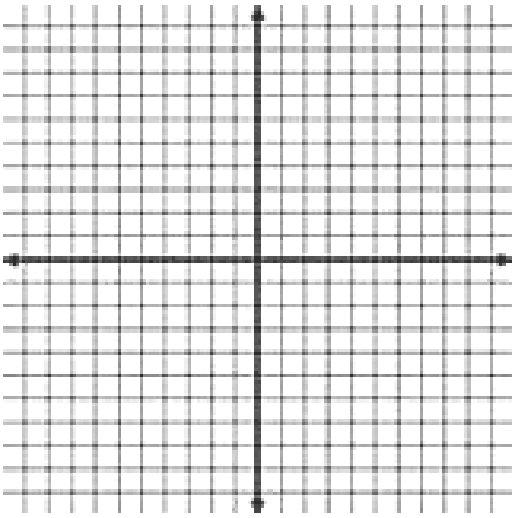
When you are done with your homework you should be able to…

- Evaluate an iterated integral
- Use an iterated integral to find the area of a plane region

Warm-up: Sketch the region bounded by the graphs $x = \cos y,\ x = \frac{1}{2},\ \frac{\pi}{3} \le y \le \frac{7\pi}{3}$.

Then find the area.



INTEGRALS OF FUNCTIONS OF TWO VARIABLES

to *x*, you hold *y* $(y), y)$.

to *y*, you hold *x* $g_1(x))$.

Example 1: Evaluate the following integrals.

a. $\int_x^{x^2} \frac{y}{x} dy$

b. $\int_y^{\pi/2} \sin^3 x \cos y dx$

ITERATED INTEGRALS

When evaluating the integral of an integral, it is called an **iterated integral**.

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx = \int_a^b f(x,y)\Big]_{g_1(x)}^{g_2(x)} dx$$

$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dxdy = \int_c^d f(x,y)\Big]_{h_1(y)}^{h_2(y)} dy$$

Example 2: Evaluate the following iterated integrals.

a. $\int_0^1 \int_0^2 (x+y)\,dydx$

b. $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x}\,dydx$

c. $\int_0^3 \int_0^{\infty} \frac{x^2}{1+y^2}\,dydx$

AREA OF A REGION IN THE PLANE

If R is defined by $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$, where g_1 and g_2 are continuous on $[a,b]$, then the area of R is given by

1. $A = \int_a^b \int_{g_1(x)}^{g_2(x)} dydx$ (vertically simple)
2. $A = \int_c^d \int_{h_1(y)}^{h_2(y)} dxdy$ (horizontally simple)

Example 3: Use an iterated integral to find the area of the region bounded by the graphs of $y = x,\ y = 2x,\ x = 2.$

Example 4: Sketch the region *R* whose area is given by the iterated integral. Then switch the order of integration and show that both orders yield the same area.

$$\int_{-2}^{2}\int_{0}^{4-y^2} dxdy$$