When you are done with your homework you should be able to…

- π Find the absolute and relative extrema of a function of two variables
- π Use the Second Partials Test to find relative extrema of a function of two variables

Warm-up: Consider the function $f(x) = \sin x \cos x$ on the interval $(0, \pi)$.

A. Find the critical numbers.

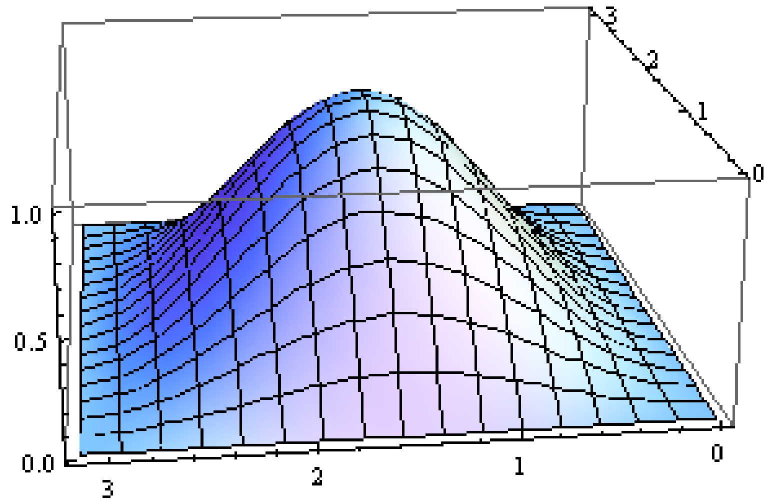
B. Apply the theorem which tests for increasing and decreasing intervals.

C. Find the open interval(s) on which the function is

a. Increasing

b. Decreasing

D. Apply the First Derivative test to identify all relative extrema. Give your result(s) as an ordered pair.



Plot3D[Sin[x]Sin[y]^2, {x, 0, Pi}, {y, 0, Pi}]

THEOREM: EXTREME VALUE THEOREM

Let f be a continuous function of two variables x and y defined on a closed bounded region R in the *xy*-plane.

1. There is at least one point in R where f takes on a minimum value.
2. There is at least one point in R where f takes on a maximum value.

DEFINITION: RELATIVE EXTREMA

Let f be a function defined on a region R containing (x_0, y_0).

1. The function f has a **relative minimum** at (x_0, y_0) if $f(x,y) \geq f(x_0, y_0)$ for all x and y in an *open* disk containing (x_0, y_0).
2. The function f has a **relative maximum** at (x_0, y_0) if $f(x,y) \leq f(x_0, y_0)$ for all x and y in an *open* disk containing (x_0, y_0).

DEFINITION: CRITICAL POINT

Let f be defined on an open region R containing (x_0, y_0). The point (x_0, y_0) is a **critical point** of f if one of the following is true.

1. $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$
2. $f_x(x_0, y_0)$ or $f_y(x_0, y_0)$ does not exist

THEOREM: RELATIVE EXTREMA OCCUR ONLY AT CRITICAL POINTS

If f has a relative extremum at (x_0, y_0) on an open region R, then (x_0, y_0) is a critical point of f.

THEOREM: SECOND PARTIALS TEST

Let f have continuous partial derivatives on an open region containing a point (a,b) for which $f_x(a,b)=0$ and $f_y(a,b)=0$. To test for relative extrema of f, consider the quantity $d = f_{xx}(a,b)f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$.

1. If $d > 0$ and $f_{xx}(a,b) > 0$, then f has a **relative minimum** at (a,b).
2. If $d > 0$ and $f_{xx}(a,b) < 0$, then f has a **relative maximum** at (a,b).
3. If $d < 0$, then $(a,b,f(a,b))$ is a **saddle point**.
4. The test is inconclusive if If $d = 0$.

Example 1: Examine the function for relative extrema and saddle points.

$$g(x,y) = xy$$

Example 2: Find the critical points and test for relative extrema. List the critical points for which the Second Partials Test fails.

$$f(x,y)=x^3+y^3-6x^2+9y^2+12x+27y+19$$

Example 3: A function f has continuous second partial derivatives on an open region containing the critical point (a,b). If $f_{xx}(a,b)$ and $f_{yy}(a,b)$ have opposite signs, what is implied?