z^8}$

c. $\left(2x^2y\sqrt[4]{8xy}\right)\left(-32xy^2\sqrt[4]{2x^2y^3}\right)$

Section 10.4: ADDING, SUBTRACTING, AND DIVIDING RADICAL EXPRESSIONS

When you are done with your 10.4 homework you should be able to…

- π Add and subtract radical expressions
- π Use the quotient rule to simplify radical expressions
- π Use the quotient rule to divide radical expressions

WARM-UP:

Simplify.

a. $\dfrac{8x^3y^5}{2x^{-2}y^2}$

b. $3xy^2\sqrt[3]{16x^2y^2}$

THE QUOTIENT RULE FOR RADICALS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and __________, then

The _______ root of a _______________ is the _____________ of the

_______ roots of the _________________ and _________________.

Example 1: Simplify using the quotient rule.

a. $\sqrt{\dfrac{20}{9}}$

b. $\sqrt[3]{\dfrac{x^6}{27y^{12}}}$

DIVIDING RADICAL EXPRESSIONS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and __________, then

To ____________ two radical expressions with the SAME ___________, divide

the radicands and retain the ______________ _____________.

Example 2: Divide and, if possible, simplify.

a. $\dfrac{\sqrt{120x^4}}{\sqrt{3x}}$

b. $\dfrac{\sqrt[3]{128x^4y^2}}{\sqrt[3]{2xy^{-4}}}$

Example 3: Perform the indicated operations.

a. $\sqrt{2}+5\sqrt{2}$

b. $-\sqrt{20x^3}+3x\sqrt{80x}$

c. $\dfrac{\sqrt{27}}{2}+\dfrac{\sqrt{75}}{7}$

d. $\dfrac{16x^4\sqrt[3]{48x^3y^2}}{8x^3\sqrt[3]{3x^2y}}-\dfrac{20\sqrt[3]{2x^3y}}{4\sqrt[3]{x^{-1}}}$

10.5: MULTIPLYING WITH MORE THAN ONE TERM AND RATIONALIZING DENOMINATORS

When you are done with your 10.5 homework you should be able to…

- π Multiply radical expressions with more than one term
- π Use polynomial special products to multiply radicals
- π Rationalize denominators containing one term
- π Rationalize denominators containing two terms
- π Rationalize numerators

WARM-UP:

Multiply.

a. $x^{\frac{1}{2}}\left(x-3\right)$

b. $\left(x^2-5\right)\left(x^2+5\right)$

c. $\left(3x-1\right)^2$

MULTIPLYING RADICAL EXPRESSIONS WITH MORE THAN ONE TERM

Radical expressions with more than one term are multiplied in much the same way

as ______________________ with more than one term are multiplied.

Example 1: Multiply.

a. $\sqrt{5}\left(x+\sqrt{10}\right)$

c. $\left(3\sqrt{3}-4\sqrt{2}\right)\left(6\sqrt{3}-10\sqrt{2}\right)$

b. $\sqrt[3]{y^2}\left(\sqrt[3]{16}-\sqrt[3]{y}\right)$

Example 2: Multiply.

a. $\left(x-\sqrt{10}\right)\left(x+\sqrt{10}\right)$

b. $\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$

c. $\left(\sqrt{3}+\sqrt{15}\right)^2$

CONJUGATES

Radical expressions that involve the ____________ and _________________ of the ____________ two terms are called ______________________.

RATIONALIZING DENOMINATORS CONTAINING ONE TERM

When you _______________ a radical expression as an ____________________ expression in which the denominator no longer contains any ________________.

When the denominator contains a _______________ radical with an *n*th root, multiply the ________________ and the ____________________ by a radical of index *n* that produces a perfect __________ power in the denominator's radicand.

Example 3: Rationalize each denominator.

a. $\frac{2}{\sqrt{3}}$

c. $\sqrt{\frac{5}{6xy}}$

b. $\sqrt[3]{\frac{13}{2}}$

d. $\frac{4x}{\sqrt[4]{8xy^3}}$

RATIONALIZING DENOMINATORS CONTAINING TWO TERMS

When the denominator contains two terms with one or more _____________ roots, **multiply the ________________ and the ____________________ by a by the ________________ of the denominator.**

Example 4: Rationalize each denominator.

a. $\dfrac{12}{1-\sqrt{3}}$

b. $\dfrac{6}{\sqrt{11}+\sqrt{5}}$

c. $\dfrac{2\sqrt{3}+7\sqrt{7}}{2\sqrt{3}-7\sqrt{7}}$

d. $\dfrac{\sqrt{x}+8}{\sqrt{x}+3}$

RATIONALIZING NUMERATORS

To rationalize a numerator, **multiply by___________ to eliminate the radical in the ____________________.**

Example 5: Rationalize each numerator.

a. $\sqrt{\frac{3}{2}}$

b. $\frac{\sqrt[3]{5x^2}}{4}$

c. $\frac{\sqrt{x}-\sqrt{2}}{x-2}$

Section 10.6: RADICAL EQUATIONS

When you are done with your homework you should be able to…

- π Solve radical equations
- π Use models that are radical functions to solve problems

WARM-UP:

Solve:

$$2x^2 - 3x = 5$$

SOLVING RADICAL EQUATIONS CONTAINING *n*th ROOTS

1. If necessary, arrange terms so that __________ radical is _______________ on one side of the equation.
2. Raise __________ sides of the equation to the _______ power to eliminate the *n*th root.
3. ___________the resulting equation. If this equation still contains radicals,

   ______________ steps 1 and 2!

4. _______________ all proposed solutions in the ________________ equation.

