MATH 126/PRECALCULUS I/GRACEY

NAME Key

EXAM 3/100 POINTS POSSIBLE

CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.

1. (6 POINTS) Write the expression as a sum and/or difference of logs. Write powers as factors.

$\log\sqrt[4]{\dfrac{x(x^2+1)}{x-12}},\ x>12$

$= \log\left(\dfrac{x(x^2+1)}{x-12}\right)^{1/4}$

$= \dfrac{1}{4}\log\left(\dfrac{x(x^2+1)}{x-12}\right)$

$= \dfrac{1}{4}\left[\log x(x^2+1) - \log(x-12)\right]$

$= \dfrac{1}{4}\left[\log x + \log(x^2+1) - \log(x-12)\right]$

2. (32 POINTS) Solve the following equations. Each problem is worth 8 points. Please give exact answers and show all work.

a. $3^{2x-7} = 27^x$

$3^{2x-7} = (3^3)^x$

$3^{2x-7} = 3^{3x}$

$2x-7 = 3x$

$-7 = x$

$\{-7\}$

b. $\log(5x+2) = 1+\log(x-9)$

$-\log(x-9) \quad -\log(x-9)$

$\log(5x+2) - \log(x-9) = 1$

$\log\left(\dfrac{5x+2}{x-9}\right) = 1$

$(x-9)10^1 = \left(\dfrac{5x+2}{x-9}\right)(x-9)$

$10x-90 = 5x+2$

$5x = 92$

$x = \dfrac{92}{5}$

$\left\{\dfrac{92}{5}\right\}$

c. $2\log_4 x = -\log_4 16$

$\log_4 x^2 = \log_4 16^{-1}$

$x^2 = 16^{-1}$

$\sqrt{x^2} = \sqrt{\dfrac{1}{16}}$

$x = \pm\dfrac{1}{4}$

$\left\{\dfrac{1}{4}\right\}$

d. $4\cdot 10^{5-x} = 7$

$\dfrac{4}{4} \quad \dfrac{7}{4}$

$\log 10^{5-x} = \log\dfrac{7}{4}$

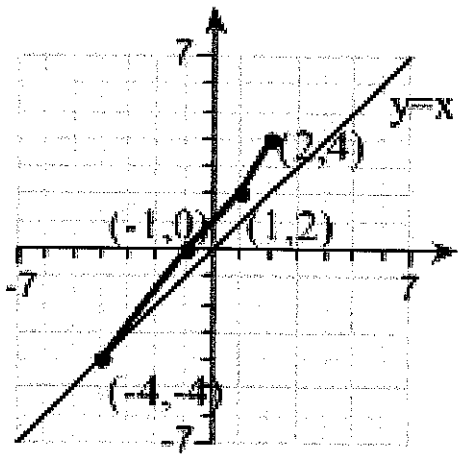
$5-x = \log\dfrac{7}{4}$

$-1(-x) = (-5+\log 7/4)(-1)$

$x = 5 - \log 7/4$

$\{5-\log 7/4\}$

3. (8 POINTS) Use the graph of $y=f(x)$ to evaluate the following:



a. $f(-4)=$ -4

b. $f(1)=$ 2

c. $f^{-1}(4)=$ 2

d. $f^{-1}(0)=$ -1

4. (6 POINTS) Express y as a function of x. The constant C is a positive number.

$\ln y=\ln x-\ln(x-8)+\ln C$

$\ln y=\ln\left(\frac{x}{x-8}\right)+\ln C$

$\ln y=\ln\left(\frac{Cx}{x-8}\right)$

$\boxed{y=\frac{Cx}{x-8}}$

5. (14 POINTS) Consider the functions $f(x)=\frac{5x+8}{x+9}$ and $g(x)=\frac{9x-8}{5-x}$.

a. (4 POINTS) Find $f(g(x))$.

$f(g(x))=f\left(\frac{9x-8}{5-x}\right)$

$=\frac{5\left(\frac{9x-8}{5-x}\right)+8}{\left(\frac{9x-8}{5-x}\right)+9}$

$=\frac{\frac{45x-40+40-8x}{5-x}}{\frac{9x-8+45-9x}{5-x}}$

$=\frac{37x}{37}\rightarrow=\boxed{x}$

b. (4 POINTS) Find $g(f(x))$.

