When you are done with your homework you should be able to…

- π Use a triple integral to find the volume of a solid region
- π Find the center of mass and moments of inertia of a solid region

Warm-up: Set up a double integral to find the volume of the solid bounded by the graphs of the equations $z=\dfrac{1}{1+y^2},\ x=0,\ x=2 \text{ and } y\geq 0.$

DEFINITION: TRIPLE INTEGRAL

If f is continuous over a bounded solid region Q, then the **triple integral of f over Q** is defined as

$$\iiint_Q f(x,y,z)\,dV=\lim_{\|\Delta\|\to 0}\sum_{i=1}^{n} f(x_i,y_i,z_i)\,\Delta V_i$$

Provided the limit exists. The **volume** of the solid region Q is given by

$$\text{Volume of } Q=\iiint_Q dV$$

THEOREM: EVALUATION BY ITERATED INTEGRALS

Let f be continuous on a solid region Q defined by

$$a \le x \le b,\ h_1(x) \le y \le h_2(x),\ g_1(x, y) \le z \le g_2(x, y)$$

where h_1, h_2, g_1, and g_2 are continuous functions. Then

$$\iiint_Q f(x, y, z)\,dV = \int_a^b \int_{h_1(x)}^{h_2(x)} \int_{g_1(x,y)}^{g_2(x,y)} f(x, y, z)\,dz\,dy\,dx$$

Example 1: Evaluate the iterated integral.

$$\int_1^4 \int_1^{e^2} \int_0^{1/(xz)} \ln z\,dy\,dz\,dx$$

Example 2: Set up a triple integral for the volume of the solid.

The solid that is the common interior below the sphere $x^2 + y^2 + z^2 = 80$ and above the paraboloid $z = \frac{1}{2}\left(x^2 + y^2\right)$

Example 3: Sketch the solid whose volume is given by the iterated integral and rewrite the integral using the indicated order of integration.

$$\int_0^2 \int_{2x}^4 \int_0^{\sqrt{y^2-4x^2}} dz\,dy\,dx$$

Rewrite using the order $dxdydz$

Example 4: List the six possible orders of integration for the triple integral over the solid region Q $\iiint_Q xyz\,dV$.

$$Q = \{(x, y, z) : 0 \le x \le 2,\ x^2 \le y \le 4,\ 0 \le z \le 6\}$$