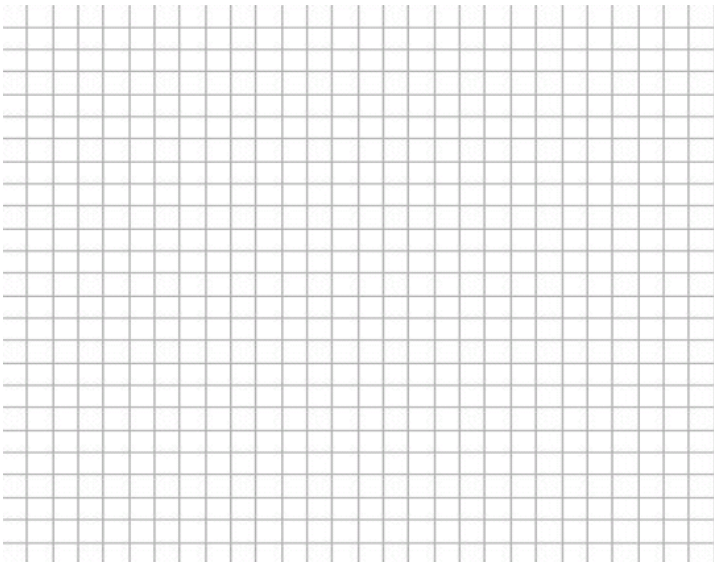
When you are done with your homework you should be able to…

- Use cylindrical coordinates to represent surfaces in space
- Use spherical coordinates to represent surfaces in space

Warm-up: Convert the rectangular equation to polar form and sketch its graph by hand.

$y^2 = 9x$



THE CYLINDRICAL COORDINATE SYSTEM

In a **cylindrical coordinate system** a point *P* in space is represented by an ordered triple (r, θ, z).

1. (r, θ) is a polar representation of the projection of *P* in the *xy*-plane.
2. z is the directed distance from (r, θ) to *P*.

Conversion Guidelines

Cylindrical to rectangular: $x = r\cos\theta,\ y = r\sin\theta,\ z = z$

Rectangular to cylindrical: $r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x},\ z = z$

Example 1: Convert the point $\left(-2, \frac{2\pi}{3}, 5\right)$ to rectangular coordinates.

Example 2: Convert the point $\left(3, \sqrt{3}, -1\right)$ to cylindrical coordinates.

Example 3: Find an equation in cylindrical coordinates for the equation $x^2 + y^2 = 8x$, given in rectangular coordinates.

THE SPHERICAL COORDINATE SYSTEM

In a **spherical coordinate system**, a point P in space is represented by an ordered triple (ρ, θ, ϕ).

1. ρ is the distance between P and the origin $\rho \geq 0$.
2. θ is the same angle used in cylindrical coordinates for $r \geq 0$.
3. ϕ is the angle *between* the positive z-axis and the line segment $\overline{OP}$, $0 \leq \phi \leq \pi$.

Note that the first and third coordinates, ρ and ϕ, are nonnegative. ρ is the lowercase Greek letter *rho* and ϕ is the lowercase Greek letter *phi*.

Conversion Guidelines

Spherical to rectangular: $x = \rho \sin\phi \cos\theta,\ y = \rho \sin\phi \sin\theta,\ z = \rho \cos\phi$

Rectangular to spherical: $\rho^2 = x^2 + y^2 + z^2,\ \tan\theta = \dfrac{y}{x},\ \phi = \arccos\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

Spherical to cylindrical $r \geq 0$: $r^2 = \rho^2 \sin^2\phi,\ \theta = \theta,\ z = \rho \cos\phi$

Cylindrical to spherical $r \geq 0$: $\rho = \sqrt{r^2 + z^2},\ \theta = \theta,\ \phi = \arccos\left(\dfrac{z}{\sqrt{r^2 + z^2}}\right)$

Example 4: Convert the point given in cylindrical coordinates $\left(3,-\frac{\pi}{4},0\right)$ to spherical coordinates.

Example 5: Find an equation in spherical coordinates for the equation $x^2+y^2-3z^2=0$, given in rectangular coordinates.