Example 1: Solve.

a. $\sqrt{5x-1}=8$

b. $\sqrt{2x+5}+11=6$

c. $x=\sqrt{6x+7}$

d. $\sqrt[3]{4x-3}-5=0$

e. $\sqrt{x+2}+\sqrt{3x+7}=1$

f. $2\sqrt{x-3}+4=x+1$

g. $2\left(x-1\right)^{\frac{1}{3}}=\left(x^2+2x\right)^{\frac{1}{3}}$

Example 2: If $f(x) = x - \sqrt{x-2}$, find all values of x for which $f(x) = 4$.

Example 3: Solve $r = \sqrt{\dfrac{A}{4\pi}}$ for A.

Example 4: Without graphing, find the x-intercept of the function $f(x)=\sqrt{2x-3}-\sqrt{2x}+1$.

APPLICATION

A basketball player's hang time is the time spent in the air when shooting a basket. The formula $t=\frac{\sqrt{d}}{2}$ models hang time, t, in seconds, in terms of the vertical distance of a player's jump, d, in feet.

When Michael Wilson of the Harlem Globetrotters slam-dunked a basketball 12 feet, his hang time for the shot was approximately 1.16 seconds. What was the vertical distance of his jump, rounded to the nearest tenth of a foot?

Section 10.7: COMPLEX NUMBERS

When you are done with your homework you should be able to...

- π Express square roots of negative numbers in terms of *i*
- π Add and subtract complex numbers
- π Multiply complex numbers
- π Divide complex numbers
- π Simplify powers of *i*

WARM-UP:

Rationalize the denominator:

a. $\dfrac{5}{\sqrt{x}}$

b. $\dfrac{3-\sqrt{x}}{3+\sqrt{x}}$

THE IMAGINARY UNIT *i*

The imaginary unit ____ is defined as

THE SQUARE ROOT OF A NEGATIVE NUMBER

If *b* is a positive real number, then

Example 1: Write as a multiple of *i.*

a. $\sqrt{-100}$

b. $\sqrt{-50}$

COMPLEX NUMBERS AND IMAGINARY NUMBERS

The set of all numbers in the form

with real numbers *a* and *b,* and *i*, the imaginary unit, is called the set of

______________ ________________. The real number ___ is called the complex

number is called an ___________________ number.

Example 2: Express each number in terms of *i* and simplify, if possible.

a. $7+\sqrt{-4}$

b. $-3-\sqrt{-27}$

ADDING AND SUBTRACTING COMPLEX NUMBERS

1. $(a+bi)+(c+di)$ = ___________________________

2. $(a+bi)-(c+di)$ = ___________________________

Example 3: Add or subtract as indicated. Write the result in the form $a+bi$.

a. $(6+5i)+(4+3i)$

b. $(-7+3i)-(9-10i)$

MULTIPLYING COMPLEX NUMBERS

Multiplication of complex numbers is performed the same way as multiplication of ______________________, using the ______________________ property and the FOIL method. After completing the multiplication, we replace any occurrences of ______ with _____.

Example 4: Multiply.

a. $(5+8i)(4i-3)$

b. $(2+7i)(2-7i)$

c. $\left(3+\sqrt{-16}\right)^2$

CONJUGATES AND DIVISION

The ________________ of the complex number $a+bi$ is ____________. The ________________ of the complex number $a-bi$ is ____________. Conjugates are used to ______________ complex numbers. The goal of the division procedure is to obtain a real number in the ______________________. This real number becomes the denominator of _____ and _____ in _______________. By multiplying the numerator and denominator of the quotient by the ____________________ of the denominator, you will obtain this real number in the denominator.

Example 5: Divide and simplify to the form $a+bi$.

a. $\dfrac{9}{-8i}$

b. $\dfrac{3}{4+i}$

c. $\dfrac{5i}{2-3i}$

d. $\dfrac{6-3i}{4+2i}$

e. $\dfrac{1-i}{1+i}$

SIMPLIFYING POWERS OF *i*

1. Express the given power of *i* in terms of _____.

2. Replace _______ with ________ and simplify.

Example 6: Simplify.

a. i^{14}

b. i^{15}

c. i^{46}

d. $(-i)^6$

Section 11.1: THE SQUARE ROOT PROPERTY AND COMPLETING THE SQUARE; DISTANCE AND MIDPOINT FORMULAS

When you are done with your homework you should be able to…

- π Solve quadratic equations using the square root property
- π Complete the square of a binomial
- π Solve quadratic equations by completing the square
- π Solve problems using the square root property
- π Find the distance between two points
- π Find the midpoint of a line segment

WARM-UP:

Solve.

a. $(x-1)^2 = 4$

b. $(x-5)^2 = 0$

THE SQUARE ROOT PROPERTY

If u is an algebraic expression and d is a nonzero real number, then

if ______________, then _____________ or _______________.

Equivalently,

if ______________, then _____________ or _______________.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^2 = 9$

d. $x^2 - 10x + 25 = 1$

b. $2x^2 - 10 = 0$

e. $3(x+2)^2 = 36$

c. $4x^2 + 49 = 0$

COMPLETING THE SQUARE

If $x^2 + bx$ is a binomial, then by adding $\left(\frac{b}{2}\right)^2$, which is the square of _______ the _________________ of _____, a perfect square trinomial will result.