$g(f(x))=g\left(\frac{5x+8}{x+9}\right)$

$=\frac{9\left(\frac{5x+8}{x+9}\right)-8}{5-\frac{5x+8}{x+9}}$

$=\frac{45x+72-8x-72}{5x+45-5x-8}$

$=\frac{37x}{37}$

$=\boxed{x}$

c. (4 POINTS) What is the domain of $f(g(x))$? Please give your result in interval notation

$x+9\neq 0$ / $5-x\neq 0$

$x\neq -9$ / $x\neq 5$

$\boxed{\text{Domain}:(-\infty,5)\cup(5,\infty)}$

d. (2 POINTS) Are f and g inverses of each other? CIRCLE ONE: (YES) NO

6. (4 POINTS) Evaluate the expression below. Round your result to four decimal places.

$\log_\pi 23 = \frac{\ln 23}{\ln \pi}$

≈ 2.7391

7. (6 POINTS) Find functions f and g so that $H = f \circ g$.

$H(x) = \sqrt{x-5}$

a. $f(x) = \sqrt{x}$ b. $g(x) = x-5$

8. (8 POINTS) The formula $D = 5e^{-0.9h}$ can be used to find the number of milligrams D of a certain drug that is in a patient's bloodstream h hours after the drug was administered. When the number of milligrams reaches 4, the drug is to be administered again. What is the time between injections? If necessary, please round to two decimal places.

$D = 4$

$\frac{4}{5} = \frac{5e^{-0.9h}}{5}$

$\ln \frac{4}{5} = \ln e^{-0.9h}$

$\frac{\ln 4/5}{-0.9} = \frac{-0.9h}{-0.9}$

$h \approx 0.25$ hr or every 15 minutes

9. (8 POINTS) Determine whether the function given by the table is linear, exponential, or neither. If the function is linear, find a linear function that models the data; if it is exponential, find an exponential function that models the data.

x	$f(x)$
-2	½
0	2
1	4
2	8

-1 not included!

Check for linear:

$m = \frac{8-4}{2-1} = 4$

$m = \frac{4-2}{1-0} = 2$

not linear

Check for exponential:

$\frac{8}{4} = 2$

$\frac{4}{2} = 2$

exponential

$y = 2 \cdot 2^x$

common ratio = 2, $y = Ca^x$, $C = 2, a = 2$

10. (10 POINTS) Reacting with water in an acidic solution at a particular temperature, compound A decomposes into compounds B and C according to the law of uninhibited decay. An initial amount of 0.60 M of compound A decomposes to 0.55 M in 30 minutes. Do not round until the final answer. Then round to the nearest hundredth as needed.

a. (4 POINTS) Give the mathematical function which models this situation.

$N(t) = N_0 e^{kt}$, 30 minutes = ½ hour

Find k

$\frac{0.55}{0.6} = \frac{0.6e^{k \cdot 1/2}}{0.6}$

$\frac{55}{60} = e^{k/2}$

$\ln \frac{11}{12} = \ln e^{k/2}$

$\ln \frac{11}{12} = k/2$

$2 \ln 11/12 = k$

$N(t) = 0.6e^{(2\ln 11/12)t}$

b. (3 POINTS) How much of compound A will remain after 3 hours?

$N(3) = 0.6e^{(2\ln 11/12) \cdot 3}$

$N(3) \approx 0.36 M$

c. (3 POINTS) How long will it take until 0.10 M of compound A remains?

$N(t) = 0.6e^{(2\ln 11/12)t}$

$\frac{.10}{0.6} = \frac{0.6e^{(2\ln(11/12))t}}{0.6}$

$\ln \frac{1}{6} = \ln e^{(2\ln 11/12)t}$

$\frac{\ln \frac{1}{6}}{2\ln 11/12} = \frac{(2\ln 11/12)t}{2\ln(11/12)}$

$t \approx 10.30$ hrs