2/4/11

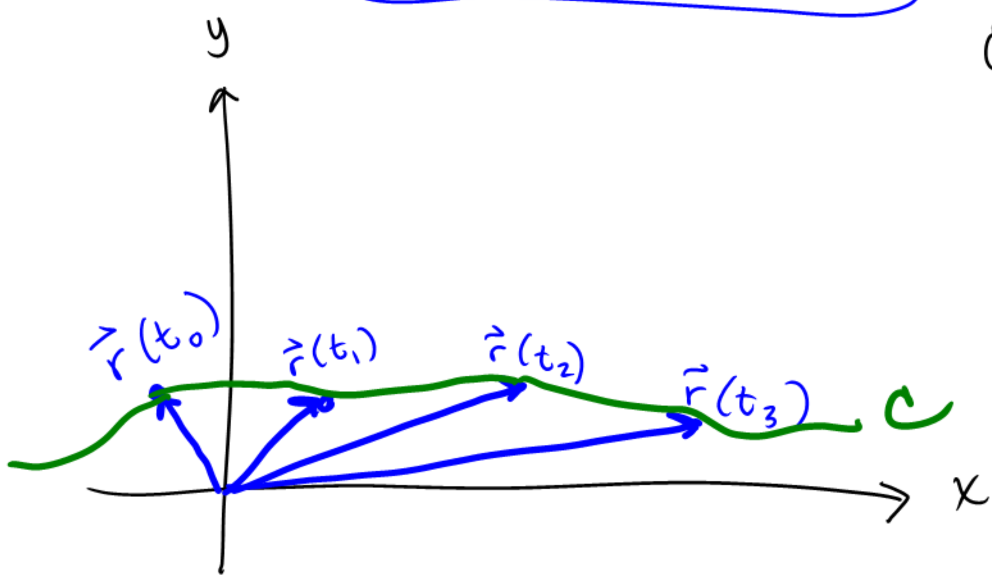
- Lecture 12.1

Monday

Lecture 12.2

→ practice your derivatives

curve in a plane



When you are done with your homework you should be able to…

- π Analyze and sketch a space curve given by a vector-valued function
- π Extend the concepts of limits and continuity to vector-valued functions

Warm-up: Evaluate the following limits analytically.

1. $2\lim_{x\to 0}\frac{\sin 2x}{2x} = 2(1) = \boxed{2}$

L'Hôpital's Rule

D.S.

2. $\lim_{t\to 4}\frac{t^2-16}{t^2-4t} = \lim_{t\to 4}\frac{\frac{\partial}{\partial t}(t^2-16)}{\frac{\partial}{\partial t}(t^2-4t)} = \lim_{t\to 4}\frac{2t}{2t-4} = \frac{2(4)}{2(4)-4} = \boxed{2}$

D.S. 0/0

3. $\lim_{x\to\infty}\left(e^{-x}-\frac{6}{x}-\arctan x\right) = \lim_{x\to\infty}\frac{1}{e^x} - \lim_{x\to\infty}\frac{6}{x} - \lim_{x\to\infty}\arctan x$

$= 0 - 0 - \frac{\pi}{2}$

$= \boxed{-\frac{\pi}{2}}$

DEFINITION OF VECTOR-VALUED FUNCTION

A function of the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \qquad \text{plane}$$
$$= \langle f(t), g(t)\rangle$$

or

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k} \qquad \text{space}$$
$$= \langle f(t), g(t), h(t)\rangle$$

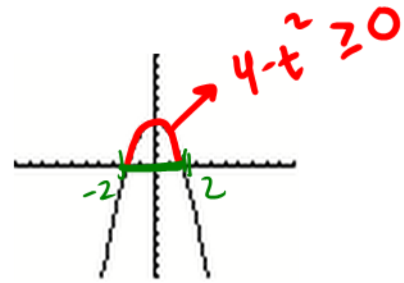
is a **vector-valued function**, where the **component functions** *f*, *g*, and *h* are real-valued functions of the parameter *t*. The domain is considered to be the intersection of the domains of the component functions *f*, *g*, and *h*, unless stated otherwise.

Example 1: Find the domain of the vector-valued function.

$$\mathbf{r}(t)=\sqrt{4-t^2}\,\mathbf{i}+t^2\mathbf{j}-6t\mathbf{k}$$

$f(t)=\sqrt{4-t^2}$, D: $[-2,2]$
$g(t)=t^2$, D: $(-\infty,\infty)$
$h(t)=-6t$, D: $(-\infty,\infty)$

Domain $\vec{r}(t)$: $[-2,2]$



Example 2: Sketch the curve represented by the vector-valued function.

