When you are done with your homework you should be able to…

- π Understand the definition of and sketch a parametric surface
- π Find a set of parametric equations to represent a surface
- π Find a normal vector and a tangent plane to a parametric surface
- π Find the area of a parametric surface

Warm-up:

1. Find the unit tangent vector $\mathbf{T}(t)$ and find a set of parametric equations for the line tangent to the space curve $\mathbf{r}(t)=\langle 2\sin t, 2\cos t, 4\sin^2 t\rangle$ at the point $(1,\sqrt{3},1)$.

How do you represent a curve in the plane by a vector-valued function?

How do you represent a curve in space by a vector-valued function?

DEFINITION OF PARAMETRIC SURFACE

Let x, y and z be functions of u and v that are continuous on a domain *D* in the *uv*-plane. The set of points (x, y, z) given by

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k} \qquad \text{Parametric surface}$$

is called a **parametric surface**. The equations

$$x = x(u,v), \quad y = y(u,v), \quad \text{and} \quad z = z(u,v) \qquad \text{Parametric equations}$$

are the **parametric equations** for the surface.

Example 1: Find the rectangular equation for the surface by eliminating the parameters from the vector-valued function. Identify the surface and sketch its graph.

$$\mathbf{r}(u,v) = 2u\cos v\mathbf{i} + 2u\sin v\mathbf{j} + \frac{1}{2}u^2\mathbf{k}$$

Example 2: Find a vector-valued function whose graph is the indicated surface.

The plane $x+y+z=6$

Example 3: Write a set of parametric equations for the surface of revolution obtained by revolving the graph of $y=x^{3/2},\ \ 0\le x\le 4$ about the *x*-axis.

Example 4: Find an equation of the tangent plane to the surface represented by the vector-valued function $\mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + \sqrt{uv}\mathbf{k}$ at the point $(1,1,1)$.

AREA OF A PARAMETRIC SURFACE

Let S be a smooth parametric surface $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$ defined over an open region D in the *uv*-plane. If each point on the surface S corresponds to exactly one point in the domain D, then the **surface area** of S is given by

$$\text{Surface Area} = \int_S\int dS = \int_D\int \left\| \mathbf{r}_u \times \mathbf{r}_v \right\| dA$$

$$\text{where } \mathbf{r}_u = \frac{dx}{du}\mathbf{i} + \frac{dy}{du}\mathbf{j} + \frac{dz}{du}\mathbf{k} \text{ and } \mathbf{r}_v = \frac{dx}{dv}\mathbf{i} + \frac{dy}{dv}\mathbf{j} + \frac{dz}{dv}\mathbf{k}$$

Example 5: Find the area of the surface over the part of the paraboloid $\mathbf{r}(u,v) = 4u\cos v\mathbf{i} + 4u\sin v\mathbf{j} + u^2\mathbf{k}$, where $0 \le u \le 2 \text{ and } 0 \le v \le 2\pi$.