$x^2 + bx$ __________=__________________

Example 2: Find $\left(\frac{b}{2}\right)^2$ for each expression.

a. $x^2 + 2x$

b. $x^2 - 12x$

c. $x^2 + 5x$

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Consider a quadratic equation in the form $ax^2 + bx + c$.

1. If $a \neq 1$, divide both sides of the equation by ______.
2. Isolate $x^2 + bx$.
3. Add _________ to BOTH sides of the equation.
4. Factor and simplify.
5. Apply the square root property.
6. Solve.
7. Check your solution(s) in the ____________________ equation.

Example 3: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^2+8x-2=0$

b. $x^2-3x-5=0$

c. $3x^2 - 6x = -2$

d. $4x^2 - 2x + 5 = 0$

A FORMULA FOR COMPOUND INTEREST

Suppose that an amount of money, ____, is invested at interest rate, ____, compounded annually. In ____ years, the amount, ____, or balance, in the account is given by the formula

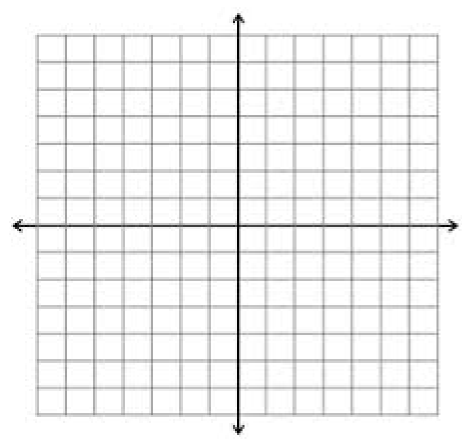
Example 4: You invested $4000 in an account whose interest is compounded annually. After 3 years, the amount, or balance, in the account is $4300. Find the annual interest rate.

THE PYTHAGOREAN THEOREM

The sum of the squares of the _______________ of the ______________ of a ______________ triangle equals the ______________ of the _____________ of the ____________________.

If the legs have lengths _____ and _____, and the hypotenuse has length ____, then

Example 5: The doorway into a room is 4 feet wide and 8 feet high. What is the diameter of the largest circular tabletop that can be taken through this doorway diagonally?



THE DISTANCE FORMULA

The distance, ____, between the points ___________ and _________ in the rectangular coordinate system is

Example 6: Find the distance between each pair of points.

a. $(5,1)$ and $(8,-2)$

b. $\left(2\sqrt{3},\sqrt{6}\right)$ and $\left(-\sqrt{3},5\sqrt{6}\right)$

THE MIDPOINT FORMULA

Consider a line segment whose endpoints are ______________ and ____________.

The coordinates of the segment's midpoints are

Example 7: Find the midpoint of the line segment with the given endpoints.

a. $(10,4)$ and $(2,6)$

b. $\left(-\frac{2}{5},\frac{7}{15}\right)$ and $\left(-\frac{2}{5},-\frac{4}{15}\right)$

Section 11.2: THE QUADRATIC FORMULA

When you are done with your homework you should be able to…

- π Solve quadratic equations using the quadratic formula
- π Use the discriminant to determine the number and type of solutions
- π Determine the most efficient method to use when solving a quadratic equation
- π Write quadratic equations from solutions
- π Use the quadratic formula to solve problems

WARM-UP:

Solve for *x* by completing the square and applying the square root property.

$$ax^2+bx+c=0$$

THE QUADRATIC FORMULA

The solutions of a quadratic equation in standard form $ax^2+bx+c=0$, with $a\neq 0$, are given by the **quadratic formula**:

STEPS FOR USING THE QUADRATIC FORMULA

1. Write the quadratic equation in ___________ form (__________________).

2. Determine the numerical values for ____, _____, and _____.

3. Substitute the values of _____, _____, and ____ into the quadratic formula and ________________ the expression.

4. Check your solution(s) in the ________________ equation.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $4x^2+3x=2$

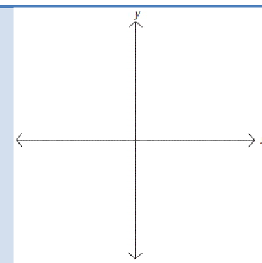
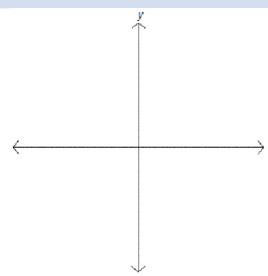
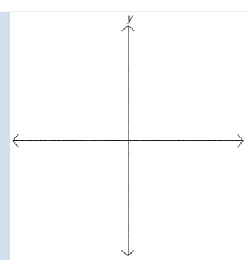
b. $3x^2 = 4x - 6$

c. $2x(x+4) = 3x - 3$

d. $x^2 + 5x - 10 = 0$

THE DISCRIMINANT

The quantity ________________, which appears under the ______________ sign in the ______________ formula, is called the __________________. The discriminant determines the _______________ and _________ of solutions of quadratic equations.

