3/18/11

- Practice Test
- HW ?'s

Monday

- Exam 3/Ch. 13.1–13.9
- HW due 13.1, 13.3–13.9 (13.2 extra credit)

Next Wednesday and Friday: no class

→ out-of-class assignment is due Monday, 3/29

13.1

(69) $f(x,y,z) = x - y + z,\ c = 1$

$x - y + z = 1$

$z = 0 \rightarrow$ xy-trace

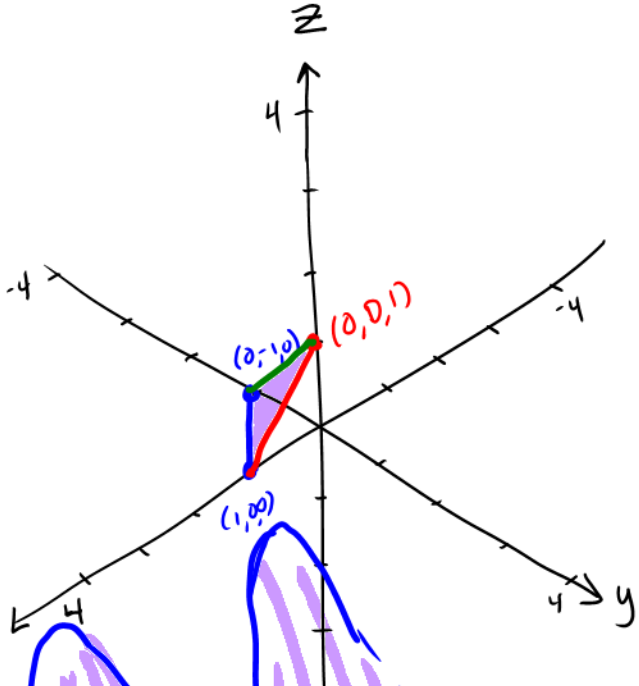
$x - y = 1$

$y = 0 \rightarrow$ xz-trace

$x + z = 1$

$x = 0$, yz-trace

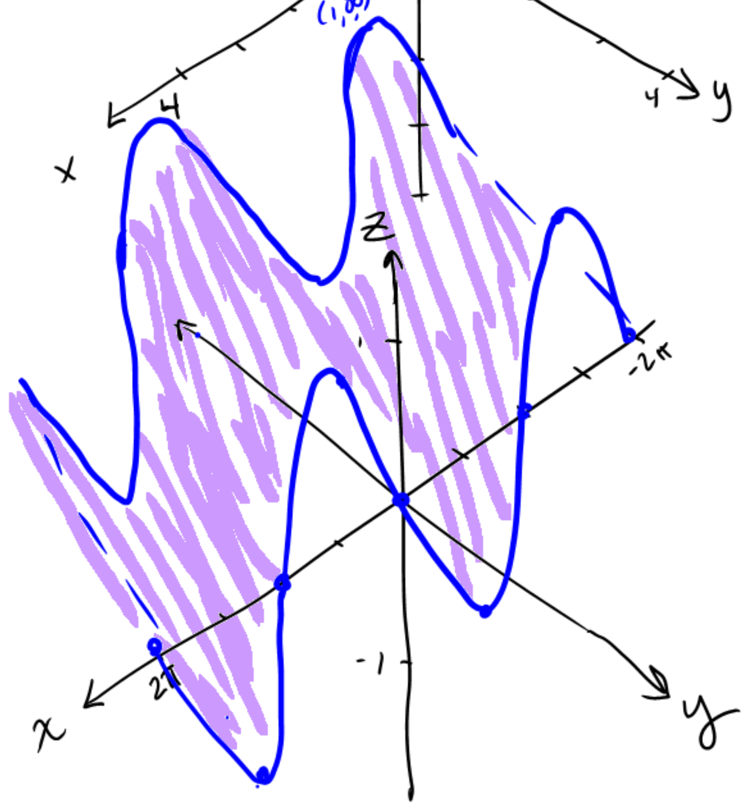
$-y + z = 1$



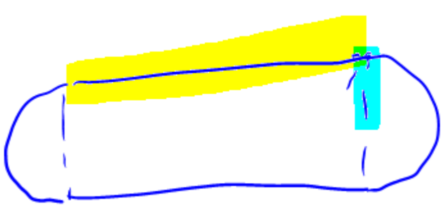
(74) $f(x,y,z) = \sin x - z$

$\sin x - z = 0$

$z = \sin x$



(82)



$V = \pi r^2 \ell + \frac{4}{3}\pi r^3$

13.3

(17) $z = e^x \tan y$

$z_x = e^x \tan y$ $\quad z_y = e^x \sec^2 y$

$z_{xx} = e^x \tan y$ $\quad z_{yy} = 2e^x \sec y \cdot \sec y \tan y$

$z_{xy} = e^x \sec^2 y$ $\quad z_{yy} = 2e^x \sec^2 y \tan y$

13.4

like 44 $\quad f(x,y) = 5x - 10y + y^2$

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$$

$$= [5(x+\Delta x) - 10(y+\Delta y) + (y+\Delta y)^2] - [5x - 10y + y^2]$$

$$= 5x + 5\Delta x - 10y - 10\Delta y + y^2 + 2y\Delta y + (\Delta y)^2 - 5x + 10y - y^2$$

$$= 5\Delta x - 10\Delta y + 2y\Delta y + (\Delta y)(\Delta y)$$

$$= f_x(x,y) \cdot \Delta x - f_y(x,y) \cdot \Delta y + 0 \cdot \Delta x + (\Delta y)(\Delta y)$$

$$= f_x(x,y)\Delta x - f_y(x,y) \cdot \Delta y + \varepsilon_1 \cdot \Delta x + \varepsilon_2 \cdot \Delta y$$

$\varepsilon_1 = 0, \quad \varepsilon_2 = \Delta y$

Since $\varepsilon_1 \to 0, \varepsilon_2 \to 0$ as $(0, \Delta y) \to (0,0)$, it follows that f is differentiable at every point in the domain.

