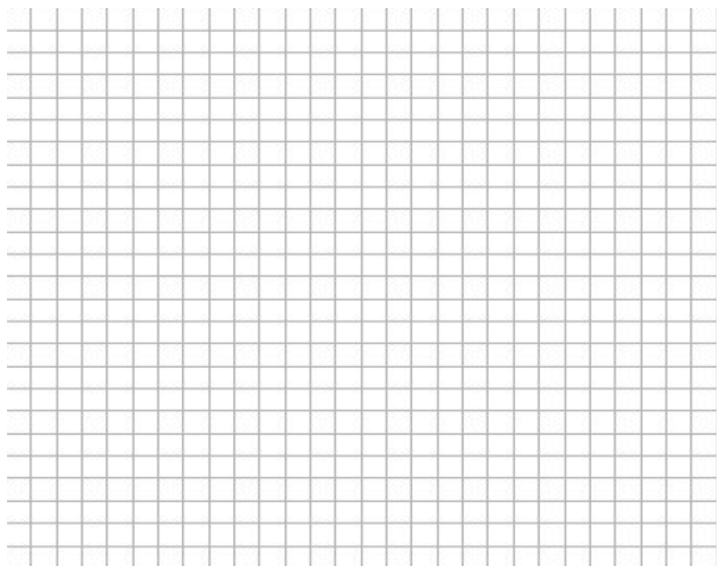
When you are done with your homework you should be able to…

- π Recognize and write equations for cylindrical surfaces
- π Recognize and write equations for quadric surfaces
- π Recognize and write equations for surfaces of revolution

Warm-up: Find the volume of the region bounded by the graphs $y=4,\ x=4,\ x=0,\ \text{and } y=0$ which has been rotated about the x-axis. Graph the resulting solid.



**DEFINITION OF A CYLINDER**

Let $C$ be a curve in a plane and let $L$ be a line not in a parallel plane. The set of all lines parallel to $L$ and intersecting $C$ is called a **cylinder**. $C$ is called the **generating curve (aka directrix)** of the cylinder and the parallel lines are called **rulings**.

## EQUATIONS OF CYLINDERS

The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only the variables corresponding to the other two axes.

Example 1: Sketch the surface represented by each equation.

a) $y = z^2$

b) $z = \cos x$

## QUADRIC SURFACE

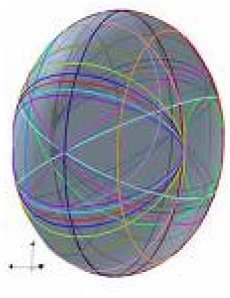
The equation of a **quadric surface** in space is a second-degree equation of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

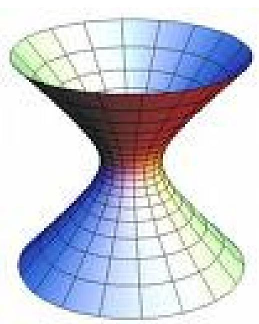
There are six basic types of quadric surfaces:

**Ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, and hyperbolic paraboloid.**

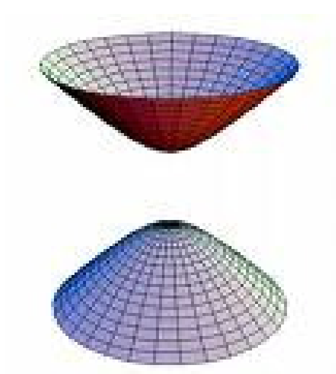
Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Trace Plane



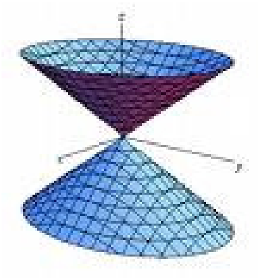
Hyperboloid (1 sheet) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Trace Plane



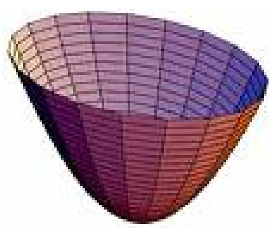
Hyperboloid (2 sheets) $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Trace Plane



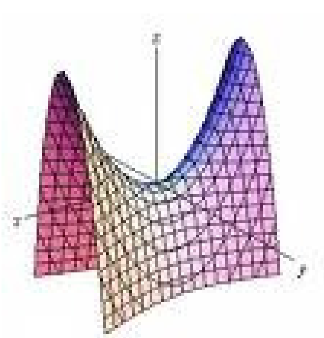
Elliptic Cone $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ Trace Plane



Elliptic Paraboloid $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Trace Plane



Hyperbolic Paraboloid $z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$ Trace Plane



Example 2: Identify and sketch the quadric surface.

$$\frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{25}=1$$

**SURFACE OF REVOLUTION**

If the graph of a radius function *r* is revolved about one of the coordinate axes, the equation of the resulting surface of revolution has one of the following forms:

1. Revolved about the x-axis: $y^2+z^2=\left[r(x)\right]^2$
2. Revolved about the y-axis: $x^2+z^2=\left[r(y)\right]^2$
3. Revolved about the z-axis: $x^2+y^2=\left[r(z)\right]^2$

Example 3: Find an equation for the surface of revolution generated by revolving the curve $z=3y$ in the yz-plane about the y-axis.