When you are done with your homework you should be able to…

- π Find equations of tangent planes and normal lines to surfaces
- π Find the angle of inclination of a plane in space
- π Compare the gradients $\nabla f(x, y)$ and $\nabla F(x, y)$

Warm-up: Find the general equation of the plane containing the points (2, 1, 1), (0, 4, 1), and (-2, 1, 4).

DEFINITION OF TANGENT PLANE AND NORMAL LINE

Let F be differentiable at the point $P(x_0, y_0, z_0)$ on the surface given by $F(x, y, z) = 0$ such that $\nabla F(x_0, y_0, z_0) \neq \mathbf{0}$.

1. The plane through P that is normal to $\nabla F(x_0, y_0, z_0)$ is called the ***tangent plane to S at P***.
2. The line through *P* having the direction of $\nabla F(x_0, y_0, z_0)$ is called the ***normal line to S at P***.

Example 1: Find a unit normal vector to the surface at the given point. (*HINT:* normalize the gradient vector $\nabla F(x, y, z)$).

$$x^2 + y^2 + z^2 = 11, \text{ at the point } P(3,1,1)$$

THEOREM: EQUATION OF TANGENT PLANE

If F is differentiable at (x_0, y_0, z_0), then an equation of the tangent plane to the surface is given by $F(x, y, z) = 0$ at (x_0, y_0, z_0) is

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

Example 2: Find an equation of the tangent plane to the surface at the given point.

$$h(x, y) = \ln\sqrt{x^2 + y^2}, \text{ at the point } P(3, 4, \ln 5)$$

Example 3: Find an equation of the tangent plane and find symmetric equations of the normal line to the surface at the given point.

$$z = \arctan\frac{y}{x}, \text{ at the point } \left(1, 1, \frac{\pi}{4}\right)$$

Example 4: Find the path of a heat-seeking particle placed at the point in space $(2,2,5)$ with a temperature field $T(x,y,z)=100-3x-y-z^2$.

THE ANGLE INCLINATION OF A PLANE

$$\cos\theta=\frac{|\mathbf{n}\cdot\mathbf{k}|}{\|\mathbf{n}\|}$$

THEOREM: GRADIENT IS NORMAL TO LEVEL SURFACES

If F is differentiable at (x_0,y_0,z_0) and $\nabla F(x_0,y_0,z_0)\neq\mathbf{0}$, then $\nabla F(x_0,y_0,z_0)$ is normal to the level surface through (x_0,y_0,z_0).