Today

Lecture 13.1

Wednesday

13.2, 13.3

When you are done with your homework you should be able to…

- π Understand the notation for a function of several variables
- π Sketch the graph of a function of two variables
- π Sketch level curves for a function of two variables
- π Sketch level surfaces for a function of three variables

Warm-up: Find two functions such that the composition $h(x) = (f \circ g)(x) = \sin^2 x$

$f(x) =$ x^2

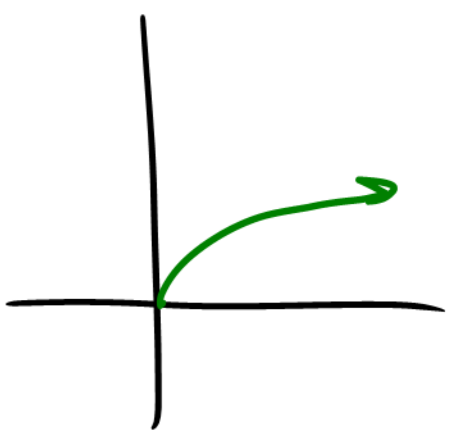
$g(x) =$ $\sin x$

DEFINITION: A FUNCTION OF TWO VARIABLES

Let *D* be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in *D* there corresponds a unique real number $f(x, y)$, then f is called a **function of *x* and *y***. The set *D* is the **domain of *f***, and the corresponding set of values for $f(x, y)$ is the **range** of *f*.

Consider $f(x) = \sqrt{x}$

Domain: $[0, \infty)$



Example 1: Find and simplify the function values.

$$g(x, y) = \ln|x + y|$$

a. $g(2,3) = \ln|2+3| = \boxed{\ln 5}$

b. $g(e,0) = \ln|e+0| = \ln e = \boxed{1}$

c. $g(0,1) = \ln|0+1| = \ln 1 = \boxed{0}$

Example 2: Describe the domain and range of each function.

a. $f(x, y) = \arccos\left(\frac{y}{x}\right)$

Consider...

$-1 \le \frac{y}{x} \le 1$, $x \ne 0$

Domain: $\{(x,y) \mid x, y \in \mathbb{R}, -1 \le \frac{y}{x} \le 1, x \ne 0\}$

Range: $\{f(x,y) \mid f(x,y) \in \mathbb{R}, 0 \le f(x,y) \le \pi\}$

b. $g(x, y) = x\sqrt{y}$

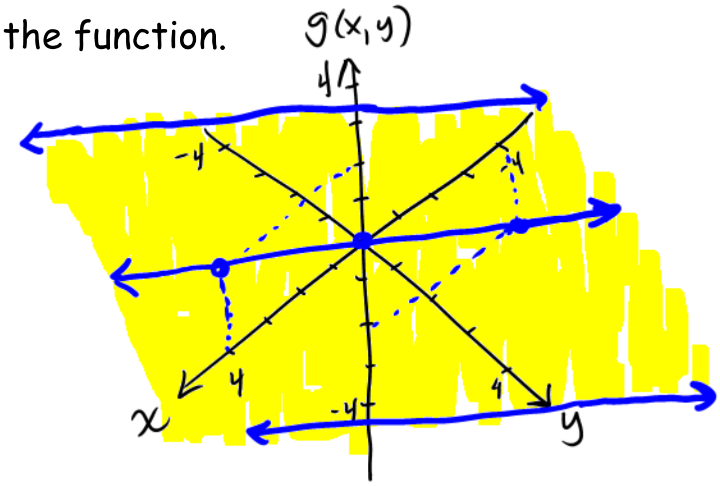
Domain: $\{(x,y) \mid x, y \in \mathbb{R}, y \ge 0\}$

Range: $\{z \mid z \in \mathbb{R}\}$

Example 3: Sketch the surface given by the function.

