When you are done with your homework you should be able to…

- π Analyze and sketch a space curve given by a vector-valued function
- π Extend the concepts of limits and continuity to vector-valued functions

Warm-up: Evaluate the following limits analytically.

1. $\lim_{x\to 0}\frac{\sin 2x}{x}$

2. $\lim_{t\to 4}\frac{t^2-16}{t^2-4t}$

3. $\lim_{x\to\infty}\left(e^{-x}-\frac{6}{x}-\arctan x\right)$

DEFINITION OF VECTOR-VALUED FUNCTION

A function of the form

$$\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j} \qquad \text{plane}$$
$$=\langle f(t),g(t)\rangle$$
or
$$\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j}+h(t)\mathbf{k} \qquad \text{space}$$
$$=\langle f(t),g(t),h(t)\rangle$$

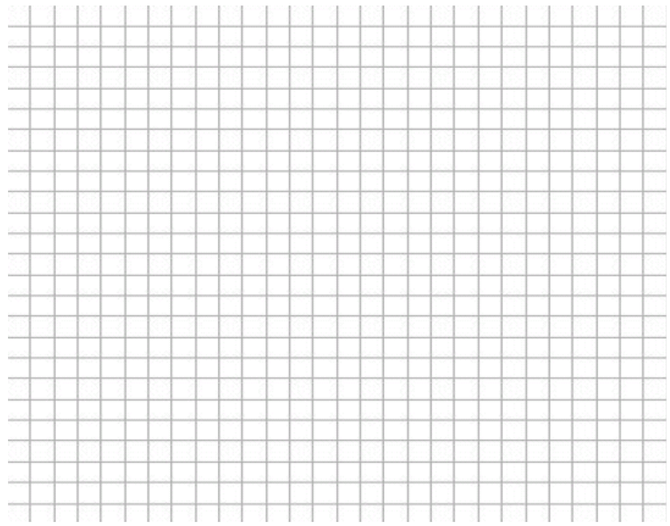
is a **vector-valued function**, where the **component functions** *f, g,* and *h* are real-valued functions of the parameter *t*. The domain is considered to be the intersection of the domains of the component functions *f, g,* and *h,* unless stated otherwise.

Example 1: Find the domain of the vector-valued function.

$$\mathbf{r}(t)=\sqrt{4-t^2}\,\mathbf{i}+t^2\mathbf{j}-6t\mathbf{k}$$

Example 2: Sketch the curve represented by the vector-valued function.

a) $\mathbf{r}(t)=(1-t)\mathbf{i}+\sqrt{t}\,\mathbf{j}$



b) $\mathbf{r}(t)=(3\cos t)\mathbf{i}+(4\sin t)\mathbf{j}+\dfrac{t}{2}\mathbf{k}$

DEFINITION OF THE LIMIT OF A VECTOR-VALUED FUNCTION

1. If $\mathbf{r}$ is a vector-valued function such that $\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j}$, then
$$\lim_{t\to a}\mathbf{r}(t)=\left[\lim_{t\to a}f(t)\right]\mathbf{i}+\left[\lim_{t\to a}g(t)\right]\mathbf{j}$$
provided *f* and *g* have limits as $t\to a$.
2. If $\mathbf{r}$ is a vector-valued function such that $\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j}+h(t)\mathbf{k}$, then
$$\lim_{t\to a}\mathbf{r}(t)=\left[\lim_{t\to a}f(t)\right]\mathbf{i}+\left[\lim_{t\to a}g(t)\right]\mathbf{j}+\left[\lim_{t\to a}h(t)\right]\mathbf{k}$$
provided *f*, *g* and *h* have limits as $t\to a$.

DEFINITION OF CONTINUITY OF A VECTOR-VALUED FUNCTION

A vector-valued function $\mathbf{r}$ is **continuous at the point** given by $t=a$ if the limit of $\mathbf{r}(t)$ exists as $t\to a$ and $\lim_{t\to a}\mathbf{r}(t)=\mathbf{r}(a)$.

A vector-valued function $\mathbf{r}$ is **continuous on an interval** *I* if it is continuous at every point in the interval.

Example 3: Evaluate the limit and determine the interval(s) on which the vector-valued function is continuous.

$$\lim_{t\to 1}\left((\ln t)\mathbf{i}-\left(\frac{1-t^2}{1-t}\right)\mathbf{j}+(\arcsin t)\mathbf{k}\right)$$