1/31/11

- Review Ch. 11

Wednesday

- Exam 1/Ch. 11
- HW 11.1–11.7 is due

* use an actual 3×5 notecard in your writing both sides

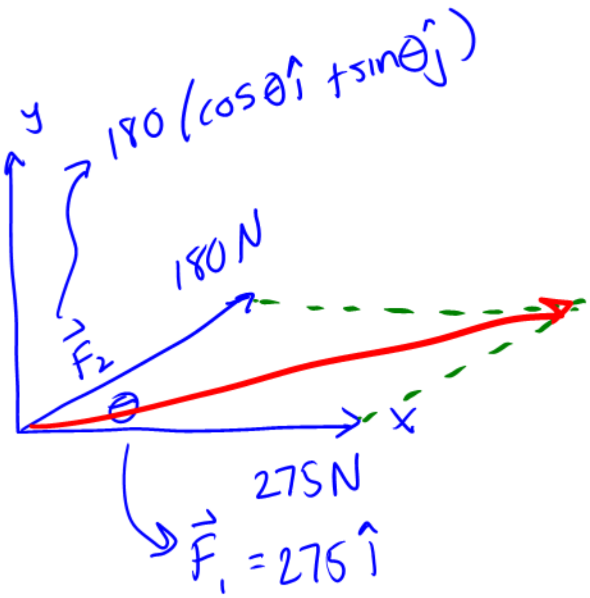
* no sample problems

Friday

12.1

11.1: 82, 47

(82)



a) $\theta = 30^\circ$

$\vec{F} = \vec{F}_1 + \vec{F}_2$

$= 275\hat{i} + 180(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j})$

$= 275\hat{i} + \overset{90}{\cancel{180}}(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j})$

$= \underbrace{(275 + 90\sqrt{3})}_{430.88}\hat{i} + 90\hat{j}$

Direction $\alpha = \arctan\frac{90}{430.88}$

$\alpha \approx 11.8^\circ$

magnitude: $\sqrt{(430.88)^2 + 90^2}$

≈ 440.18 N

b) Instead of $\theta = 30^\circ$, $\theta = \theta$

$\vec{F} = (275 + 180\cos\theta)\hat{i} + 180\sin\theta\hat{j}$

direction:

$\alpha = \arctan\frac{180\sin\theta}{275 + 180\cos\theta}$

magnitude: $\sqrt{(275 + 180\cos\theta)^2 + (180\sin\theta)^2}$

---

(47) want $\|\vec{v}\| = 6$ in the direction $\vec{u} = \langle 0, 3\rangle$

unit vector: $\frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle 0,3\rangle}{\sqrt{0^2+3^2}} = \frac{\langle 0,3\rangle}{3} = \langle 0,1\rangle$

now mult. unit vector by 6: $6\langle 0,1\rangle = \boxed{\langle 0,6\rangle}$

11.2: 44, 77, 96

(x_0, y_0, z_0) is the center

(44) $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

$$\frac{4x^2+4y^2+4z^2-24x-4y+8z-23}{4}=\frac{0}{4}$$

$$\left(x^2-6x+(-3)^2\right)+\left(y^2-y+\left(-\tfrac{1}{2}\right)^2\right)+\left(z^2+2z+(1)^2\right)=\frac{23}{4}+9+\frac{1}{4}+1$$

$$\boxed{(x-3)^2+\left(y-\tfrac{1}{2}\right)^2+(z+1)^2=16}$$

center: $\left(3, \frac{1}{2}, -1\right)$

radius: 4

(77) $\underset{A}{(2,9,1)}, \underset{B}{(3,11,4)}, \underset{C}{(0,10,2)}, \underset{D}{(1,12,5)}$

$\vec{AB}: \langle 1, 2, 3\rangle$

$\vec{AC}: \langle -2, 1, 1\rangle$

$\vec{AD}: \langle -1, 3, 4\rangle$

$\vec{BC}: \langle -3, -1, -2\rangle$

$\vec{BD}: \langle -2, 1, 1\rangle$

$\vec{CD}: \langle 1, 2, 3\rangle$

$\vec{AB}=\vec{CD}$ and

$\vec{AC}=\vec{BD}$

so we have a parallelogram

---

7 mag, dir. $\langle -4, 6, 2\rangle$

(96) $\vec{v}=7\left(\dfrac{2\langle -2,3,1\rangle}{|2|\sqrt{(-2)^2+(3)^2+(1)^2}}\right)$

$\vec{v}=7\dfrac{\langle -2,3,1\rangle}{\sqrt{14}}$

$\vec{v}=\dfrac{\cancel{7}\sqrt{14}}{2\,\cancel{14}}\langle -2,3,1\rangle$

$$\boxed{\vec{v}=\frac{\sqrt{14}}{2}\langle -2,3,1\rangle}$$

11.3: 71, 69, 80

71 $\vec{F} = -48000\hat{j}$

$\vec{v} = \cos 10°\hat{i} + \sin 10°\hat{j}$

$$\vec{w}_1 = \frac{\vec{F}\cdot\vec{v}}{\|\vec{v}\|^2}\cdot\vec{v}$$

$$\vec{w}_1 = \frac{\langle 0, -48000\rangle\cdot\langle \cos 10°, \sin 10°\rangle}{\left(\sqrt{\cos^2 10° + \sin^2 10°}\right)^2}$$

