4/8/11

- warm up using 14.6 w.s.
- Lecture 14.6

Prep for Monday

- Have questions ready about converting to polars
- Lecture 14.7

Wednesday

Review

Friday 4/15/11

Exam 4 / 14.1-14.3, 14.5-14.7

14.2 41

$z = x^2 + y^2, \; x^2 + y^2 = 4, \; z = 0$

1st quadrant

$V = 4\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)\, dy\, dx$

$V = 4\int_0^2 \left[x^2 y + \frac{y^3}{3}\right]_0^{\sqrt{4-x^2}} dx$

$V = 4\int_0^2 \left[x^2(4-x^2)^{1/2} + \frac{1}{3}(4-x^2)^{3/2}\right] dx$

$V = 4\int_0^{\pi/2} \left[(4\sin^2\theta)(2\cos\theta)(2\cos\theta\, d\theta)\right] + \frac{4}{3}\int_0^{\pi/2} (8\cos^3\theta)(2\cos\theta\, d\theta)$

$V =$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$(4-x^2)^{3/2} = 8\cos^3\theta$

$x^2 = 4\sin^2\theta$

limits: $x = 0$

$0 = 2\sin\theta$

$\theta = 0$

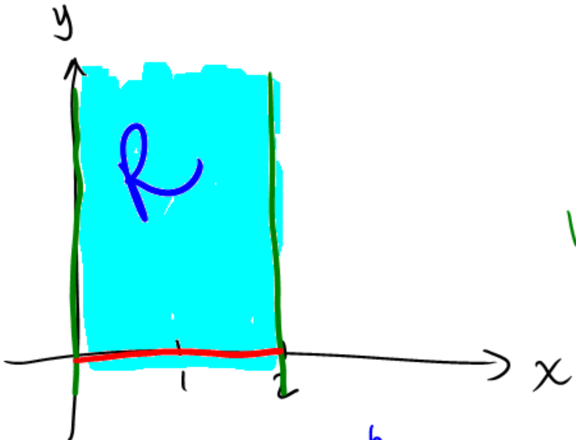
$2 = 2\sin\theta$

$1 = \sin\theta \rightarrow \theta = \frac{\pi}{2}$

When you are done with your homework you should be able to…

- π Use a triple integral to find the volume of a solid region
- π Find the center of mass and moments of inertia of a solid region

Warm-up: Set up a double integral to find the volume of the solid bounded by the graphs of the equations $z=\dfrac{1}{1+y^2}$, $x=0$, $x=2$ and $y\ge 0$.



$$V=\int_0^2\int_0^{\infty}\frac{1}{1+y^2}\,dy\,dx$$

$$V=\int_0^2 \frac{\pi}{2}\,dx$$

$$V=\frac{\pi}{2}\left(x\Big|_0^2\right.$$

$$=\frac{\pi}{2}(2-0)$$

$$=\boxed{\pi \text{ cubic units}}$$

$$\int_0^{\infty}\frac{1}{1+y^2}\,dy=\lim_{b\to\infty}\int_0^b\frac{dy}{1+y^2}=\lim_{b\to\infty}\arctan y\Big|_0^b=\lim_{b\to\infty}\arctan b-\lim_{b\to\infty}\arctan 0$$

$$=\frac{\pi}{2}-0=\frac{\pi}{2}$$

DEFINITION: TRIPLE INTEGRAL

If f is continuous over a bounded solid region Q, then the **triple integral of f over Q** is defined as

$$\iiint_Q f(x,y,z)\,dV=\lim_{\|\Delta\|\to 0}\sum_{i=1}^{n} f(x_i,y_i,z_i)\,\Delta V_i$$

Provided the limit exists. The **volume** of the solid region Q is given by

$$\text{Volume of } Q=\iiint_Q dV$$

THEOREM: EVALUATION BY ITERATED INTEGRALS

Let f be continuous on a solid region Q defined by

$$a \le x \le b,\ h_1(x) \le y \le h_2(x),\ g_1(x,y) \le z \le g_2(x,y)$$

where $h_1,\ h_2,\ g_1,$ and g_2 are continuous functions. Then

$$\iiint_Q f(x,y,z)\,dV = \int_a^b \int_{h_1(x)}^{h_2(x)} \int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dz\,dy\,dx$$

Example 1: Evaluate the iterated integral.

$$\int_1^4 \int_1^{e^2} \int_0^{1/(xz)} \ln z\,dy\,dz\,dx = \int_1^4 \int_1^{e^2} (\ln z)\,y \Big|_0^{1/xz} dz\,dx$$

$$= \int_1^4 \int_1^{e^2} \frac{\ln z}{xz}\,dz\,dx$$

$$= \int_1^4 \frac{(\ln z)^2}{2x} \Big|_1^{e^2} dx$$

$$= \int_1^4 \left[\frac{(\ln e^2)^2}{2x} - \frac{(\ln 1)^2}{2x}\right] dx$$

$$= \int_1^4 \frac{2}{x}\,dx$$

$$= 2\ln|x| \Big|_1^4$$

$$= 2(\ln 4 - \overset{0}{\ln 1})$$

$$= 2\ln 4$$

$$= \boxed{\ln 16}$$

$\ln e^2 = 2$

$\ln 1 = 0$

Example 2: Set up a triple integral for the volume of the solid.