DISCRIMINANT b^2-4ac	**KINDS OF SOLUTIONS TO** $ax^2+bx+c=0$	**GRAPH OF** $y=ax^2+bx+c$
$b^2-4ac>0$		 2 ______________
$b^2-4ac=0$		 1 _______________
$b^2-4ac<0$		 NO ______________

Example 2: Compute the discriminant. Then determine the number and type of solutions.

a. $2x^2 - 4x + 3 = 0$ b. $4x^2 = 20x - 25$ c. $x^2 + 2x - 3 = 0$

DESCRIPTION AND FORM OF THE QUADRATIC EQUATION	MOST EFFICIENT SOLUTION METHOD
$ax^2 + bx + c = 0$, and $ax^2 + bx + c$ can be easily factored.	
$ax^2 + c = 0$ The quadratic equation has no ______ term (_______).	
$u^2 = d$; u is a first-degree polynomial.	
$ax^2 + bx + c = 0$, and $ax^2 + bx + c$ cannot factored or the factoring is too difficult.	

THE ZERO-PRODUCT PRINCIPLE IN REVERSE

If _____ or _______, then _____________.

Example 3: Write a quadratic equation with the given solution set.

a. $\{-2, 6\}$

b. $\{-\sqrt{3}, \sqrt{3}\}$

c. $\{2+i, 2-i\}$

Example 4: The hypotenuse of a right triangle is 6 feet long. One leg is 2 feet shorter than the other. Find the lengths of the legs.

Section 11.3: QUADRATIC FUNCTIONS AND THEIR GRAPHS

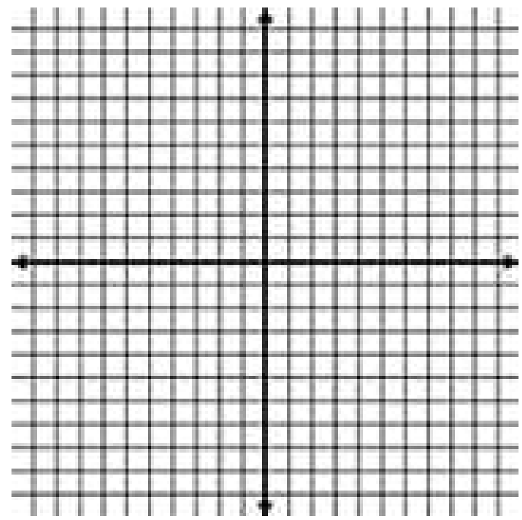
When you are done with your homework you should be able to…

- π Recognize characteristics of parabolas
- π Graph parabolas in the form $f(x)=a(x-h)^2+k$
- π Graph parabolas in the form $f(x)=a(x-h)^2+k$
- π Determine a quadratic function's minimum or maximum value
- π Solve problems involving a quadratic function's minimum or maximum value

WARM-UP: Graph the following functions by plotting points.

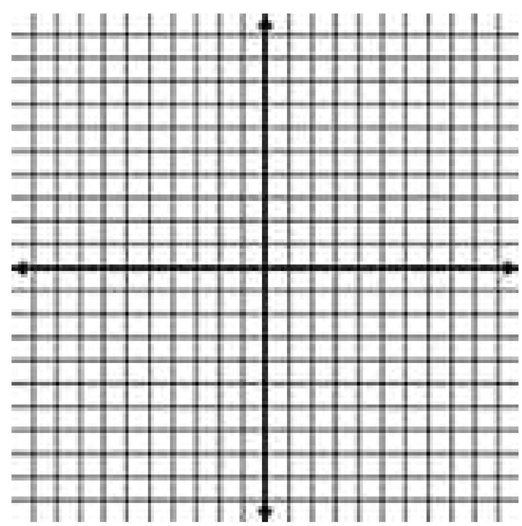
a. $f(x)=x^2$

x	$f(x)=x^2$	(x,y)



b. $f(x)=-x^2$

x	$f(x)=-x^2$	(x,y)



QUADRATIC FUNCTIONS IN THE FORM $f(x) = a(x-h)^2 + k$

The graph of

is a ___________________ whose _____________ is the point ____________.

The parabola is ___________________ with respect to the line __________. If

___________, the parabola opens upwards; if ____________, the parabola opens

_____________________.

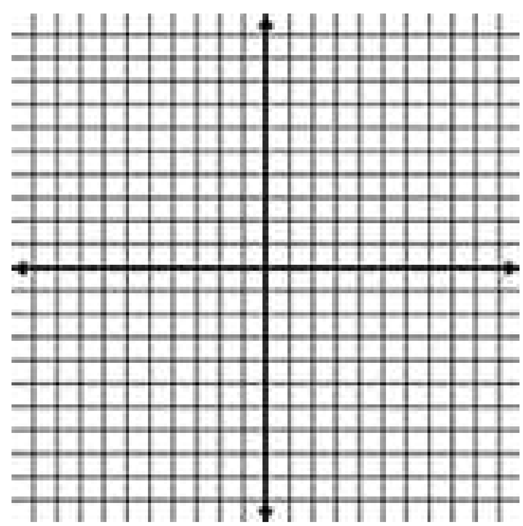
$$f(x) = a(x-h)^2 + k$$

GRAPHING QUADRATIC FUNCTIONS WITH EQUATIONS IN THE FORM $f(x)=a(x-h)^2+k$

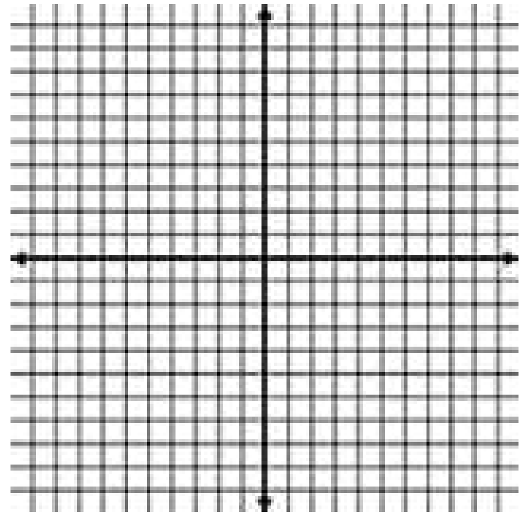
1. Determine whether the ________________ opens _______________ or ____________________. If ___________ the parabola opens upward and if ________________ the parabola opens ______________________.
2. Determine the ________________ of the parabola. The vertex is _________.
3. Find any _________________ by solving _________________.
4. Find the ______________ by computing _______.
5. Plot the _________________, the ______________, and additional points as necessary. Connect these points with a _______________ curve that is shaped like a _____________ or an inverted bowl.

