When you are done with your homework you should be able to…

- π Use a double integral to represent the volume of a solid region
- π Use properties of double integrals
- π Evaluate a double integral as an iterated integral

Warm-up: Evaluate the iterated integral $\int_0^{\pi}\int_0^{\pi/2} \sin^2 x \cos^2 y\, dy dx$.

ACTIVITY: The table below shows values of a function f over a square region R. Divide the region into 16 equal squares and select (x_i, y_i) to be the point in the ith square closest to the origin. Compare this approximation with that obtained by using the point in the ith square furthest from the origin.

x \ y	0	1	2	3	4
0	32	31	28	23	16
1	31	30	27	22	15
2	28	27	24	19	12
3	23	22	19	14	7
4	16	15	12	7	0

DEFINITION: DOUBLE INTEGRAL

If f is defined on a closed, bounded region R in the *xy*-plane, then the **double integral of *f* over *R*** is given by

$$\int_R\int f(x,y)\,dA = \lim_{\|\Delta\|\to 0}\sum_{i=1}^{n} f(x_i,y_i)\,\Delta A_i$$

provided the limit exists. If the limit exists, then f is **integrable** over R.

VOLUME OF A SOLID REGION

If f is integrable over a plane region R and $f(x,y)\ge 0$ for all (x,y) in R, then the volume of the solid region that lies above R and below the graph of f is defined as

$$V = \int_R\int f(x,y)\,dA$$

Example 1: Sketch the region R and evaluate the iterated integral $\int_R\int f(x,y)\,dA$.

$$\int_0^4\int_{\frac{1}{2}y}^{\sqrt{y}} x^2y^2\,dx\,dy$$

PROPERTIES OF DOUBLE INTEGRALS

Let f and g be continuous over a closed, bounded plane region R, and let c be a constant.

1. $\int_R\int cf(x,y)\,dA = c\int_R\int f(x,y)\,dA$
2. $\int_R\int [f(x,y) \pm g(x,y)]\,dA = \int_R\int f(x,y)\,dA \pm \int_R\int g(x,y)\,dA$
3. $\int_R\int f(x,y)\,dA \ge 0, \quad \text{if } f(x,y) \ge 0$
4. $\int_R\int f(x,y)\,dA \ge \int_R\int g(x,y)\,dA, \quad \text{if } f(x,y) \ge g(x,y)$
5. $\int_R\int f(x,y)\,dA = \int_{R_1}\int f(x,y)\,dA + \int_{R_2}\int f(x,y)\,dA, \quad$ where R is the union of two nonoverlapping subregions R_1 and R_2

THEOREM: FUBINI'S THEOREM

Let f be continuous on a plane region R.

1. If R is defined by $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$, where g_1 and g_2 are continuous on $[a,b]$, then
$$\int_R\int f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} dydx$$
2. If R is defined by $c \le y \le d$ and $\mathrm{h}_1(y) \le x \le h_2(y)$, where h_1 and h_2 are continuous on $[c,d]$, then
$$\int_R\int f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} dxdy$$

Example 2: Set up an integrated integral for both orders of integration, and use the more convenient order to evaluate over the region R.

$$\int_R\int xe^y\,dA,$$

R: triangle bounded by $y = 4 - x,\ y = 0,\ x = 0$

Example 3: Set up a double integral to find the volume of the solid bounded by the graphs of the equations $x^2 + z^2 = 1,\ y^2 + z^2 = 1,\ \text{first octant}$.

Example 4: Find the average value of $f(x, y)$ over the region *R* where

$$\text{Average value} = \frac{1}{A}\int_R\int f(x, y)\,dA, \text{ where } A \text{ is the area of } R.$$

$f(x, y) = xy.$

R: rectangle with vertices $(0,0)$, $(4,0)$, $(4,2)$ and $(0,2)$.