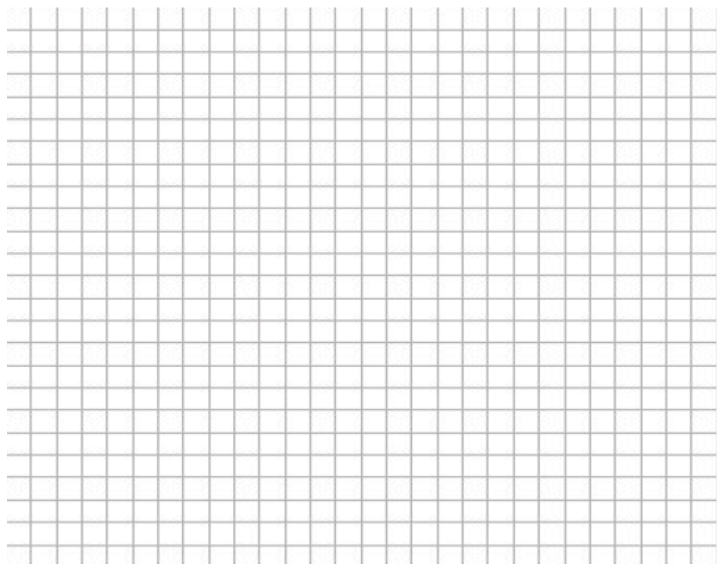
When you are done with your homework you should be able to…

- π Write a set of parametric equations for a line in space
- π Write a linear equation to represent a plane in space
- π Sketch the plane given by a linear equation
- π Find the distance between points, planes, and lines in space

Warm-up: Graph the following parametric curve, indicating the orientation.

$$x-3=\cos^2\theta,\ \text{and}\ y=\sin^2\theta,\ 0\le\theta<2\pi$$



In the plane _______________ is used to determine an equation of a line. In space, it is convenient to use _________________ to determine the equation of a line.

THEOREM: PARAMETRIC EQUATIONS OF A LINE IN SPACE

A line *L* parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P = (x_1, y_1, z_1)$ is represented by the **parametric equations**

$$x = x_1 + at, \; y = y_1 + bt, \text{ and } z = z_1 + ct$$

If the direction numbers *a, b,* and *c* are all nonzero, you can eliminate the parameter *t* to obtain **symmetric equations** of the line.

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Example 1: Find equations of the line which passes through the point $(-3, 0, 2)$ and is parallel to the vector $\mathbf{v} = 6\mathbf{j} + 3\mathbf{k}$ in

a) Parametric form

b) Symmetric form

THEOREM: STANDARD EQUATION OF A PLANE IN SPACE

The plane containing the point (x_1, y_1, z_1) and having normal vector $\mathbf{n} = \langle a, b, c \rangle$ can be represented, in **standard form**, by the equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

The **general form** is given by the equation

$$ax + by + cz + d = 0$$

THEOREM: DISTANCE BETWEEN A POINT AND A PLANE

The distance between a plane and a point Q (not in the plane) is

$$D = \left\| \text{proj}_{\mathbf{n}} \overrightarrow{PQ} \right\| = \frac{\left| \overrightarrow{PQ} \cdot \mathbf{n} \right|}{\|\mathbf{n}\|}$$

where P is a point in the plane and $\mathbf{n}$ is normal to the plane. Other forms of this distance from a point $Q(x_0, y_0, z_0)$ and the plane given by $ax + by + cz + d = 0$ are as follows:

$$D = \frac{\left| a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1) \right|}{\sqrt{a^2 + b^2 + c^2}} \quad \text{or} \quad D = \frac{\left| ax_0 + by_0 + cz_0 \right|}{\sqrt{a^2 + b^2 + c^2}}$$

Example 2: Find an equation of the plane passing through the point $(1,0,-3)$ perpendicular to the vector $\mathbf{n}=\mathbf{k}$.

THEOREM: DISTANCE BETWEEN A POINT AND A LINE IN SPACE

The distance between a point Q and a line in space is given by

$$D=\frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\left\|\mathbf{u}\right\|}$$

where $\mathbf{u}$ is a direction vector for the line and P is a point on the line.

Example 3: Find the distance between the point $(3,2,1)$ and the plane $x-y+2z=4$.