Example 1: Use the vertex and intercepts to sketch the graph of each quadratic function. Use the graph to identify the function's range.

a. $f(x)=(x-1)^2-2$



b. $f(x)=2(x+2)^2-1$



THE VERTEX OF A PARABOLA WHHOSE EQUATION IS $f(x)=ax^2+bx+c$

The parabola's vertex is ________________________. The __________________ is

________________ and the ______________________ is found by substituting the

_____________________ into the parabola's equation and _____________________

the function at this value of _________.

Example 2: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

a. $f(x)=3x^2-12x+1$

b. $f(x)=-2x^2+7x-4$

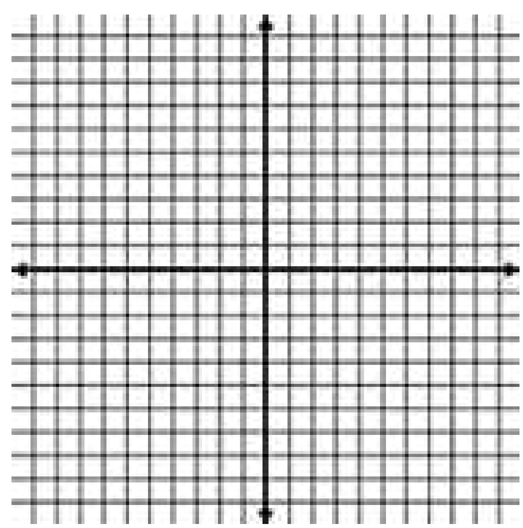
c. $f(x)=-3(x-2)^2+12$

GRAPHING QUADRATIC FUNCTIONS WITH EQUATIONS IN THE FORM $f(x) = ax^2 + bx + c$

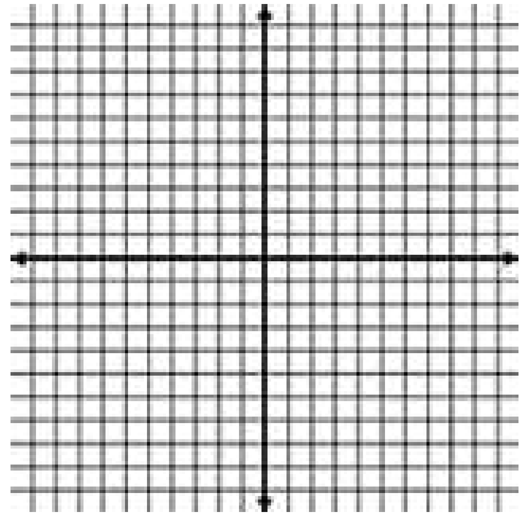
1. Determine whether the ________________ opens _______________ or ___________________. If ___________ the parabola opens upward and if ________________ the parabola opens ______________________.
2. Determine the ___________ of the parabola. The vertex is ______________.
3. Find any _________________ by solving _________________.
4. Find the ______________ by computing _______.
5. Plot the _________________, the ______________, and additional points as necessary. Connect these points with a _______________ curve that is shaped like a _____________ or an inverted bowl.

Example 3: Use the vertex and intercepts to sketch the graph of each quadratic function. Use the graph to identify the function's range.

a. $f(x) = x^2 - 2x - 15$



b. $f(x) = 5 - 4x - x^2$



MINIMUM AND MAXIMUM: QUADRATIC FUNCTIONS

Consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If _______, then ______ has a ___________________ that occurs at _______.

   This __________________ ____________ is ________________.

2. If _______, then ______ has a ___________________ that occurs at _______.

   This __________________ ____________ is ________________.

In each case, the value of _____ gives the ___________________ of the minimum or maximum value. The value of _____, or ____________, gives that minimum or maximum value.

Example 4: Among all pairs of numbers whose sum is 20, find a pair whose product is as large as possible. What is the maximum product?

Example 5: You have 200 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

Section 11.4: EQUATIONS QUADRATIC IN FORM

When you are done with your homework you should be able to…

- π Solve equations that are quadratic in form

WARM-UP: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $-5x^2+x=3$

b. $x^2=x-6$

EQUATIONS WHICH ARE QUADRATIC IN FORM

An equation that is ___________________ in ____________ is one that can be

expressed as a quadratic equation using an appropriate ____________________.

In an equation that is quadratic in form, the ____________________ factor in one

term is the __________________ of the variable factor in the other variable

term. The third term is a ___________________. By letting ____ equal the

variable factor that reappears squared, a quadratic equation in ____ will result.

Solve this quadratic equation for ____ using the methods you learned earlier.

