MATH 126/PRECALCULUS I/GRACEY

NAME Key

EXAM 2/75 POINTS POSSIBLE

<u>YOUR GRAPHING CALCULATOR SHOULD ONLY BE USED TO CHECK YOUR RESULTS. CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.</u>

1. (5 POINTS) Let $f(x) = -2x^2 + 4x - 1$.
   a. (2 POINTS) Determine, without graphing, whether the given polynomial function has a maximum value or minimum value. Explain.

$a = -2 < 0 \longrightarrow \boxed{\text{maximum}}$

opens downward

   b. (3 POINTS) Find the minimum value or maximum value without using your calculator. <u>Do not use your graphing calculator and show all work.</u>

$-\frac{b}{2a} = -\frac{4}{2(-2)} = 1$

$\boxed{\text{maximum is 1 at } (1,1)}$

$f(1) = -2(1)^2 + 4(1) - 1 = 1$

2. (3 POINTS) Determine whether the function is a polynomial function. If it is, state the degree. If it is not, explain why not.

$f(x) = -\frac{1}{3}x^5 + x^2$

$\boxed{f \text{ is a polynomial function of degree 5}}$

3. (8 POINTS) Solve the following inequality. <u>Do not use your graphing calculator and show all work</u>.

$x^3 - 64x \le 0$

① Find zeros of $f(x) = x^3 - 64x$

$0 = x^3 - 64x$

$0 = x(x^2 - 64)$

$0 = x$ or $0 = x^2 - 64$

$\sqrt{64} = \sqrt{x^2}$

$\pm 8 = x$

② Number line sign chart

$(-\infty,-8)$ | $(-8,0)$ | $(0,8)$ | $(8,\infty)$

$-$ | $+$ | $-$ | $+$

-8 | 0 | 8

$f(-7) = (-7)^3 - 64(-7) = 105 > 0$

$f(-9) = (-9)^3 - 64(-9) = -153 < 0$

$f(1) = (1)^3 - 64(1) = -63 < 0$

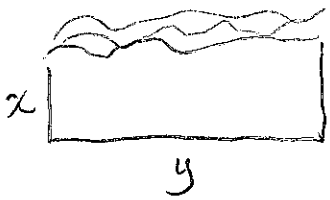
$f(9) = (9)^3 - 64(9) = 153$

③ Conclusion

$\boxed{(-\infty, -8] \cup [0, 8]}$

4. (6 POINTS) Farmer Ed has 3,500 meters of fencing, and wants to enclose a rectangular plot that borders on a river. If Farmer Ed does not fence the side along the river, what is the largest area that can be enclosed?

① Analysis



$2x + y = 3500$

$y = 3500 - 2x$

② Find Maximum

$A(x,y) = xy$

$A(x) = x(3500 - 2x)$

$A(x) = -2x^2 + 3500x$

$-\dfrac{b}{2a} = -\dfrac{3500}{2(-2)} = 875$

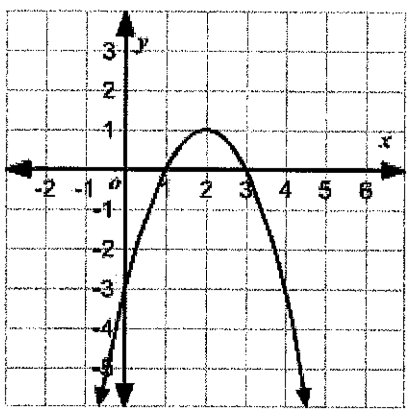
$A(875) = -2(875)^2 + 3500(875)$

$A(875) = 1531250$

③ Conclusion

The largest area that can be enclosed is $1{,}531{,}250\ m^2$.

5. (6 POINTS) Determine the quadratic function whose graph is given below.



Vertex: $(2,1)$, $(0,-3)$ is a point on the graph

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-2)^2 + 1$

$-3 = a(0-2)^2 + 1$

$-3 = 4a + 1$

$-4 = 4a \rightarrow a = -1$

$\boxed{f(x) = -(x-2)^2 + 1}$

6. (4 POINTS) Determine whether the function is linear or nonlinear. If it is linear, determine the equation of the line.

x	y
−2	4
−1	4
0	4
1	4
2	4

↑

all the same y-values w/different x-values means a horizontal line.

linear

$m = 0$

$\boxed{f(x) = 4}$

7. (8 POINTS) Form a polynomial $f(x)$ with real coefficients having the given degree and zero.

