1/26/11

- Warm-up
- Lecture 11.6

Friday

- Lecture 11.7

*review polars

Prep for Monday

Finish 11.1–11.7 HW so you know what questions to ask me

Next Wed

- Exam 1/ Ch. 11
- 11.1–11.7 HW due

When you are done with your homework you should be able to…

- π Recognize and write equations for cylindrical surfaces
- π Recognize and write equations for quadric surfaces
- π Recognize and write equations for surfaces of revolution

Warm-up: Find the volume of the region bounded by the graphs $y = 4,\ x = 4,\ x = 0,$ and $y = 0$ which has been rotated about the x-axis. Graph the resulting solid.

disk:

$V = \pi \int_0^4 (4)^2 \, dx$

$V = \pi \int_0^4 16 \, dx$

$V = 16\pi (x \big|_0^4$

$V = 16\pi (4-0)$

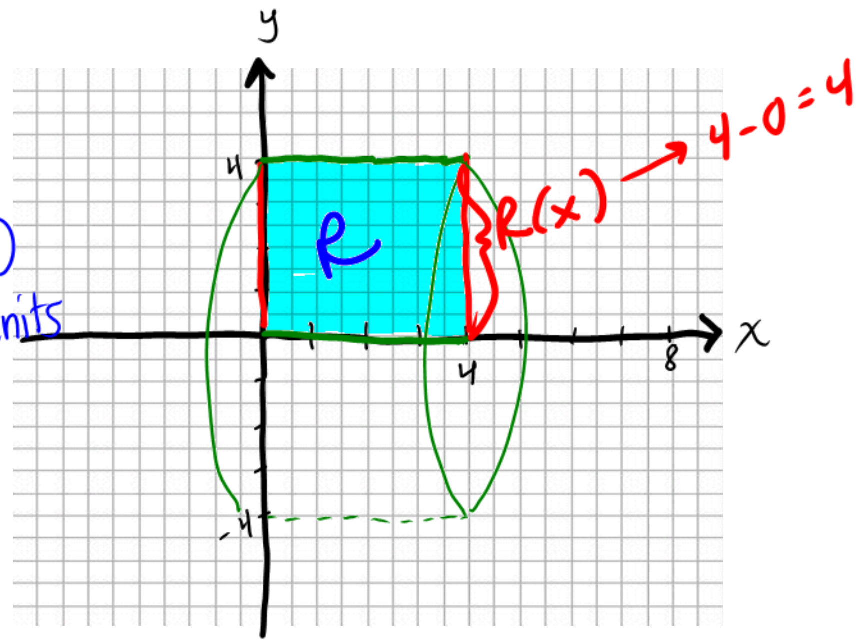
$\boxed{V = 64\pi \text{ cu.units}}$

Geom.

$V = \pi r^2 h$

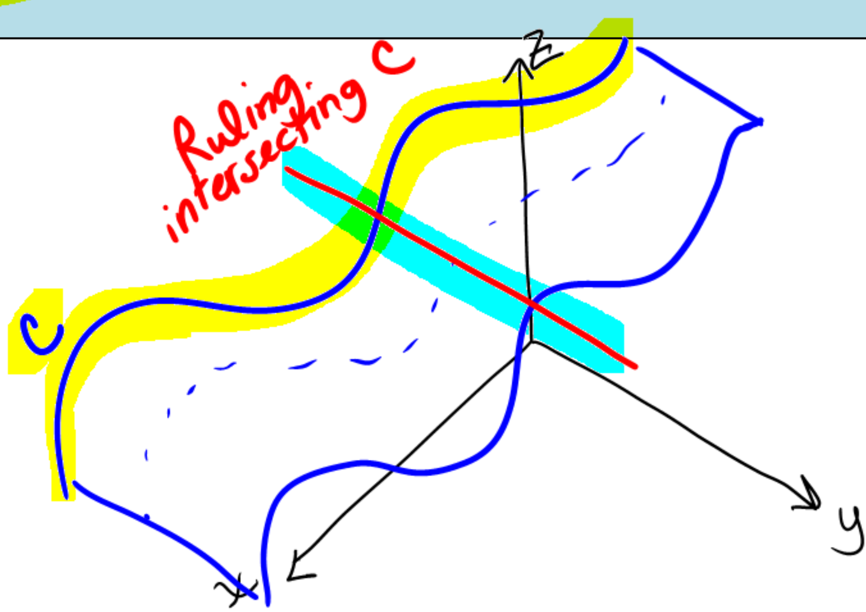
$V = \pi (4)^2 \cdot (4)$

$V = 64\pi$ cu.units



DEFINITION OF A CYLINDER

Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a **cylinder**. C is called the **generating curve (aka directrix)** of the cylinder and the parallel lines are called **rulings**.



