When you are done with your homework you should be able to…

- π Find and use partial derivatives of a function of two variables
- π Find and use partial derivatives of a function of three or more variables
- π Find higher-order partial derivatives of a function of two or three variables

Warm-up: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

1. $f(x)=\dfrac{3x^2-x+2}{\sqrt{x}}$

2. $g(x)=(5x-3)^2$

3. $f(x)=\cos\left(x-\dfrac{\pi}{4}\right)$

DEFINITION: PARTIAL DERIVATIVES OF A FUNCTION OF TWO VARIABLES

If $z = f(x, y)$ then the **first partial derivatives** of f with respect to x and y are f_x and f_y defined by

$$f_x(x, y) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$
$$f_y(x, y) = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

provided the limit exists.

Example 1: Find the partial derivatives f_x and f_y of the following functions.

a. $f(x, y) = x^2 - 2y^2 + 4$

b. $z = \sin 5x \cos 5y$

c. $f(x, y) = \int_x^y (2t + 1)\, dt + \int_y^x (2t - 1)\, dt$

NOTATION FOR FIRST PARTIAL DERIVATIVES FOR $z = f(x, y)$

Example 2: Use the limit definition to find the first partial derivatives with respect to x, y and z.

$$f(x, y, z) = 3x^2 y - 5xyz + 10yz^2$$

PARTIAL DERIVATIVES OF A FUNCTION OF THREE OR MORE VARIABLES

If $w = f(x, y, z)$ then the **first partial derivatives** of f with respect to x, y and z are defined by

$$\frac{dw}{dx} = f_x(x, y, z) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y, z) - f(x, y, z)}{\Delta x}$$
$$\frac{dw}{dy} = f_y(x, y, z) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y, z) - f(x, y, z)}{\Delta y}$$
$$\frac{dw}{dz} = f_z(x, y, z) = \lim_{\Delta z \to 0} \frac{f(x, y, z+\Delta z) - f(x, y, z)}{\Delta z}$$

provided the limit exists.

Example 3: Find f_x, f_y and f_z at the given point.

$$f(x, y, z) = \frac{xy}{x+y+z}, \quad (3,1,-1)$$

HIGHER ORDER PARTIAL DERIVATIVES

1. Differentiate twice with respect to *x*.

2. Differentiate twice with respect to *y*.

3. Differentiate first with respect to *x* and then with respect to *y*.

4. Differentiate first with respect to *y* and then with respect to *x*.

Example 4: Find the four second partial derivatives.

a. $z = \ln\left(x - y\right)$

b. $z = \arctan\left(\dfrac{y}{x}\right)$

THEOREM: EQUALITY OF MIXED PARTIAL DERIVATIVES

If f is a function of x and y such that f_{xy} and f_{yx} are continuous on an open disk R, then, for every (x, y) in R,

$$f_{xy}(x, y) = f_{yx}(x, y)$$

Example 5: Find the slopes of the surface in the *x*- and *y*-directions at the given point.

$h(x, y) = x^2 - y^2, \quad (-2, 1, 3)$