1/28/11

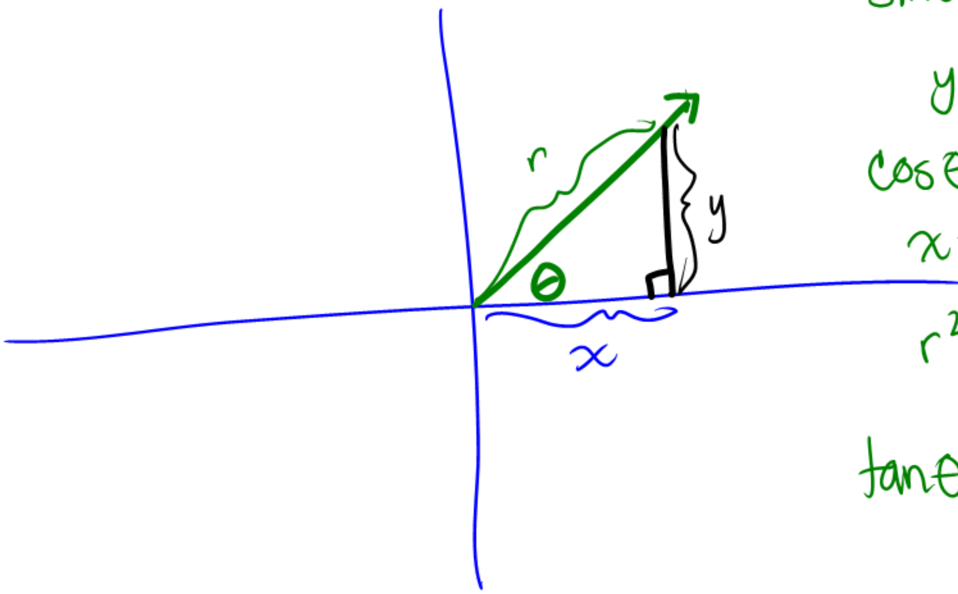
- Warmup
- Lecture 11.7

Prep for Monday

- Finish 11.1-11.7 HW and have questions ready

Wednesday

- Exam 1/ Ch. 11
- 11.1-11.7 HW is due



$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

When you are done with your homework you should be able to…

- π Use cylindrical coordinates to represent surfaces in space
- π Use spherical coordinates to represent surfaces in space

Warm-up: Convert the rectangular equation to polar form and sketch its graph by hand.

