

When you are done with your homework you should be able to...

- $\pi$  Describe the velocity and acceleration associated with a vector-valued function
- $\pi$  Use a vector-valued function to analyze projectile motion

Warm-up: Consider the circle given by  $\mathbf{r}(t) = (\cos \omega t)\mathbf{i} + (\sin \omega t)\mathbf{j}$ . Use a graphing calculator in parametric mode to graph this circle for several values of  $\omega$ .

How does  $\omega$  affect the velocity of the terminal point as it traces out the curve?

For a given value of  $\omega$ , does the speed appear constant?

Does the acceleration appear constant?

**DEFINITIONS OF VELOCITY AND ACCELERATION**

If  $x$  and  $y$  are twice differentiable functions of  $t$ , and  $\mathbf{r}$  is a vector-valued function given by  $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ , then the velocity vector, acceleration vector, and speed at time  $t$  are as follows:

$$\underline{\text{Velocity}} = \mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}$$

$$\underline{\text{Acceleration}} = \mathbf{a}(t) = \mathbf{r}''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j}$$

$$\underline{\text{Speed}} = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$$

For motion along a space curve, the definitions are as follows:

$$\underline{\text{Velocity}} = \mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$$

$$\underline{\text{Acceleration}} = \mathbf{a}(t) = \mathbf{r}''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j} + z''(t)\mathbf{k}$$

$$\underline{\text{Speed}} = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$

Example 1: The position vector  $\mathbf{r}(t) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j}$  describes the path of an object moving in the  $xy$ -plane. Sketch a graph of the path and sketch the velocity and acceleration vectors at the point  $(3,0)$ .

Example 2: The position vector  $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 2t^{3/2}\mathbf{k}$  describes the path of an object moving in space. Find the velocity, speed and acceleration of the object.

### THEOREM: POSITION FUNCTION FOR A PROJECTILE

Neglecting air resistance, the path of a projectile launched from an initial height  $h$  with initial speed  $v_0$  and angle of elevation  $\theta$  is described by the vector function

$$\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + \left[ h + (v_0 \sin \theta)t - \frac{1}{2}gt^2 \right]\mathbf{j}$$

where  $g$  is the gravitational constant.

Example 3: Determine the maximum height and range of a projectile fired at a height 3 feet above the ground with an initial velocity of 900 feet per second and at an angle of  $45^\circ$  above the horizontal.

Example 4: A baseball is hit from a height of 2.5 feet above the ground with an initial velocity of 140 feet per second and at an angle of  $22^\circ$  above the horizontal. Use a graphing utility to graph the path of the ball and determine whether it will clear a ten foot high fence located 375 feet from home plate.

Example 5: Find the maximum speed of a point on the circumference of an automobile tire of radius one foot when the automobile is traveling at 55 mph. Compare this speed with the speed of the automobile. Use the following formula for the cycloid:

$$\mathbf{r}(t) = b(\omega t - \sin \omega t)\mathbf{i} + b(1 - \cos \omega t)\mathbf{j}$$

$\omega$  is the constant angular velocity of the circle and

$b$  is the radius of the circle.