4/6/11

- Finish 14.3
- Start 14.5

Friday

- Finish 14.5
- Start 14.6

Monday

- Finish 14.6
- Lecture 14.7

Exam to Friday: 4/15

When you are done with your homework you should be able to…

- π Use a double integral to find the area of a surface

Warm-up: Find the area of the parallelogram with vertices $A=(2,-3,1)$, $B=(6,5,-1)$, $C=(3,-6,4)$ and $D=(7,2,2)$. Hint: Section 11.4

DEFINITION: SURFACE AREA

If f and its first partial derivatives are continuous on the closed region R in the *xy*-plane, then the **area of the surface *S*** given by $z=f(x,y)$ over R is given by

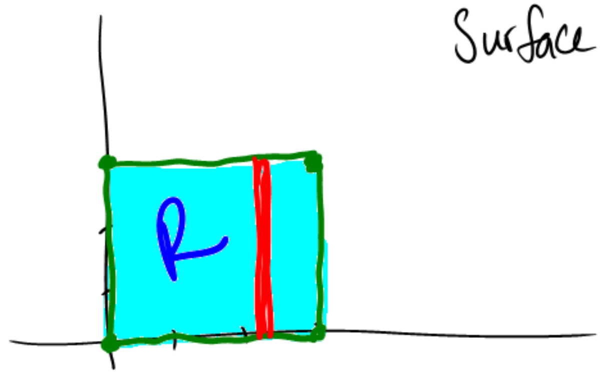
$$\text{Surface Area} = \int_R\int dS$$

$$= \int_R\int \sqrt{1+\left[f_x(x,y)\right]^2+\left[f_y(x,y)\right]^2}\,dA$$

Example 1: Find the area of the surface given by $z=f(x,y)$ over the region R.

$$f(x,y)=15+2x-3y$$

R: square with vertices $(0,0)$, $(3,0)$, $(0,3)$, $(3,3)$



$$\text{Surface Area} = \iint_R \sqrt{1+[f_x(x,y)]^2+[f_y(x,y)]^2}\, dA$$

$$= \int_0^3\int_0^3 \sqrt{1+(2)^2+(3)^2}\, dy\,dx$$

$$= \int_0^3\int_0^3 \sqrt{14}\, dy\,dx$$

$$= \sqrt{14}\int_0^3 y\Big|_0^3 dx$$

$$= 3\sqrt{14}\int_0^3 dx$$

$$= 3\sqrt{14}\; x\Big|_0^3 = \boxed{9\sqrt{14}\text{ sq. units}}$$

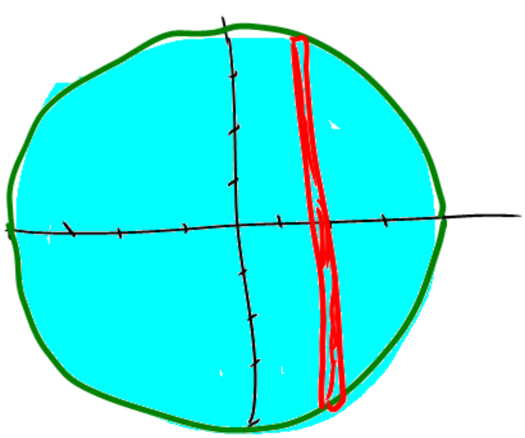
$f_x(x,y)=2$

$f_y(x,y)=-3$

Example 2: Find the area of the surface given by $z=f(x,y)$ over the region R.