$y^2 = 9x$

$(r\sin\theta)^2 = 9(r\cos\theta)$

$r^2\sin^2\theta = 9r\cos\theta$

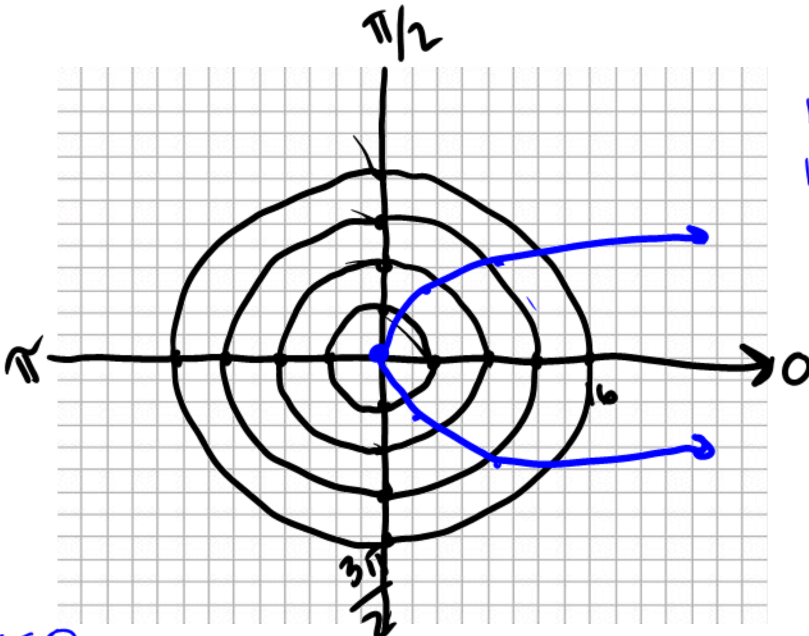
$r^2\sin^2\theta - 9r\cos\theta = 0$

$r(r\sin^2\theta - 9\cos\theta) = 0$

$r = 0$ $\quad r\sin^2\theta - 9\cos\theta = 0$

$r\sin^2\theta = 9\cos\theta$

$r = 9\cot\theta\csc\theta$



r	θ
12.7	π/4
12.7	−π/4
6	π/3
6	−π/3
31.2	π/6
31.2	−π/6

THE CYLINDRICAL COORDINATE SYSTEM

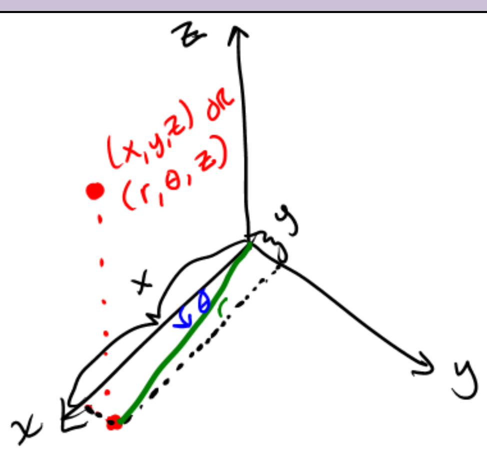
In a **cylindrical coordinate system** a point *P* in space is represented by an ordered triple (r, θ, z).

1. (r, θ) is a polar representation of the projection of *P* in the *xy*-plane.
2. z is the directed distance from (r, θ) to *P*.

Conversion Guidelines

Cylindrical to rectangular: $x = r\cos\theta,\ y = r\sin\theta,\ z = z$

Rectangular to cylindrical: $r^2 = x^2 + y^2,\ \tan\theta = \dfrac{y}{x},\ z = z$



Example 1: Convert the point $\left(-2, \frac{2\pi}{3}, 5\right)$ to rectangular coordinates.

r θ z

$x = r\cos\theta$
$x = -2\cos\frac{2\pi}{3}$
$x = -2\left(-\frac{1}{2}\right)$
$x = 1$

$y = r\sin\theta$
$y = -2\sin\frac{2\pi}{3}$
$y = -2\left(\frac{\sqrt{3}}{2}\right)$
$y = -\sqrt{3}$

$z = z$
$z = 5$

$\left(-2, \frac{2\pi}{3}, 5\right) = \boxed{\left(1, -\sqrt{3}, 5\right)}$

Example 2: Convert the point $\left(3, \sqrt{3}, -1\right)$ to cylindrical coordinates.

QI

(x, y, z)

$r^2 = x^2 + y^2$
$r^2 = (3)^2 + (\sqrt{3})^2$
$r^2 = 12$
$r = \cancel{\pm} 2\sqrt{3}$

$\tan\theta = \frac{y}{x}$
$\tan\theta = \frac{\sqrt{3}}{3}$
$\tan\theta = \frac{1/2}{\sqrt{3}/2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\theta = \frac{\pi}{6}$

$z = z$
$z = -1$

$(3, \sqrt{3}, -1) = \boxed{\left(2\sqrt{3}, \frac{\pi}{6}, -1\right)}$

Example 3: Find an equation in cylindrical coordinates for the equation $x^2 + y^2 = 8x$, given in rectangular coordinates.

$r^2 = 8(r\cos\theta)$
$r^2 - 8r\cos\theta = 0$
$r(r - 8\cos\theta) = 0$
$\cancel{r = 0}$ or $\boxed{r = 8\cos\theta}$

included in

$r^2\cos^2\theta + r^2\sin^2\theta$
$= r^2(\cos^2\theta + \sin^2\theta)$
$= r^2(1)$
$= r^2$

THE SPHERICAL COORDINATE SYSTEM

In a **spherical coordinate system**, a point *P* in space is represented by an ordered triple (ρ, θ, ϕ).

1. ρ is the distance between *P* and the origin $\rho \geq 0$.
2. θ is the same angle used in cylindrical coordinates for $r \geq 0$.
3. ϕ is the angle *between* the positive *z*-axis and the line segment $\overline{OP}$, $0 \leq \phi \leq \pi$.

Note that the first and third coordinates, ρ and ϕ, are nonnegative. ρ is the lowercase Greek letter *rho* and ϕ is the lowercase Greek letter *phi*.

