When you are done with your homework you should be able to…

- π Understand and use the Divergence Theorem
- π Use the Divergence Theorem to calculate flux

Warm-up: Find the flux of $\mathbf{F}$ through S, $\int_S\int \mathbf{F}\cdot\mathbf{N}\,dS$, where $\mathbf{N}$ is the upward unit normal vector to S.

$$\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} - 2z\mathbf{k}$$

$$S:\ z = \sqrt{a^2 - x^2 - y^2}$$

THEOREM: THE DIVERGENCE THEOREM (aka GAUSS'S THEOREM)

Let Q be a solid region bounded by a closed surface S oriented by a unit normal vector directed outward from Q. If $\mathbf{F}$ is a vector field whose component functions have continuous partial derivatives in Q, then

$$\int_S\int \mathbf{F}\cdot\mathbf{N}\,dS = \iiint_Q \operatorname{div}\mathbf{F}\,dV$$

Example 1: Verify the Divergence Theorem by evaluating $\int_S \int \mathbf{F} \cdot \mathbf{N} dS$ as a surface integral and as a triple integral.

$\mathbf{F}(x, y, z) = xy\mathbf{i} + z\mathbf{j} + (x + y)\mathbf{k}$

S: surface bounded by the planes $y = 4$, and $z = 4 - x$ and the coordinate planes

Example 2: Use the Divergence Theorem to evaluate $\int_S\int \mathbf{F}\cdot\mathbf{N}dS$ and find the outward flux of $\mathbf{F}$ through the surface of the solid bounded by the graphs of the equations.

$$\mathbf{F}(x,y,z)=xyz\mathbf{j}$$
$$S:\ x^2+y^2=9,\ z=0,\ z=4$$

Example 3: Use the Divergence Theorem to evaluate $\int_S\int \mathbf{F}\cdot\mathbf{N}\,dS$ and find the outward flux of $\mathbf{F}$ through the surface of the solid bounded by the graphs of the equations.

$$\mathbf{F}(x,y,z)=2(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})$$
$$S:\ z=\sqrt{4-x^2-y^2},\ z=0$$