a) $\mathbf{r}(t)=(1-t)\mathbf{i}+\sqrt{t}\,\mathbf{j}$

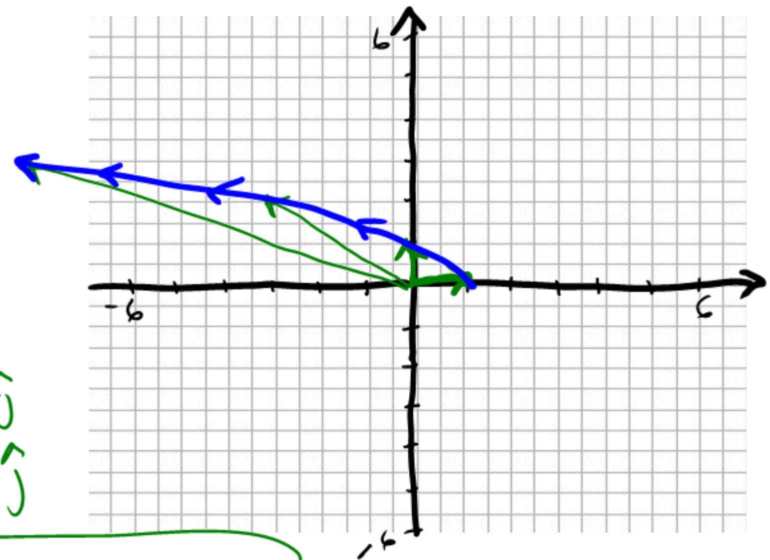
D: $[0,\infty)$

$\vec{r}(0)=(1-(0))\hat{i}+\sqrt{0}\,\hat{j}=\hat{i}$

$\vec{r}(1)=(1-1)\hat{i}+\sqrt{1}\,\hat{j}=\hat{j}$

$\vec{r}(4)=(1-4)\hat{i}+\sqrt{4}\,\hat{j}=-3\hat{i}+2\hat{j}$

$r(9)=(1-9)\hat{i}+\sqrt{9}\,\hat{j}=-8\hat{i}+3\hat{j}$



b) $\mathbf{r}(t)=(3\cos t)\mathbf{i}+(4\sin t)\mathbf{j}+\frac{t}{2}\mathbf{k}$

D: $(-\infty,\infty)$

$\vec{r}(0)=3\cos 0\,\hat{i}+4\sin 0\,\hat{j}+\frac{0}{2}\hat{k}$

$=3\hat{i}$

$r(\frac{\pi}{2})=4\hat{j}+\frac{\pi}{4}\hat{k}$

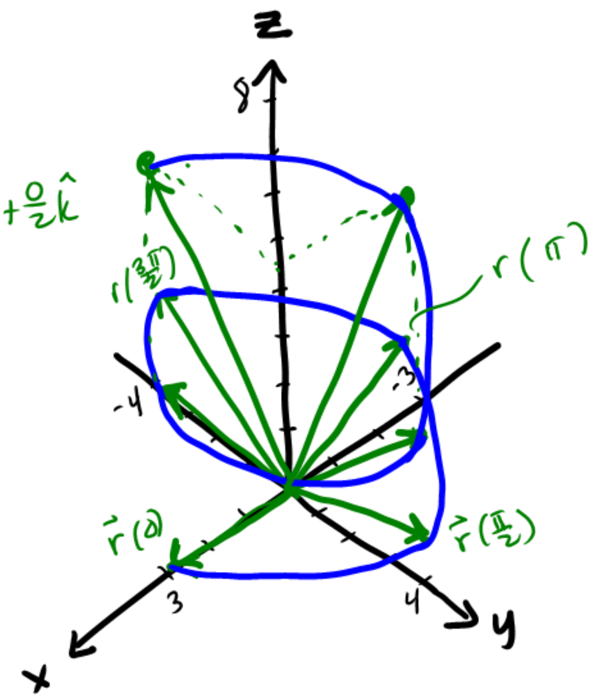
$r(\pi)=-3\hat{i}+\frac{\pi}{2}\hat{k}$

$r(\frac{3\pi}{2})=-4\hat{j}+\frac{3\pi}{4}\hat{k}$

$r(2\pi)=3\hat{i}+\pi\hat{k}$

$r(\frac{5\pi}{2})=4\hat{j}+\frac{5\pi}{4}\hat{k}$

$r(3\pi)=-3\hat{i}+\frac{3\pi}{2}\hat{k}$



$r(\frac{7\pi}{2})=-4\hat{j}+\frac{7\pi}{4}$

DEFINITION OF THE LIMIT OF A VECTOR-VALUED FUNCTION

1. If $\mathbf{r}$ is a vector-valued function such that $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$, then
$$\lim_{t\to a}\mathbf{r}(t) = \left[\lim_{t\to a} f(t)\right]\mathbf{i} + \left[\lim_{t\to a} g(t)\right]\mathbf{j}$$
provided *f* and *g* have limits as $t \to a$.
2. If $\mathbf{r}$ is a vector-valued function such that $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, then
$$\lim_{t\to a}\mathbf{r}(t) = \left[\lim_{t\to a} f(t)\right]\mathbf{i} + \left[\lim_{t\to a} g(t)\right]\mathbf{j} + \left[\lim_{t\to a} h(t)\right]\mathbf{k}$$
provided *f*, *g* and *h* have limits as $t \to a$.

DEFINITION OF CONTINUITY OF A VECTOR-VALUED FUNCTION

A vector-valued function $\mathbf{r}$ is **continuous at the point** given by $t = a$ if the limit of $\mathbf{r}(t)$ exists as $t \to a$ and $\lim_{t\to a}\mathbf{r}(t) = \mathbf{r}(a)$.

A vector-valued function $\mathbf{r}$ is **continuous on an interval** *I* if it is continuous at every point in the interval.

Example 3: Evaluate the limit and determine the interval(s) on which the vector-valued function is continuous.

$$\lim_{t\to 1}\left((\ln t)\mathbf{i} - \left(\frac{1-t^2}{1-t}\right)\mathbf{j} + (\arcsin t)\mathbf{k}\right)$$

$$\boxed{\text{continuous on } (0,1).}$$

$f(t) = \ln t$

$D: (0, \infty)$

$g(t) = \dfrac{1-t^2}{1-t}$

$D: (-\infty, 1) \cup (1, \infty)$

$h(t) = \arcsin t$

$D: [-1, 1]$

$= (\ln 1)\hat{i} - \left(\lim_{t\to 1} \dfrac{\cancel{-}2t}{\cancel{-}1}\right)\hat{j} + \frac{\pi}{2}\hat{k}$

$= 0\hat{i} - 2(1)\hat{j} + \frac{\pi}{2}\hat{k}$

$= -2\hat{j} + \frac{\pi}{2}\hat{k}$