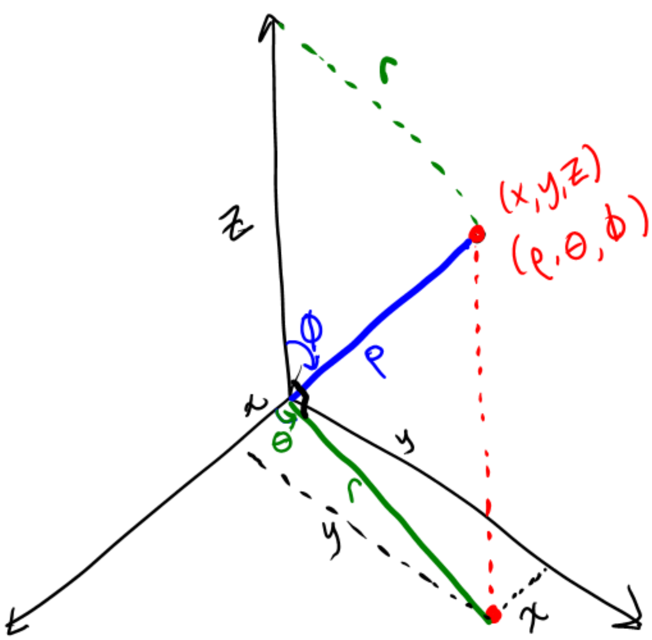
Conversion Guidelines

Spherical to rectangular: $x = \rho \sin\phi \cos\theta,\ y = \rho \sin\phi \sin\theta,\ z = \rho\cos\phi$

Rectangular to spherical: $\rho^2 = x^2 + y^2 + z^2,\ \tan\theta = \dfrac{y}{x},\ \phi = \arccos\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)$

Spherical to cylindrical $r \geq 0$: $r^2 = \rho^2 \sin^2\phi,\ \theta = \theta,\ z = \rho\cos\phi$

Cylindrical to spherical $r \geq 0$: $\rho = \sqrt{r^2 + z^2},\ \theta = \theta,\ \phi = \arccos\left(\dfrac{z}{\sqrt{r^2+z^2}}\right)$



$\sin\phi = \dfrac{r}{\rho}$, $r = \rho\sin\phi$

$r = \sqrt{x^2+y^2}$

$\dfrac{y}{r} = \sin\theta \rightarrow y = r\sin\theta$

$y = \rho\sin\phi\sin\theta$

$\dfrac{z}{\rho} = \cos\phi$

$z = \rho\cos\phi$

$\dfrac{x}{r} = \cos\theta$

$x = r\cos\theta$

$x = \rho\sin\phi\cos\theta$

Example 4: Convert the point given in cylindrical coordinates $\left(3, -\frac{\pi}{4}, 0\right)$ to spherical coordinates.

cylindrical to spherical: $\rho = \sqrt{r^2 + z^2}$

$\theta = \theta$

$\phi = \arccos\left(\frac{z}{\sqrt{r^2+z^2}}\right)$

$\rho = \sqrt{(3)^2 + 0^2}$, $\theta = -\frac{\pi}{4}$, $\phi = \arccos 0$

$\rho = 3$

$\phi = \frac{\pi}{2}$

$$\boxed{\left(3, -\frac{\pi}{4}, 0\right) = \left(3, -\frac{\pi}{4}, \frac{\pi}{2}\right)}$$

Example 5: Find an equation in spherical coordinates for the equation $x^2 + y^2 - 3z^2 = 0$, given in rectangular coordinates.

$x^2 + y^2 - 3z^2 = 0$

$(x)^2 = (\rho \sin\phi \cos\theta)^2 = \rho^2 \sin^2\phi \cos^2\theta$

$(y)^2 = (\rho \sin\phi \sin\theta)^2 = \rho^2 \sin^2\phi \sin^2\theta$

$(z)^2 = (\rho \cos\phi)^2 = \rho^2 \cos\phi$

$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta - 3\rho^2 \cos^2\phi = 0$

$\rho^2 \sin^2\phi \, (\overbrace{\cos^2\theta + \sin^2\theta}^{1}) - 3\rho^2 \cos^2\phi = 0$

$\rho^2 (\sin^2\phi - 3\cos^2\phi) = 0$

$\rho^2 = 0$ or $\sin^2\phi = 3\cos^2\phi$ $\rightarrow$ $\sqrt{\tan^2\phi} = \sqrt{3}$

$\tan\phi = \sqrt{3}$

$\rho = 0$ included in $\rightarrow$ $\boxed{\phi = \frac{\pi}{3}}$