When you are done with your homework you should be able to…

- π Write the component form of a vector
- π Perform vector operations and interpret the results geometrically
- π Write a vector as a linear combination of standard unit vectors
- π Use vectors to solve problems involving force or velocity

Warm-up: Find the distance between the points (2, 1) and (4, 7).

What is a scalar quantity?

Give examples of quantities which can be characterized by a scalar.

What is a vector?

Give examples of quantities which are represented by vectors.

How do you find the length, aka magnitude, aka norm, of a vector?

What makes two vectors equivalent?

DEFINITION OF COMPONENT FORM OF A VECTOR IN THE PLANE

If $\mathbf{v}$ is a vector in the plane whose initial point is the origin and whose terminal point is (v_1, v_2), then the **component form** $\mathbf{v}$ is given by

$$\mathbf{v} = \langle v_1, v_2 \rangle.$$

The coordinates v_1 and v_2 are called the **components** of $\mathbf{v}$. If both the initial point and the terminal point lie at the origin, then $\mathbf{v}$ is called the **zero vector** and is denoted by $\mathbf{0} = \langle 0, 0 \rangle$.

Example 1: Sketch the vector whose initial point is the origin and whose terminal point is (3, -2).

DEFINITIONS OF VECTOR ADDITION AND SCALAR MULTIPLICATION

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let c be a scalar.

1. The **vector sum** of $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$.
2. The **scalar multiple** of c and $\mathbf{u}$ is the vector $c\mathbf{u} = \langle cu_1, cu_2 \rangle$.
3. The **negative** of $\mathbf{v}$ is the vector $-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle$.
4. The **difference** of $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$.

Example 2: Find the component form and length of the vector $\mathbf{v}$ that has initial point (-1, 4) and terminal point (7, 3). Find the norm of $\mathbf{v}$.

Example 3: Let $\mathbf{u} = \langle -1, -3 \rangle$ and $\mathbf{v} = \langle 2, -8 \rangle$ find the following vectors. Illustrate the vector operations geometrically.

a) $\mathbf{u} - \mathbf{v}$

b) $-2\mathbf{v}$

THEOREM: PROPERTIES OF VECTOR OPERATIONS

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in the plane, and let *c* and *d* be scalars.

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(d\mathbf{u}) = (cd)\mathbf{u}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $1(\mathbf{u}) = \mathbf{u}$, and $0(\mathbf{u}) = \mathbf{0}$

THEOREM: LENGTH OF A SCALAR MULTIPLE

Let $\mathbf{v}$ be a vector, and let c be a scalar. Then

$$\|c\mathbf{v}\| = |c|\|\mathbf{v}\|.$$

THEOREM: UNIT VECTOR IN THE DIRECTION OF $\mathbf{v}$

If $\mathbf{v}$ is a nonzero vector in the plane, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Has length 1 and the same direction as **v**.

Example 4: Find a unit vector in the direction of $\mathbf{v} = \langle 7, -5 \rangle$. Verify that it has length 1.

Standard Unit Vectors

$\mathbf{i} = \langle \ , \ \rangle$ and $\mathbf{j} = \langle \ , \ \rangle$

Example 5: Let $\mathbf{u}$ be the vector with initial point (-4, 1) and terminal point (3, -1) and let $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$. Write each vector as a linear combination of **i** and **j**.

a) $\mathbf{u}$

b) $\mathbf{w} = 4\mathbf{u} - 2\mathbf{v}$

Example 6: The vector **v** has a magnitude of 2 and makes an angle of $\frac{\pi}{3}$ with the positive *x*-axis. Write **v** as a linear combination of the unit vectors **i** and **j**.