EXAM 1/CHAPTER 11
100 POINTS POSSIBLE

NAME Key

LEAVE ALL ANSWERS EXACT UNLESS THE PROBLEM INDICATES OTHERWISE
SHOW ALL WORK IN ORDER TO EARN FULL CREDIT

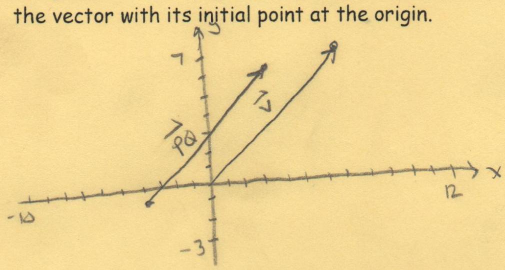
1. (10 POINTS) The initial and terminal points of a vector $\mathbf{v}$ are $(-4,-1)$ and $(3,6)$, respectively.
   a. (2 POINTS) Write the vector $\mathbf{v}$ in component form.

   $\vec{v} = \langle 3-(-4), 6-(-1) \rangle \rightarrow \boxed{\vec{v} = \langle 7,7 \rangle}$

   b. (1 POINT) Write the vector $\mathbf{v}$ as the linear combination of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$.

   $\boxed{\vec{v} = 7\hat{i} + 7\hat{j}}$

   c. (4 POINTS) Sketch on the same two-dimensional coordinate plane
      i. the **given** directed line segment (be sure to label your axes) and
      ii. the vector with its initial point at the origin.



   d. (3 POINTS) Find a **unit vector** $\mathbf{u}$ in the direction of $\mathbf{v}$.

   $$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 7,7 \rangle}{\sqrt{7^2+7^2}} = \frac{\langle 7,7 \rangle}{\sqrt{2\cdot 49}} = \frac{\langle 7,7 \rangle}{7\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}\langle 1,1 \rangle}$$

2. (6 POINTS) Consider the following vectors: $\mathbf{u} = \langle -4,3,0 \rangle$, $\mathbf{v} = \langle 1,0,-4 \rangle$ and $\mathbf{w} = \langle -1,2,3 \rangle$.
   a. (3 POINTS) Find $\mathbf{z} = 2\mathbf{u} - \mathbf{v} - \frac{1}{2}\mathbf{w}$.

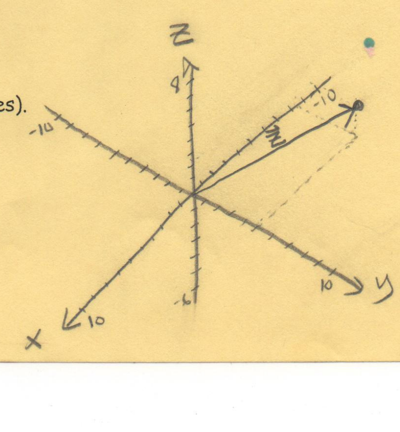
   $\vec{z} = 2\langle -4,3,0 \rangle - \langle 1,0,-4 \rangle - \frac{1}{2}\langle -1,2,3 \rangle$

   $\vec{z} = \langle -8,6,0 \rangle + \langle -1,0,4 \rangle + \langle \frac{1}{2},-1,-\frac{3}{2} \rangle \rightarrow \boxed{\vec{z} = \langle -\frac{17}{2}, 5, \frac{5}{2} \rangle}$

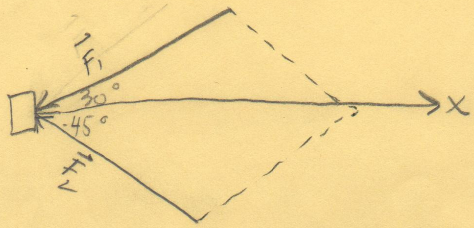
   b. (1 POINT) Find a vector which is parallel to $\mathbf{z}$.

   $\boxed{2\vec{z} = \langle -17, 10, 5 \rangle}$

   c. (2 POINTS) Sketch the vector $\mathbf{z}$ (be sure to label your axes).



3. (7 POINTS) Forces with magnitudes of 500 pounds and 200 pounds act on a machine part at angles 30° and -45°, respectively, with the x-axis. Find the direction and magnitude of the **resultant force**. Round to the nearest tenth.