$$\vec{w}_1 = \frac{(-48000\sin 10°)\langle \cos 10°, \sin 10°\rangle}{1^2}$$

$\|\vec{w}_1\| = 8335.1\ lb$

$\|\vec{w}_2\| = \|\vec{F} - \vec{w}_1\|$

$\approx \boxed{47270.8\ lbs}$

69 $\vec{u} = \langle 3, 1, -2\rangle$

$\vec{v} = \langle v_1, v_2, v_3\rangle$

$-\vec{v} = \langle -v_1, -v_2, -v_3\rangle$

$\vec{u}\cdot\vec{v} = 0, \quad \vec{u}\cdot -\vec{v} = 0$

$\langle 3, 1, -2\rangle\langle v_1, v_2, v_3\rangle = 0$

$3v_1 + v_2 - 2v_3 = 0$

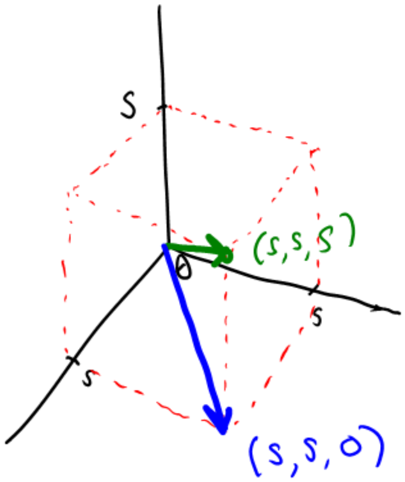
try $\vec{v} = \langle 1, -1, 1\rangle$

and $-\vec{v} = \langle -1, 1, -1\rangle$

yaay!

80

s : side



$\vec{v}_1 = \langle s,s,s \rangle,\ \|\vec{v}_1\| = \sqrt{s^2+s^2+s^2} = s\sqrt{3}$

$\vec{v}_2 = \langle s,s,0 \rangle,\ \|\vec{v}_2\| = \sqrt{s^2+s^2+0^2} = s\sqrt{2}$

$\cos\theta = \dfrac{\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{3}}$

$\cos\theta = \dfrac{\sqrt{6}}{3}$

$\boxed{\theta \approx 35.26^\circ}$

11.4: 32

32 $A = \|\vec{u} \times \vec{v}\|$, $\vec{u}$ and $\vec{v}$ are adjacent sides

$A(2,-3,1), B(6,5,-1), C(7,2,2), D(3,-6,4)$

$\vec{u} = \overrightarrow{AB} = \langle 4, 8, -2 \rangle$

$\vec{v} = \overrightarrow{AC} = \langle 5, 5, 1 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 8 & -2 \\ 5 & 5 & 1 \end{vmatrix} = (8+10)\hat{i} - (4+10)\hat{j} + (20-40)\hat{k}$$

$= 18\hat{i} - 14\hat{j} - 20\hat{k}$

$= 2(9\hat{i} - 7\hat{j} - 10\hat{k})$

$\|\vec{u} \times \vec{v}\| = |2|\sqrt{81+49+100} = \boxed{2\sqrt{230}}$

11.5: 53

53 $\frac{x-1}{-2} = y-4 = z$ and $\frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}$

$\vec{u} = \langle -2, 1, 1 \rangle$

$\vec{v} = \langle -3, 4, -1 \rangle$

$\vec{u} \times \vec{v} = -5 \langle \overset{a}{1}, \overset{b}{1}, \overset{c}{1} \rangle$

Point of intersection occurs when $t = 1$ (see below)

$\frac{x-1}{-2} \rightarrow x = 1 - 2t$ $\quad$ $\frac{x-2}{-3} \Rightarrow x = 2 - 3t$

$y - 4 \rightarrow y = 4 + t$ $\quad$ $\frac{y-1}{4} \Rightarrow y = 1 + 4t$

$z \rightarrow z = t$ $\quad$ $\frac{z-2}{-1} \Rightarrow z = 2 - t$

$1 - 2t = 2 - 3t$
$t = 1$

$4 + t = 1 + 4t$
$-3t = -3$
$t = 1$

$t = 2 - t$
$2t = 2$
$t = 1$

$x = 1 - 2(1) = -1$
$y = 4 + (1) = 5$
$z = (1)$

$(-1, 5, 1)$

$$\boxed{(x+1) + (y-5) + (z-1) = 0}$$

or $\boxed{x + y + z = 5}$

(38) $2x+3y+4z=4$

let $x=0$, $y=0$, $z=1$

$P(0,0,1)$

Let $x=0, z=0$, $y=4/3$

$Q(0,4/3,0)$

Let $y=0, z=0$

$2x=4$

$x=2$

$R(2,0,0)$

$\vec{PQ} = \langle 0, 4/3, -1\rangle$

$\vec{PR} = \langle 2, 0, -1\rangle$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4/3 & -1 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= -4/3\,\hat{i} - 2\hat{j} - \tfrac{8}{3}\hat{k}$$

$$= -\tfrac{1}{3}\langle 4, 6, 8\rangle$$

$$= -\tfrac{2}{3}\langle 2, 3, 4\rangle$$

The components of the cross product are proportional to the coefficents of the variables in the equation of the plane $\Rightarrow$ the cross product is parallel to the vector normal to the plane.