(14) $f(x,y) = \frac{y}{x}$

a) $f(2,1) = \frac{1}{2}$

$f(2.1, 1.05) = \frac{1.05}{2.1} = \frac{1}{2}$

$\Delta z = \frac{1}{2} - \frac{1}{2} = 0$

b) $dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$

$dz = -\frac{y}{x^2}\Delta x + \frac{1}{x}\Delta y$

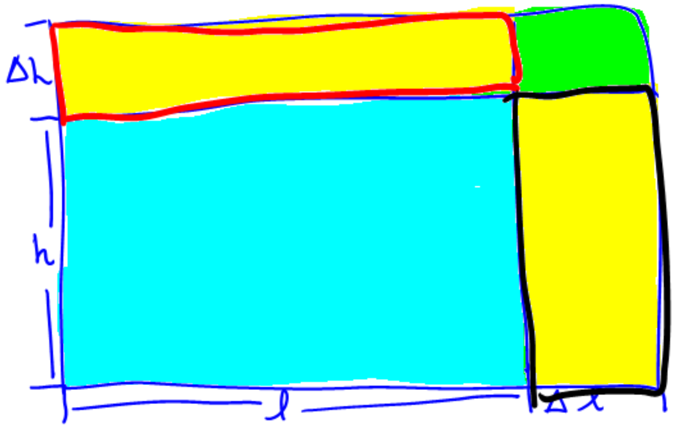
$\Delta x = 2.1 - 2 = .1$

$\Delta y = 1.05 - 1 = .05$

$dz = -\frac{1}{(2)^2} \cdot (.1) + \frac{1}{2}(.05)$

$dz = 0$

25



$A = lh$

$dA = \frac{\partial A}{\partial l} dl + \frac{\partial A}{\partial h} dh$

$dA = h\,dl + l\,dh$

green and yellow regions

$\Delta A = (h+\Delta h)(l+\Delta l) - hl$

$\Delta A - dA$

13.9

(4) Step 1: Analysis

Let S be the square of the distance from the point (0,0,2) to the surface $z = \sqrt{1-2x-2y}$

Step 2: Primary equation

$S = (x-0)^2 + (y-0)^2 + (z-2)^2$

$S = x^2 + y^2 + (z-2)^2$

Step 3: Reduce primary

$S = x^2 + y^2 + \left(\sqrt{1-2x-2y} - 2\right)^2$

Step 4: optimize

$S_x(x,y) = 2x + 2\left(\sqrt{1-2x-2y} - 2\right) \cdot \frac{-2}{2\sqrt{1-2x-2y}}$

$S_x(x,y) = 2\left(x - \frac{\sqrt{1-2x-2y} - 2}{\sqrt{1-2x-2y}}\right)$

$S_y(x,y) = 2\left(y - \frac{\sqrt{1-2x-2y} - 2}{\sqrt{1-2x-2y}}\right)$

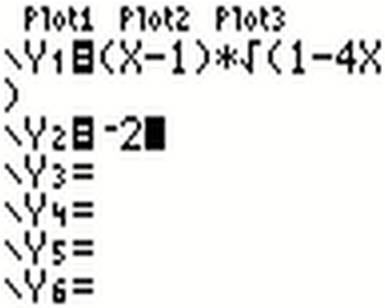
so $x = y$

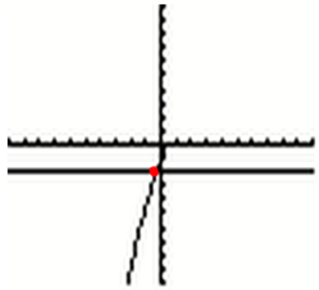
$x - \frac{\sqrt{1-2x-2x} - 2}{\sqrt{1-2x-2x}} = 0$

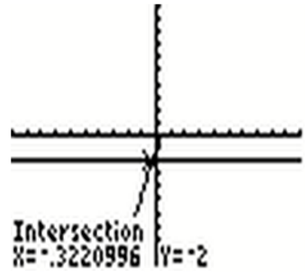
$\sqrt{1-4x}\left(x - \frac{\sqrt{1-4x} - 2}{\sqrt{1-4x}}\right) = (0)\sqrt{1-4x}$

$x\sqrt{1-4x} - \sqrt{1-4x} + 2 = 0$

$(x-1)\sqrt{1-4x} = -2$







$x = y \approx -0.3221$

$$S = x^2 + y^2 + \left(\sqrt{1-2x-2y} - 2\right)^2$$

$$S = (-.3221)^2 + (-.3221)^2 + \left(\sqrt{1-2(-.3221)-2(-.3221)} - 2\right)^2$$

$$= .1037 + .1037 + \left(\sqrt{2.2884} - 2\right)^2$$

$$= .2074 + .2374$$

$$= .4448$$

$$d = \sqrt{.4448} \approx \boxed{0.667 \text{ units}}$$