$\vec{F}_1 = 500(\cos 30°\,\hat{i} + \sin 30°\,\hat{j}) = 500\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) = 250(\sqrt{3}\,\hat{i} + \hat{j})$

$\vec{F}_2 = 200(\cos(-45°)\,\hat{i} + \sin(-45°)\,\hat{j}) = 200\left(\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right) = 100\sqrt{2}(\hat{i} - \hat{j})$

Resultant Force: $\|\vec{F}_1 + \vec{F}_2\| = \boxed{\sqrt{(250\sqrt{3}+100\sqrt{2})^2 + (250-100\sqrt{2})^2} \text{ pounds}}$

$\boxed{\theta = \arctan \frac{250-100\sqrt{2}}{250\sqrt{3}+100\sqrt{2}}}$

approximations:

$\boxed{\|\vec{F}_1+\vec{F}_2\| \approx 584.6 \text{ pounds} \quad \theta \approx 10.7°}$

4. (9 POINTS) Consider the vectors $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.

a. (3 POINTS) Determine whether $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, parallel, or neither.

$\vec{u}\cdot\vec{v} = (3)(2) + (2)(-3) + (1)(0)$

$\vec{u}\cdot\vec{v} = 0$

$\boxed{\text{orthogonal}}$

b. (3 POINTS) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}$

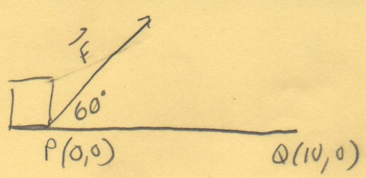
$= \vec{0}$

c. (3 POINTS) Find the angle between $\mathbf{u}$ and $\mathbf{v}$. Give your result in radians.

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} \rightarrow \cos\theta = 0$

$\boxed{\theta = \frac{\pi}{2}}$

5. (6 POINTS) An object is pulled 10 feet across a floor, using a force of 45 pounds. The direction of the force is 60° above the horizontal. Find the work done.



$\vec{F} = 45(\cos 60°\,\hat{i} + \sin 60°\,\hat{j})$

$\vec{F} = 45\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$\vec{PQ} = 10\hat{i}$

$W = \vec{F}\cdot\vec{PQ} = \left(\frac{45}{\cancel{2}}\right)(\cancel{10}^{5}) + \left(\frac{45\sqrt{3}}{2}\right)(0) = \boxed{225 \text{ foot-pounds}}$

6. (4 POINTS) Find the direction cosines of the vector $\mathbf{u} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$.

$\|\vec{u}\| = \sqrt{(2)^2 + (8)^2 + (-2)^2} = \sqrt{72} = 6\sqrt{2}$

$\cos\alpha = \frac{u_1}{\|\vec{u}\|} = \frac{\cancel{2}^{1}}{\cancel{6}_{3}\sqrt{2}} = \boxed{\frac{1}{3\sqrt{2}} \text{ or } \frac{\sqrt{2}}{6}}$

$\cos\beta = \frac{u_2}{\|\vec{u}\|} = \frac{\cancel{8}^{4}}{\cancel{6}_{3}\sqrt{2}} = \boxed{\frac{4}{3\sqrt{2}} \text{ or } \frac{2\sqrt{2}}{3}}$

$\cos\gamma = \frac{u_3}{\|\vec{u}\|} = -\frac{\cancel{2}^{1}}{\cancel{6}_{3}\sqrt{2}} = \boxed{-\frac{1}{3\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{6}}$

7. (10 POINTS) Consider the line which passes through the point (0, -1, 2) parallel to the vector $\mathbf{v} = \langle -2, 5, -3 \rangle$

a. (5 POINTS) Find sets of parametric equations of the line.

$a = -2, \ b = 5, \ c = -3$

$x = x_1 + at$ | $y = y_1 + bt$ | $z = z_1 + ct$

$x = 0 - 2t$ | $y = -1 + 5t$ | $z = 2 - 3t$

$x = -2t$

b. (5 POINTS) Find sets of symmetric equations of the line.

$$\boxed{\frac{x}{-2} = \frac{y+1}{5} = \frac{z-2}{-3}}$$

8. (10 POINTS) Find an equation of the plane which passes through (0, 1, 0), (1, 2, 3) and (-1, 6, 2).