a. $g(x,y)=\left(\frac{1}{2}\right)x$

$z=\frac{1}{2}x$



b. $z=\frac{1}{2}\sqrt{x^2+y^2}$

top half of $z^2=\frac{x^2}{4}+\frac{y^2}{4}$

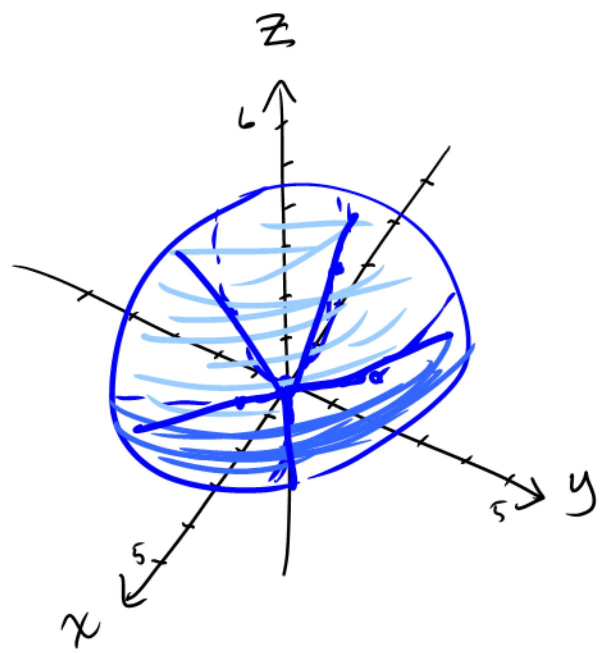
which is an elliptic cone

If $z=1$, $x^2+y^2=4$

(2,0,1) (0,-2,1)

(0,2,1)

(-2,0,1)

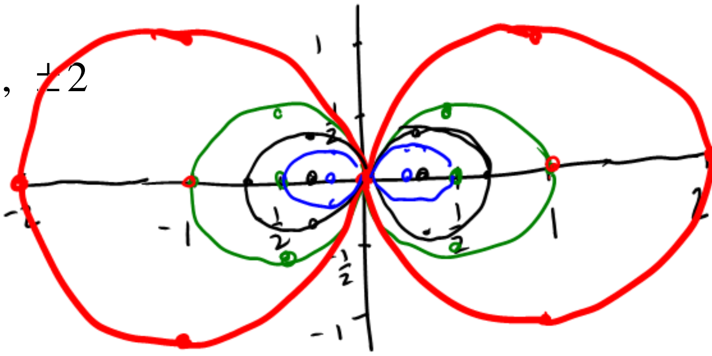


LEVEL CURVES

We can also visualize a function of two variables using a scalar field. This involves assigning a scalar value to *z*. This is then assigned to the point (*x, y*).

Example 4: Describe the level curves of the function. Sketch the level curves for the given c-values.

$$\underbrace{f(x,y)}_{c}=\frac{x}{x^2+y^2},\quad c=\pm\frac{1}{2},\ \pm 1,\ \pm\frac{3}{2},\ \pm 2$$



$$c=\frac{x}{x^2+y^2}$$

$$c(x^2+y^2)=x$$

$$x^2+y^2=\frac{x}{c}$$

$$x^2-\frac{x}{c}+\left(-\frac{1}{2c}\right)^2=-y^2+\left(-\frac{1}{2c}\right)^2$$

$$\left(x-\frac{1}{2c}\right)^2+y^2=\left(\frac{1}{2c}\right)^2$$

these are circles centered at $\left(\frac{1}{2c},0\right)$, w/radius $\frac{1}{2c}$

$c=1 \to \left(x-\frac{1}{2}\right)^2+y^2=\left(\frac{1}{2}\right)^2$

$c=\frac{1}{2} \to (x-1)^2+y^2=1^2$

$c=\frac{3}{2} \to \left(x-\frac{1}{3}\right)^2+y^2=\left(\frac{1}{3}\right)^2$

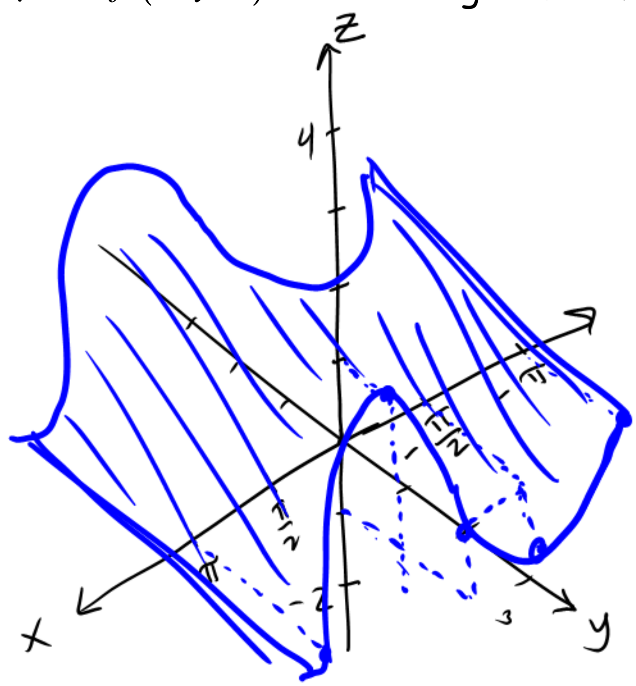
$c=2 \to \left(x-\frac{1}{4}\right)^2+y^2=\left(\frac{1}{4}\right)^2$

Example 5: Sketch the graph of the level surface $f(x,y,z)=c$ at the given value of c.

