When you are done with your homework you should be able to…

- π Evaluate a surface integral as a double integral
- π Evaluate a surface integral for a parametric surface
- π Determine the orientation of a surface
- π Understand the concept of a flux integral

Warm-up: Find the principal unit normal vector to the curve $\mathbf{r}(t)=\ln t\mathbf{i}+(t+1)\mathbf{j}$ when $t=2$.

EVALUATING A SURFACE INTEGRAL

Let S a surface with equation $z=g(x,y)$ and let R be its projection onto the *xy*-plane. If g, g_x, and g_y are continuous on R and f is continuous on S, then the <u>**surface integral of *f* over *S***</u> is

$$\int_S\int f(x,y,z)\,dS=\int_R\int f(x,y,g(x,y))\sqrt{1+[g_x(x,y)]^2+[g_y(x,y)]^2}\,dA$$

Example 1: Evaluate $\int_S\int (x-2y+z)\,dS$.

$$S:\ z=\frac{2}{3}x^{3/2},\quad 0\le x\le 1,\quad 0\le y\le x$$

Example 2: Evaluate $\int_S\int f(x,y)\,dS$.

$$f(x,y)=x+y$$
$$S:\ \mathbf{r}(u,v)=4u\cos v\,\mathbf{i}+4u\sin v\,\mathbf{j}+3u\mathbf{k}$$
$$0\le u\le 4,\quad 0\le v\le \pi$$

Example 3: Evaluate $\int_S\int f(x,y,z)\,dS$.

$$f(x,y,z)=\frac{xy}{z}$$
$$S:\ z=x^2+y^2,\quad 4\le x^2+y^2\le 16$$

DEFINITION OF FLUX INTEGRAL

Let $\mathbf{F}(x,y,z)=M\mathbf{i}+N\mathbf{j}+P\mathbf{k}$ where M, N, and P have continuous first partial derivatives on the surface S oriented by a unit normal vector $\mathbf{N}$. The **flux integral** of $\mathbf{F}$ across S is given by

$$\int_S\int \mathbf{F}\cdot\mathbf{N}\,dS$$

THEOREM: EVALUATING A FLUX INTEGRAL

Let S be an oriented surface given by $z = g(x, y)$ and let R be its projection onto the *xy*-plane.

$$\int_S\int \mathbf{F}\cdot\mathbf{N}dS = \int_R\int \mathbf{F}\cdot\left[-g_x(x,y)\mathbf{i} - g_y(x,y)\mathbf{j} + \mathbf{k}\right]dA \quad \text{oriented upward}$$

$$\int_S\int \mathbf{F}\cdot\mathbf{N}dS = \int_R\int \mathbf{F}\cdot\left[g_x(x,y)\mathbf{i} + g_y(x,y)\mathbf{j} - \mathbf{k}\right]dA \quad \text{oriented downward}$$

Example 4: Find the flux of $\mathbf{F}$ through S, $\int_S\int \mathbf{F}\cdot\mathbf{N}dS$, where $\mathbf{N}$ is the upward unit normal vector to S.

$$\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j}$$
$$S: \ 2x + 3y + z = 6, \ \text{first octant}$$