$\vec{u} = \langle 1-0, 2-1, 3-0 \rangle = \langle 1, 1, 3 \rangle$

$\vec{v} = \langle -1-0, 6-1, 2-0 \rangle = \langle -1, 5, 2 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -1 & 5 & 2 \end{vmatrix} = (2-15)\hat{i} - (2+3)\hat{j} + (5+1)\hat{k} = \underset{a}{-13\hat{i}} \underset{b}{-5\hat{j}} + \underset{c}{6\hat{k}}$$

$\boxed{-13(x-0) - 5(y-1) + 6(z-0) = 0}$

$-13x - 5y + 5 + 6z = 0$

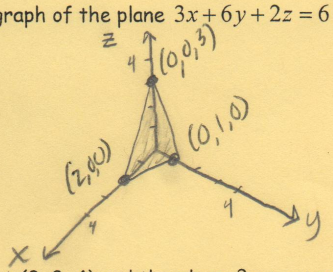
$\boxed{-13x - 5y + 6z = -5}$

9. (6 POINTS) Label any intercepts and sketch a graph of the plane $3x + 6y + 2z = 6$.

$x = 0, y = 0, z = 3, \ (0,0,3)$

$x = 0, z = 0, y = 1, \ (0,1,0)$

$y = 0, z = 0, x = 2, \ (2,0,0)$



10. (6 POINTS) Find the distance between the point (2, 8, 4) and the plane $2x + y + z = 5$.

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$x_0 \ y_0 \ z_0$

$d = -(ax_1 + by_1 + cz_1)$

let $x = 0, y = 0 \rightarrow z = 5$ ← point in the plane

$(0,0,5) \rightarrow (x_1, y_1, z_1)$

$d = (1)(5) = 5$

$$D = \frac{|2(2) + (1)(8) + (1)(4) - 5|}{\sqrt{(2)^2 + (1)^2 + (1)^2}}$$

$D = \frac{|11|}{\sqrt{6}} \rightarrow \boxed{D = \frac{11}{\sqrt{6}} \text{ or } \frac{11\sqrt{6}}{6}}$

11. (8 POINTS) Identify the quadric surface.

a. $\frac{x^2}{2}-\frac{y^2}{3}-\frac{z^2}{5}=1$ hyperboloid of 2 sheets

b. $z^2-x^2-\frac{y^2}{4}=1$ hyperboloid of 2 sheets

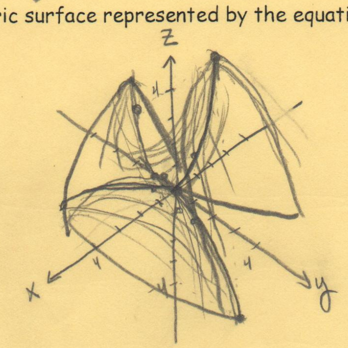
c. $x=\frac{y^2}{81}-\frac{z^2}{9}$ hyperbolic paraboloid

d. $\frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{25}=1$ ellipsoid

12. (8 POINTS) Sketch the quadric surface represented by the equation $z=x^2-y^2$. Be sure to label your axes.

$x=0,\ z=-y^2$

$y=0,\ z=x^2$



13. (10 POINTS) Consider the rectangular equation $x^2+y^2=4z^2$.

a. (5 POINTS) Convert the rectangular equation to an equation in cylindrical coordinates.

$$\boxed{r^2=4z^2}$$

b. (5 POINTS) Convert the rectangular equation to an equation in spherical coordinates.

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta=4\rho^2\cos^2\phi$$

$$\rho^2\sin^2\phi\underbrace{(\cos^2\theta+\sin^2\theta)}_{1}=4\rho^2\cos^2\phi$$

$$\rho^2\sin^2\phi-4\rho^2\cos^2\phi=0$$

$$\rho^2(\sin^2\phi-4\cos^2\phi)=0$$

$$\rho^2=0 \text{ or } \sin^2\phi=4\cos^2\phi$$

$$\rho=0 \qquad \tan^2\phi=4$$

$$\tan\phi=2 \rightarrow \boxed{\phi=\arctan 2}$$

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