When you are done with your homework you should be able to…

- π Find a unit tangent vector at a point on a space curve
- π Find the tangential and normal components of acceleration

Warm-up: Consider the two curves given by $y_1 = 1 - x^2$ and $y_2 = x^2 - 1$.

a. Find the unit tangent vectors to each curve at their points of intersection.

b. Find the angles ($0 \le \theta \le 90^\circ$) between the curves at their points of intersection.

DEFINITION OF UNIT TANGENT VECTOR

Let C be a smooth curve represented by $\mathbf{r}$ on an open interval I. The **unit tangent vector** $\mathbf{T}(t)$ at t is defined to be

$$\mathbf{T}(t)=\frac{\mathbf{r}'(t)}{\left\|\mathbf{r}'(t)\right\|},\ \mathbf{r}'(t)\neq\mathbf{0}$$

The **tangent line to a curve** at a point is the line passing through point and parallel to the unit tangent vector.

Example 1: Find the unit tangent vector to the curve $\mathbf{r}(t)=e^t\cos t\mathbf{i}+e^t\mathbf{j}$ when $t=0$.

Example 2: Consider the space curve $\mathbf{r}(t)=\left\langle t,t,\sqrt{4-t^2}\right\rangle$ at the point $\left(1,1,\sqrt{3}\right)$. a.

a. Find the unit tangent vector at the given point.

b. Find a set of parametric equations for the line tangent to the space curve at the given point.

DEFINITION: PRINCIPAL UNIT NORMAL VECTOR

Let *C* be a smooth curve represented by $\mathbf{r}$ on an open interval *I*. If $\mathbf{T}'(t) \neq \mathbf{0}$, then the **principal unit normal vector** $\mathbf{N}(t)$ at t is defined to be

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\left\| \mathbf{T}'(t) \right\|}$$

At any point on a curve, a unit normal vector is orthogonal to the unit tangent vector. The principal unit normal vector points in the direction in which the curve is turning.

Example 3: Find the principal unit normal vector to the curve $\mathbf{r}(t) = \ln t\,\mathbf{i} + (t+1)\,\mathbf{j}$ at the time $t = 2$.

THEOREM: ACCELERATION VECTOR

If $\mathbf{r}(t)$ is the position vector for a smooth curve *C* and $\mathbf{N}(t)$ exists, then the acceleration vector $\mathbf{a}(t)=a_{\mathbf{T}}\mathbf{T}(t)+a_{\mathbf{N}}\mathbf{N}(t)$ lies in the plane determined by $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

THEOREM: TANGENTIAL AND NORMAL COMPONENTS OF ACCELERATION

If $\mathbf{r}(t)$ is the position vector for a smooth curve *C* and $\mathbf{N}(t)$ exists, then the **tangential and normal components of acceleration are as follows**:

$$a_{\mathbf{T}} = D_t\left[\|\mathbf{v}\|\right] = \mathbf{a}\cdot\mathbf{T} = \frac{\mathbf{v}\cdot\mathbf{a}}{\|\mathbf{v}\|}$$

$$a_{\mathbf{N}} = \|\mathbf{v}\|\|\mathbf{T}'(t)\| = \mathbf{a}\cdot\mathbf{N} = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|} = \sqrt{\|\mathbf{a}\|^2 - a_{\mathbf{T}}^{\;2}}$$

Note that $a_{\mathbf{N}} \geq 0$. The normal component of acceleration is also called the **centripetal component of acceleration.**

Example 4: Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, $a_{\mathbf{T}}$, and $a_{\mathbf{N}}$ for the plane curve $\mathbf{r}(t)=e^t\mathbf{i}+e^{-t}\mathbf{j}+t\mathbf{k}$ at the time $t=0$.