Then use your substitution to find the values for the _________________ in the

___________________ equation.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^4 - 13x^2 + 36 = 0$

b. $x^4 + 4x^2 = 5$

c. $x + \sqrt{x} - 6 = 0$

d. $(x+3)^2 + 7(x+3) - 18 = 0$

e. $x^{-2} - 6x^{-1} = -4$

Section 12.1: EXPONENTIAL FUNCTIONS

When you are done with your homework you should be able to…

- π Evaluate exponential functions
- π Graph exponential functions
- π Evaluate functions with base *e*
- π Use compound interest formulas

WARM-UP:

Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

$$\left(x^2-2\right)^2-\left(x^2-2\right)=6$$

DEFINITION OF AN EXPONENTIAL FUNCTION

The **exponential function ____ with base ____** is defined by

where ____ is a _____________ constant other than ___ (____ and ____) and

___ is any real number.

Example 1: Determine if the given function is an exponential function.

a. $f(x) = 3^x$

b. $g(x) = (-4)^{x+1}$

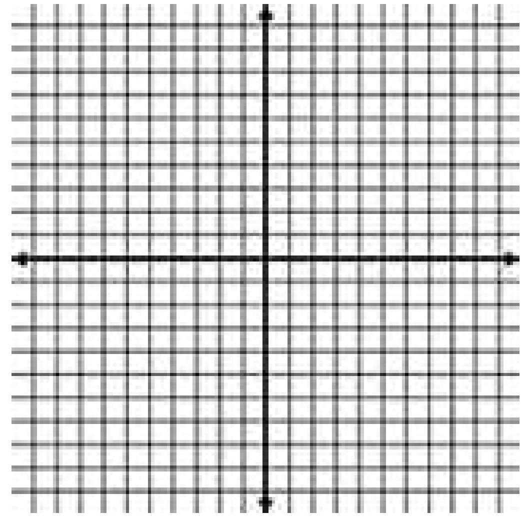
Example 2: Evaluate the exponential function at $x = -2,\ 0,\ \text{and}\ 2$.

a. $f(x) = 2^x$

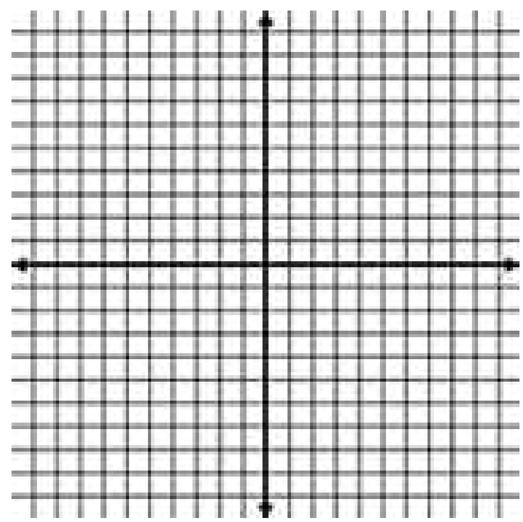
b. $g(x) = \left(\frac{1}{3}\right)^x$

Example 3: Sketch the graph of each exponential function.

a. $f(x) = 3^x$



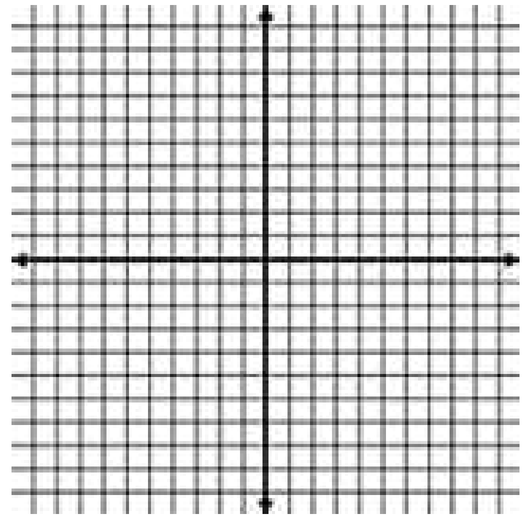
b. $g(x) = 3^{-x}$



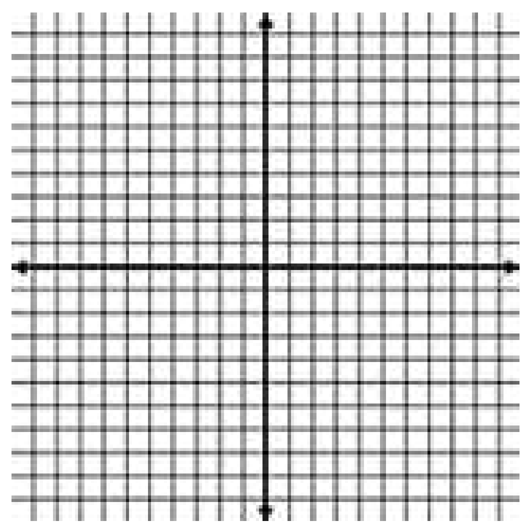
How are these two graphs related?

Example 4: Sketch the graph of each exponential function.

a. $f(x)=2^x$



b. $g(x)=2^{x+1}$



How are these two graphs related?