$$f(x,y)=xy$$

$$R=\{(x,y)\mid x^2+y^2\le 16\}$$



$$\text{Surface Area} = \int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{1+x^2+y^2}\, dy\,dx$$

$$= \frac{1}{2}\int_0^{2\pi}\int_0^4 2r\sqrt{1+r^2}\, dr\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{2}{3}(1+r^2)^{3/2}\Big|_0^4 d\theta$$

$$= \frac{1}{3}\left(17^{3/2}-1\right)\int_0^{2\pi} d\theta$$

$$= \frac{1}{3}\left(17^{3/2}-1\right)\theta\Big|_0^{2\pi}$$

$$= \boxed{\frac{2\pi}{3}\left(17^{3/2}-1\right)\text{ sq. units}}$$

$f_x = y$

$f_y = x$

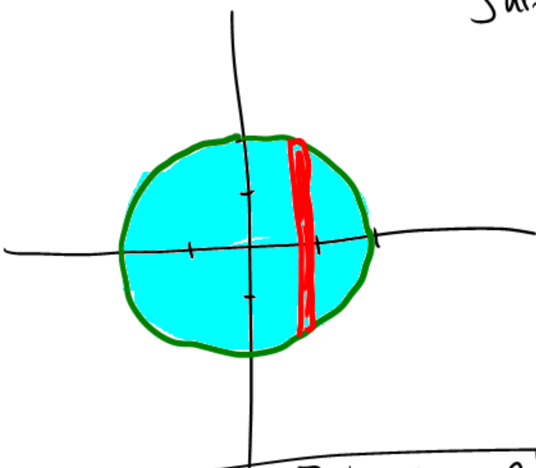
$x^2+y^2=16$

$y^2=16-x^2$

$y=\pm\sqrt{16-x^2}$

Example 3: Find the area of the surface.

The portion of the cone $z = 2\sqrt{x^2+y^2}$ inside the cylinder $x^2+y^2=4$.



$$\text{Surface Area} = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1+(f_x)^2+(f_y)^2}\; dy\,dx$$

$$= \int_0^{2\pi}\int_0^{2} r\cdot\sqrt{5}\; dr\,d\theta$$

$$= \int_0^{2\pi} \frac{\sqrt{5}}{2} r^2 \Big|_0^2 \, d\theta$$

$$= \frac{4\sqrt{5}}{2}\int_0^{2\pi} d\theta$$

$$= 2\sqrt{5}\,\theta\Big|_0^{2\pi}$$

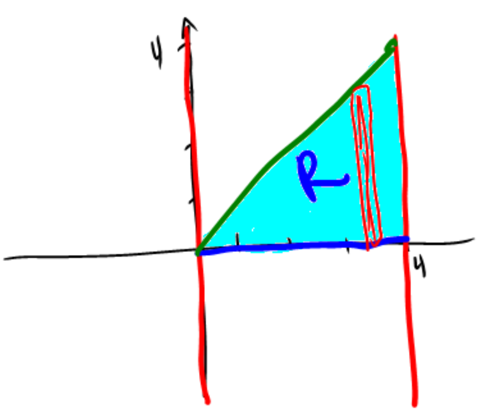
$$= \boxed{4\pi\sqrt{5}\text{ sq. units}}$$

$$f_x = 2\left[\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}\right] \qquad (f_x)^2 = \frac{4x^2}{x^2+y^2}$$

$$f_y = 2\left[\frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}\right] \qquad (f_y)^2 = \frac{4y^2}{x^2+y^2}$$

$$(f_x)^2+(f_y)^2 = \frac{4x^2+4y^2}{x^2+y^2} = \frac{4\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}}$$

Example 4: Set up a double integral that gives the area of the surface on the graph of $f(x,y) = e^{-x}\sin y,\ R = \{(x,y)\,|\,0 \le x \le 4,\ 0 \le y \le x\}$.



$$\boxed{\text{Surface Area} = \int_0^4\int_0^x \sqrt{1+e^{-2x}}\; dy\,dx}$$

$$f_x = -e^{-x}\sin y$$

$$(f_x)^2 = e^{-2x}\sin^2 y$$

$$f_y = e^{-x}\cos y$$

$$(f_y)^2 = e^{-2x}\cos^2 y$$

$$(f_x)^2+(f_y)^2 = e^{-2x}(\sin^2 y+\cos^2 y)$$

$$= e^{-2x}$$