The solid that is the common interior below the sphere $x^2 + y^2 + z^2 = 80$

$z = \sqrt{80 - x^2 - y^2}$

and above the paraboloid $z = \frac{1}{2}\left(x^2 + y^2\right)$ $\to z \ge 0$

$$\boxed{V = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\frac{1}{2}(x^2+y^2)}^{\sqrt{80-x^2-y^2}} dz\,dy\,dx}$$

$x^2 + y^2 = 2z$

so... $x^2 + y^2 + z^2 = 80$

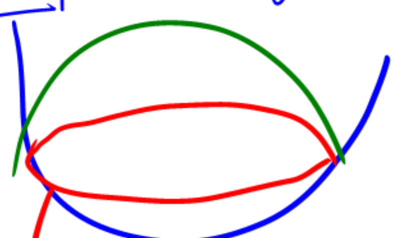
$2z + z^2 = 80$

$z^2 + 2z - 80 = 0$

$(z+10)(z-8) = 0$

$\cancel{z = -10}$ or $z = 8$

so $x^2 + y^2 = 2z \to x^2 + y^2 = 16$



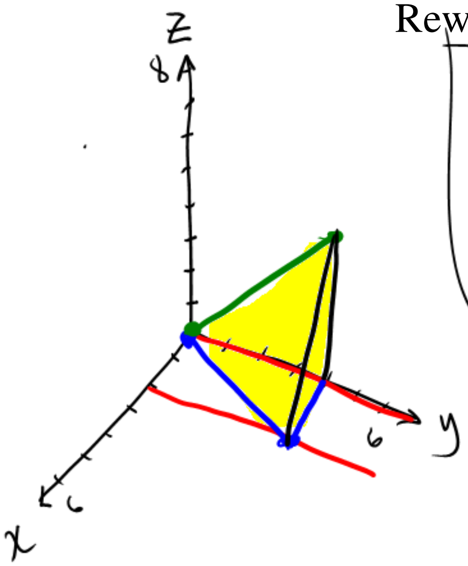
$x^2 + y^2 = 4^2$

Example 3: Sketch the solid whose volume is given by the iterated integral and rewrite the integral using the indicated order of integration.

$$\int_0^2 \int_{2x}^4 \int_0^{\sqrt{y^2-4x^2}} dz\,dy\,dx$$

Rewrite using the order $dxdydz$

$$\int_0^4 \int_z^4 \int_0^{\frac{\sqrt{y^2-z^2}}{2}} dx\,dy\,dz$$



$z=\sqrt{y^2-4x^2}$

$z=0$

$x=0 \rightarrow z=\sqrt{y^2-0}$

$z=y$ $\quad z=\pm y \rightarrow 0\le y\le 4$

$y=2x$

$x=2 \rightarrow y=2(2)$

$y=4$

$z=\sqrt{y^2-4x^2}$

$z^2=y^2-4x^2$

$x=\sqrt{\frac{y^2-z^2}{4}}$

$x=\frac{\sqrt{y^2-z^2}}{2}$

Example 4: List the six possible orders of integration for the triple integral over the solid region Q $\iiint_Q xyz\,dV$.

$x^2=y \rightarrow x=\sqrt{y}$

$$Q=\{(x,y,z): 0\le x\le 2, x^2\le y\le 4, 0\le z\le 6\}$$

$$\int_0^6\int_0^4\int_0^{\sqrt{y}} xyz\,dx\,dy\,dz$$

$$=\int_0^4\int_0^6\int_0^{\sqrt{y}} xyz\,dx\,dz\,dy$$

$$=\int_0^6\int_0^2\int_{x^2}^{4} xyz\,dy\,dx\,dz$$

$0\le y\le 4, \quad 0<x<\sqrt{y}$

$$=\int_0^2\int_0^6\int_{x^2}^{4} xyz\,dy\,dz\,dx$$

$$=\int_0^2\int_{x^2}^{4}\int_0^6 xyz\,dz\,dy\,dx$$

$$=\int_0^4\int_0^{\sqrt{y}}\int_0^6 xyz\,dz\,dx\,dy$$