When you are done with your homework you should be able to…

- π Write and evaluate double integrals in polar coordinates

Warm-up: Find the area of the region inside $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$..

Recall:

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \ \text{and} \ \tan\theta = \frac{y}{x}$$

THEOREM: CHANGE OF VARIABLES IN POLAR FORM

Let R be a plane region consisting of all points $(x, y) = (r\cos\theta, r\sin\theta)$ satisfying the conditions $0 \le g_1(\theta) \le r \le g_2(\theta),\ \alpha \le \theta \le \beta$, where $0 \le (\beta - \alpha) \le 2\pi$. If g_1 and g_2 are continuous on $[\alpha, \beta]$ and f is continuous on R, then

$$\int_R\int f(x, y)\,dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Example 1: Evaluate the double integral $\int_R \int f(r,\theta)\,dA$ and sketch the region R.

$$\int_0^{\pi/4}\int_0^4 r^2 \sin\theta\cos\theta\, dr\, d\theta$$

Example 2: Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^2\int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\, dx\, dy$$

Example 3: Use polar coordinates to set up and evaluate the double integral $\int_R\int f(x,y)\,dA$.

$$f(x,y)=e^{-\left(x^2+y^2\right)/2},\ R: x^2+y^2\le 25,\ x^2\ge 0\ .$$

Example 4: Use a double integral in polar coordinates to find the volume of the solid bounded by the graphs of the equations

$$z=\ln\left(x^2+y^2\right),\ z=0,\ x^2+y^2\ge 1,\ x^2+y^2\le 4$$