CHARACTERISTICS OF EXPONENTIAL FUNCTIONS OF THE FORM $f(x)=b^x$

1. The domain of $f(x)=b^x$ consists of all real numbers: _________. The range of $f(x)=b^x$ consists of all ______________ real numbers: _________.
2. The graphs of all exponential functions of the form $f(x)=b^x$ pass through the point ________ because ____________ (______). The _________________ is ___.
3. If _________, $f(x)=b^x$ has a graph that goes ____ to the ___________ and is an ______________ function. The greater the value of ____, the steeper the _______________.
4. If ________, $f(x)=b^x$ has a graph that goes _________ to the ________ and is a _________________ function. The smaller the value of ___, the steeper the _______________.
5. The graph of $f(x)=b^x$ approaches, but does not touch, the _____________. The line _______ is a ____________________ asymptote.

n	$\left(1+\frac{1}{n}\right)^n$
1	
2	
5	
10	
100	
1000	
10000	
100000	
1000000	
1000000000	

The irrational number ____, approximately ___________, is called the ____________ base. The function _______________ is called the _______________ exponential function.

FORMULAS FOR COMPOUND INTEREST

After _____ years, the balance ____, in an account with principal ____ and annual interest rate ____ (in decimal form) is given by the following formulas:

1. For ___ compounding interest periods per year:

2. For continuous compounding:

Example 5: Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% is the money is

a. compounded semiannually:

b. compounded monthly:

c. compounded continuously:

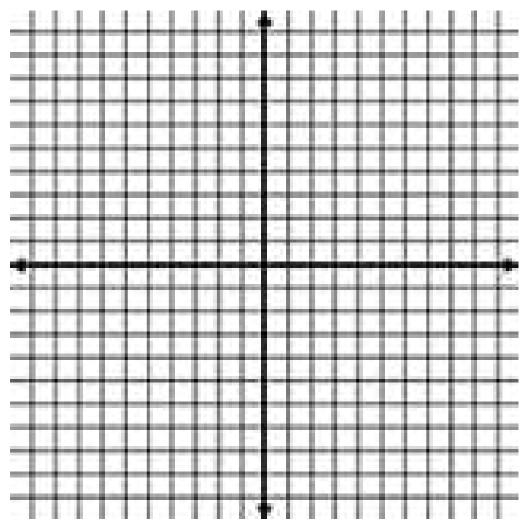
Section 12.2: LOGARITHMIC FUNCTIONS

When you are done with your homework you should be able to…

- π Change from logarithmic to exponential form
- π Change from exponential to logarithmic form
- π Evaluate logarithms
- π Use basic logarithm properties
- π Graph logarithmic functions
- π Find the domain of a logarithmic function
- π Use common logarithms
- π Use natural logarithms

WARM-UP:

Graph $y = 2^x$.



DEFINITION OF THE LOGARITHMIC FUNCTION

For _________ and ____________, __________,

___________________ is equivalent to ___________________.

The function _______________ is the **logarithmic function with base ____**.

Example 1: Write each equation in its equivalent exponential form:

a. $\log_4 x = 2$

b. $y = \log_3 81$

Example 2: Write each equation in its equivalent logarithmic form:

a. $e^y = 9$

b. $b^4 = 16$

Example 3: Evaluate.

a. $\log_5 x = 25$

b. $\log_{81} x = \frac{1}{2}$

BASIC LOGARITHMIC PROPERTIES INVOLVING 1

1. $\log_b b =$ __________ "the power to which I raise _____to get _____ is ______"

2. $\log_b 1 =$ __________ "the power to which I raise _____to get _____ is ______"

INVERSE PROPERTIES OF LOGARITHMS

For __________ and __________,

1. $\log_b b^x =$ __________

2. $b^{\log_b x} =$ __________

Example 4: Evaluate.

a. $\log_6 6$

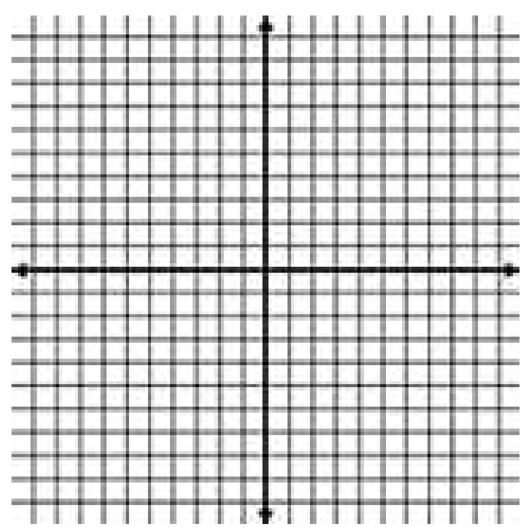
b. $\log_{12} 12^4$

c. $\log_9 1$

d. $7^{\log_7 24}$

Example 5: Sketch the graph of each logarithmic function.

$f(x) = \log_3 x$



CHARACTERISTICS OF LOGARITHMIC FUNCTIONS OF THE FORM $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: _________.

   The range of $f(x) = \log_b x$ consists of all real numbers: _________.

2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass

   through the point ________ because ___________ (______). The

   _________________ is ___. There is no __________________.

3. If _________, $f(x) = \log_b x$ has a graph that goes ____ to the __________

   and is an ______________ function.

4. If ________, $f(x) = \log_b x$ has a graph that goes _________ to the ________

   and is a ________________ function.

5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the

   _____________. The line _______ is a ___________________ asymptote.

Example 6: Find the domain.

a. $f(x)=\log_2(x-4)$

b. $f(x)=\log_5(1-x)$

COMMON LOGARITHMS

The logarithmic function with base _______ is called the **common logarithmic function.** The function ____________________ is usually expressed as ____________________. A calculator with a LOG key can be used to evaluate common logarithms.

