When you are done with your homework you should be able to…

- π Understand the concepts of increments and differentials
- π Extend the concept of differentiability to a function of two variables
- π Use a differential as an approximation

Warm-up: The measurement of a side of a square is found to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible propagated error in computing the area of the square.

DEFINITION OF TOTAL DIFFERENTIAL

If $z = f(x, y)$ and Δx and Δy are increments of x and y, then the **differentials** of the independent variables x and y are

$$dx = \Delta x \quad \text{and} \quad dy = \Delta y$$

and the **total differential** of the dependent variable z is

$$dz = \frac{dz}{dx}dx + \frac{dz}{dy}dy = f_x(x, y)\,dx + f_y(x, y)\,dy$$

Example 1: Find the total differential.

a. $z=\frac{x^2}{y}$

b. $w=e^y\cos x+z^2$

DEFINITION OF DIFFERENTIABILITY

A function f given by $z=f(x,y)$ is <u>differentiable</u> at (x_0,y_0) if Δz can be written in the form

$$\Delta z=f_x(x_0,y_0)\Delta x+f_y(x_0,y_0)\Delta y+\varepsilon_1\Delta x+\varepsilon_2\Delta y$$

where both ε_1 and $\varepsilon_2\to 0$ as $(\Delta x,\Delta y)\to(0,0)$. The function f is **<u>differentiable in a region *R*</u>** if it is differentiable at each point in *R*.

Example 2: Find $z = f(x, y)$ and use the total differential to approximate the quantity.

$$(2.03)^2(1+8.9)^3 - 2^2(1+9)^3$$

THEOREM: SUFFICIENT CONDITION FOR DIFFERENTIABILITY

If f is a function of x and y, where f_x and f_y are continuous in an open region *R*, then f is differentiable on *R*.

THEOREM: DIFFERENTIABILITY IMPLIES CONTINUITY

If a function of x and y is differentiable at (x_0, y_0) then it is continuous at (x_0, y_0).

Example 3: A triangle is measured and two adjacent sides are found to be 3 inches and 4 inches long, with an included angle of $\frac{\pi}{4}$. The possible errors in measurement are $\frac{1}{16}$ inch for the sides and 0.02 radian for the angle. Approximate the maximum possible error in the computation of the area.

Example 4: Show that the function $f(x,y)=x^2+y^2$ is differentiable by finding values for ε_1 and ε_2 as designated in the definition of differentiability, and verify that both ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0,0)$.