Degree 5; zeros: $4,\ -i,\ 7+i$

$x=4$ or $x=\pm i$ or $x=7+i$ or $x=7-i$ The complex conjugate will also be a zero

$x-4=0$ or $x-i=0$ or $x+i=0$ or $x-7-i=0$ or $x-7+i=0$

$0=(x-4)(x-i)(x+i)(x-7-i)(x-7+i)=0$

$0=(x-4)(x^2-i^2)(x^2-7x+\cancel{xi}-7x+49-\cancel{7i}-\cancel{xi}+\cancel{7i}-i^2$

$0=(x-4)(x^2-(-1))(x^2-14x+49-(-1))$

$0=(x-4)(x^2+1)(x^2-14x+50)$

$0=(x^3+x-4x^2-4)(x^2-14x+50)$

$0=(x^5-14x^4+50x^3+x^3-14x^2+50x-4x^4+56x^3-200x^2-4x^2+56x-200)$

$\boxed{f(x)=x^5-18x^4+107x^3-218x^2+106x-200}$

8. (8 POINTS) Find the real solutions of the following equation without using your graphing calculator.

$x^4+7x^3+8x^2-7x+15=0$.

possible rational zeros $\dfrac{\text{factors of } 15}{\text{factors of } 1}=\dfrac{\pm1,\pm3,\pm5,\pm15}{\pm1}=\pm1,\pm3,\pm5,\pm15$

Try:

$$\begin{array}{r|rrrrr} -1 & 1 & 7 & 8 & -7 & 15 \\ & & -1 & -6 & -2 & 9 \\ \hline & 1 & 6 & 2 & -9 & 24 \end{array}$$

no!

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 8 & -7 & 15 \\ & & -3 & -12 & 12 & -15 \\ \hline & 1 & 4 & -4 & 5 & 0 \end{array}$$

yaay!

$$\begin{array}{r|rrrr} -5 & 1 & 4 & -4 & 5 \\ & & -5 & 5 & -5 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

no

$x^4+7x^3+8x^2-7x+15=(x+3)(x^3+4x^2-4x+5)$

$0=(x+3)(x+5)(x^2-x+1)$

$x+3=0$ or $x+5=0$ or $x^2-x+1=0$

$x=-3,\ x=-5$ $\quad x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$x=\dfrac{1\pm\sqrt{-3}}{2}$ not a real zero

$\boxed{\{-5,-3\}}$

9. (4 POINTS) Use the rational zeros theorem to list the potential rational zeros of the polynomial function.
$f(x)=x^8+9x^6-5x^2-12x+4$

$\frac{\text{factors of } 4}{\text{factors of } 1} : \frac{\pm 1, \pm 2, \pm 4}{\pm 1}$

$\boxed{\text{potential rational zeros are } \pm 1, \pm 2, \pm 4}$

10. (20 POINTS)Consider the polynomial function $f(x)=x^3+9x^2-25x-33$

a. (1 POINT) Determine the end behavior of the graph of the function:

The graph of f behaves like y = __x^3__ for large values of x .

b. (7 POINTS) Find the x-intercept(s), if any. Do not use your graphing calculator and show all work.

$$\begin{array}{r|rrrr} -1 & 1 & 9 & -25 & -33 \\ & & -1 & -8 & 33 \\ \hline & 1 & 8 & -33 & 0 \end{array}$$

$\rightarrow f(x)=(x+1)(x^2+8x-33)$

$f(x)=(x+1)(x-3)(x+11)$

$0=x+1$ or $0=x-3$ or $0=x+11$

$x=-1$ $\quad x=3$ $\quad x=-11$

$\boxed{x=-11,-1,3 \text{ or } (-11,0),(-1,0),(3,0)}$

c. (1 POINT) Find the y-intercept, if any.

$f(0)=(0)^3+9(0)^2-25(0)-33=-33$

$\boxed{y=-33 \text{ or } (0,-33)}$

$f(x) = x^3 + 9x^2 - 25x - 33$

d. (4 POINTS) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis.

i. The zero(s) of f are $x = -11, -1, 3$.

ii. The smallest zero is a zero of multiplicity 1, so the graph of f crosses the x-axis at $x =$ -11.

iii. The middle zero is a zero of multiplicity 1, so the graph of f crosses the x-axis at $x =$ -1.

iv. The largest zero is a zero of multiplicity 1, so the graph of f crosses the x-axis at $x =$ 3.

e. (1 POINT) Use your graphing calculator to approximate the turning point(s) of the graph. Round the coordinates to 2 decimal places.

$\boxed{(-7.16, 240.33) \text{ and } (1.16, -48.33)}$

f. (1 POINT) Find the domain of the function. You may use either interval or set-builder notation.

$\boxed{(-\infty, \infty)}$

g. (1 POINT) Find the range of the function. You may use either interval or set-builder notation.

$\boxed{(-\infty, \infty)}$

h. (4 POINTS) Use the graph to determine where the function is increasing or decreasing. Give your results in interval notation.

i. On which interval(s) is the function increasing? Round the coordinates to 2 decimal places.

$\boxed{(-\infty, -7.16) \cup (1.16, \infty)}$

ii. On which interval(s) is the function decreasing? Round the coordinates to 2 decimal places.

$\boxed{(-7.16, 1.16)}$