Example 7: Evaluate.

a. $\log 1000$

b. $\log 0.01$

PROPERTIES OF COMMON LOGARITHMS

1. $\log 1 =$ ________
2. $\log 10 =$ ________
3. $\log 10^x =$ ______
4. $10^{\log x} =$ ______

Example 8: Evaluate.

a. $\log 10^3$

b. $10^{\log 7}$

NATURAL LOGARITHMS

The logarithmic function with base ______ is called the **natural logarithmic function.** The function ___________________ is usually expressed as ___________________. A calculator with a LN key can be used to evaluate common logarithms.

PROPERTIES OF NATURAL LOGARITHMS

1. $\ln 1 =$ ________
2. $\ln e =$ _______
3. $\ln e^{x} =$ ______
4. $e^{\ln x} =$ ______

Example 9: Evaluate.

a. $\ln \frac{1}{e^{6}}$

b. $e^{\ln 300}$

Example 10: Find the domain of $f(x) = \ln(x-4)^{2}$.

Section 12.3: PROPERTIES OF LOGARITHMS

When you are done with your 12.3 homework you should be able to…

- π Use the product rule
- π Use the quotient rule
- π Use the power rule
- π Expand logarithmic expressions
- π Condense logarithmic expressions
- π Use the change-of-base property

WARM-UP:

Simplify.

a. $5^x \cdot 5^x$

b. $\dfrac{2^{3x}}{2^x}$

THE PRODUCT RULE

Let ___, ___, and ___ be positive real numbers with ________.

The logarithm of a product is the _______ of the ________________.

Example 1: Expand each logarithmic expression.

a. $\log_6(6x)$

b. $\ln(x \cdot x)$

THE QUOTIENT RULE

Let ___, ___, and ___ be positive real numbers with _________.

The logarithm of a quotient is the _____________ of the ________________.

Example 2: Expand each logarithmic expression.

a. $\log \frac{1}{x}$

b. $\log_4 \frac{x}{2}$

THE POWER RULE

Let ___, ___, and ___ be positive real numbers with _________, and let ___ be any real number.

The logarithm of a quotient is the _____________ of the ________________.

Example 3: Expand each logarithmic expression.

a. $\log x^2$

b. $\log_5 \sqrt{x}$

PROPERTIES FOR EXPANDING LOGARITHMIC EXPRESSIONS

For______ and _______:

1. _________________ $= \log_b M + \log_b N$

2. _________________ $= \log_b M - \log_b N$

3. _________________ $= p \log_b M$

Example 4: Expand each logarithmic expression.

a. $\log x^3 \sqrt[3]{y}$

b. $\log_4 \sqrt{\frac{x}{12y^5}}$

PROPERTIES FOR CONDENSING LOGARITHMIC EXPRESSIONS

For______ and _______:

1. _____________________ $= \log_b (MN)$

2. _____________________ $= \log_b \frac{M}{N}$

3. _____________________ $= \log_b M^p$

Example 5: Write as a single logarithm.

a. $3\ln x - \frac{1}{4}\ln(x-2)$

b. $\log_4 5 + 12\log_4(x+y)$

THE CHANGE-OF-BASE PROPERTY

For any logarithmic bases ___ and ___, and any positive number ___,

The logarithm of ____ with base ____ is equal to the logarithm of ___ with any new base divided by the logarithm of ___ with that new base.

Why would we use this property?

Example 6: Use common logarithms to evaluate $\log_5 23$.

Example 7: Use natural logarithms to evaluate $\log_5 23$.

What did you find out???

Section 12.4: EXPONENTIAL AND LOGARITHMIC EQUATIONS

When you are done with your 12.4 homework you should be able to…

- π Use like bases to solve exponential equations
- π Use logarithms to solve exponential equations
- π Use exponential form to solve logarithmic equations
- π Use the one-to-one property of logarithms to solve logarithmic equations
- π Solve applied problems involving exponential and logarithmic equations

WARM-UP:

Solve.

$$\frac{x-1}{5}=\frac{2}{5}$$

SOLVING EXPONENTIAL EQUATIONS BY EXPRESSING EACH SIDE AS A POWER OF THE SAME BASE

If ________________________, then ____________________.

1. Rewrite the equation in the form ____________________.

2. Set _______________.

3. Solve for the variable.

Example 1: Solve.

a. $10^{x^2-1}=100$

b. $4^{x+1}=8^{3x}$

USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

1. Isolate the ___________________ expression.
2. Take the ____________________ logarithm on both sides for base ______. Take the ___________________ logarithm on both sides for bases other than 10.
3. Simplify using one of the following properties:
4. Solve for the variable.

Example 2: Solve.

a. $e^{2x}-6=32$

b. $\frac{3^{x-1}}{2}=5$

c. $10^{x}=120$

USING EXPONENTIAL FORM TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form __________________.
2. Use the definition of a logarithm to rewrite the equation in exponential form:

3. Solve for the variable.
4. Check proposed solutions in the _______________ equation. Include in the

   solution set only values for which __________.

Example 3: Solve.

a. $\log_3 x - \log_3(x-2) = 4$

b. $\log x + \log(x+21) = 2$

USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form ___________________. This form involves a _______________ logarithm whose coefficient is ___ on each side of the equation.

2. Use the one-to-one property to rewrite the equation without logarithms:

3. Solve for the variable.
4. Check proposed solutions in the _______________ equation. Include in the solution set only values for which __________ and ____________.

Example 4: Solve.

a. $2\log_6 x - \log_6 64 = 0$

b. $\log(5x+1) = \log(2x+3) + \log 2$