$$f(x,y,z)=\sin x - z,\quad c=0$$

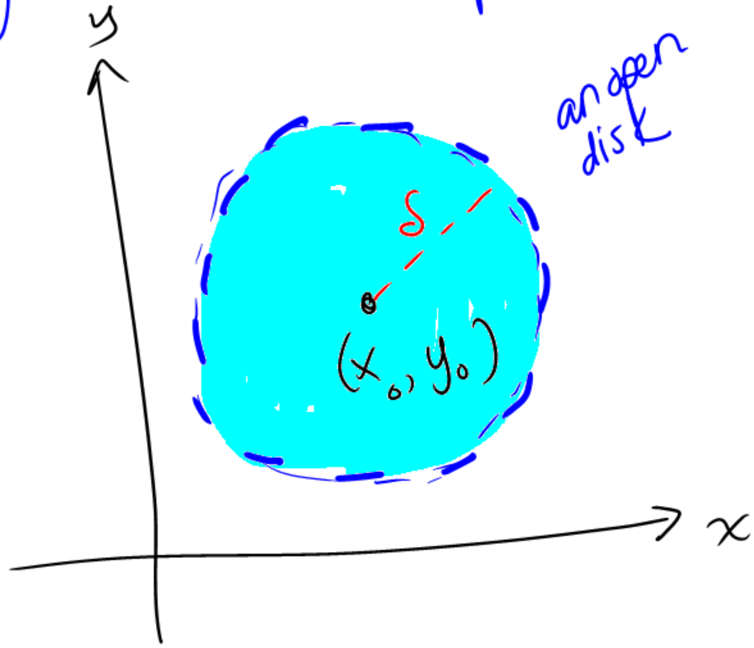
$$0=\sin x - z$$

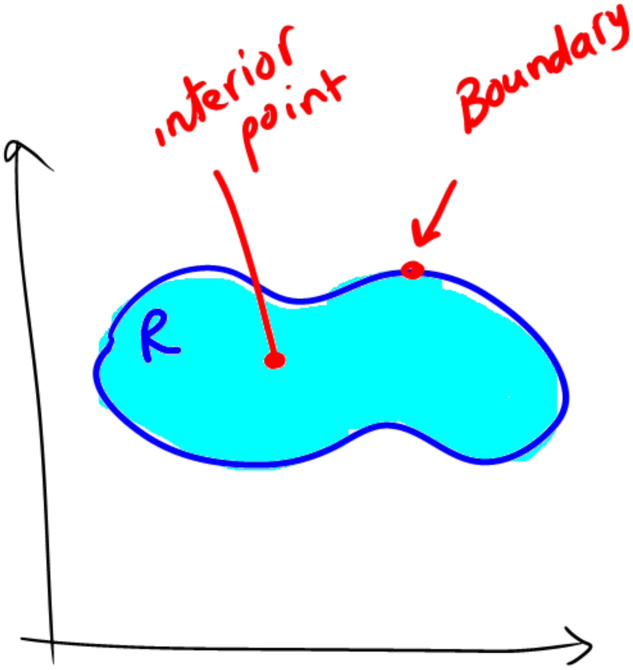
$$z=\sin x$$



13.2 Limits and Continuity

Neighborhoods in the plane



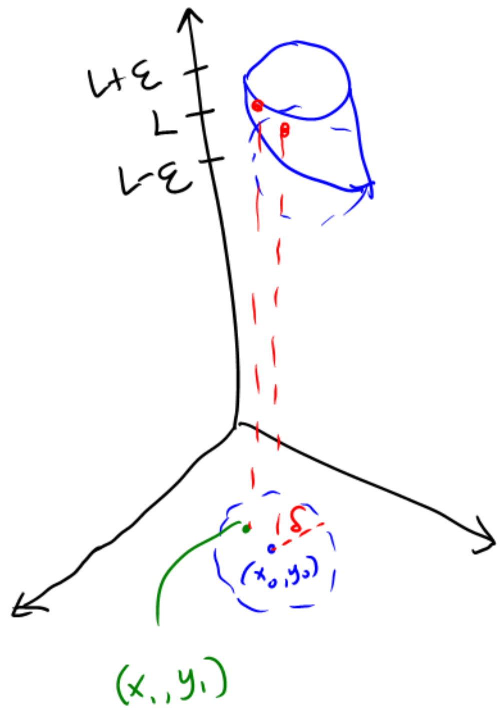


Definition of the limit of a function of 2 variables

f is a function of two variables defined, except possibly at (x_0, y_0) on an open disk centered at (x_0, y_0) and let L be a real number. Then

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$ if for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x,y) - L| < \varepsilon$ whenever

$$0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$



L+ε
L
L-ε
δ
(x₀,y₀)
(x₁,y₁)