EQUATIONS OF CYLINDERS

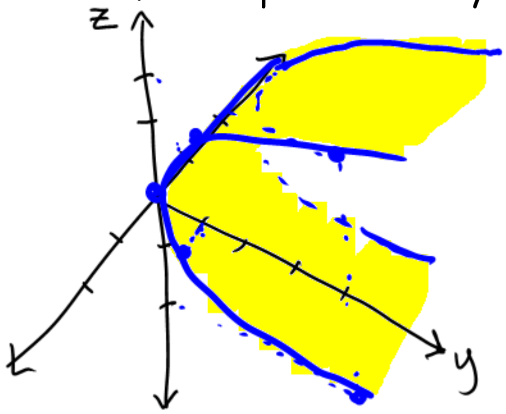
The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only the variables corresponding to the other two axes.

Example 1: Sketch the surface represented by each equation.

a) $y = z^2$

$x=0$

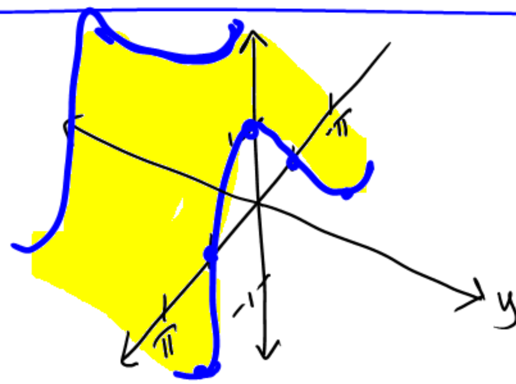
generating curve in yz-plane



b) $z = \cos x$

generating curve in xz-plane

$y=0$



QUADRIC SURFACE

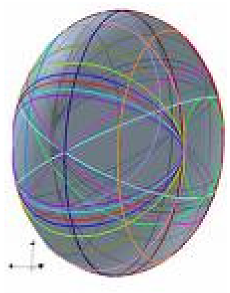
The equation of a **quadric surface** in space is a second-degree equation of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

There are six basic types of quadric surfaces:

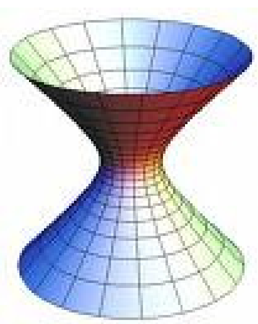
Ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, and hyperbolic paraboloid.

Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$



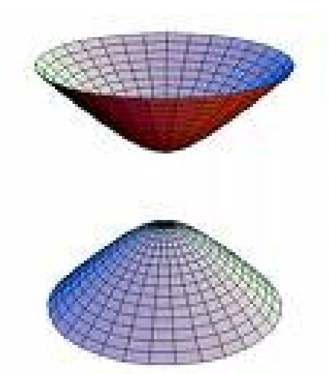
Trace	Plane
ellipse	// to xy-plane
ellipse	// to xz-plane
ellipse	// to yz-plane

Hyperboloid (1 sheet) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$



Trace	Plane
ellipse	// to xy-plane
hyperbola	// to xz-plane
hyperbola	// to yz-plane

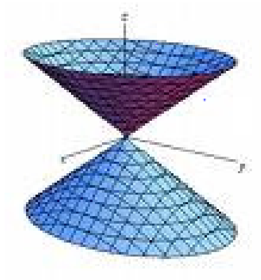
Hyperboloid (2 sheets) $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$



Trace	Plane
ellipse	// to xy-plane
hyperbola	// to xz-plane
hyperbola	// to yz-plane

Elliptic Cone

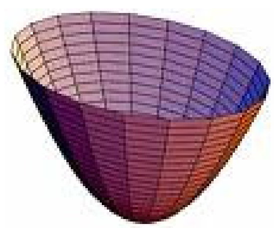
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$



Trace	Plane
ellipse	// to xy-plane
hyperbola	// to xz-plane
hyperbola	// to yz-plane

Elliptic Paraboloid

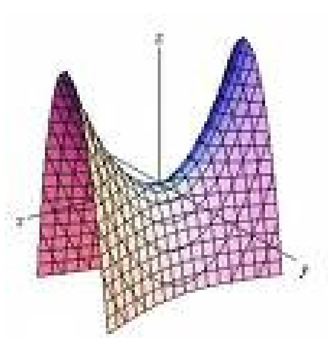
$$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$



Trace	Plane
ellipse	// to xy-plane
parabola	// to xz-plane
parabola	// to yz-plane

Hyperbolic Paraboloid

$$z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$$



Trace	Plane
hyperbola	// to xy-plane
parabola	// to xz-plane
parabola	// to yz-plane

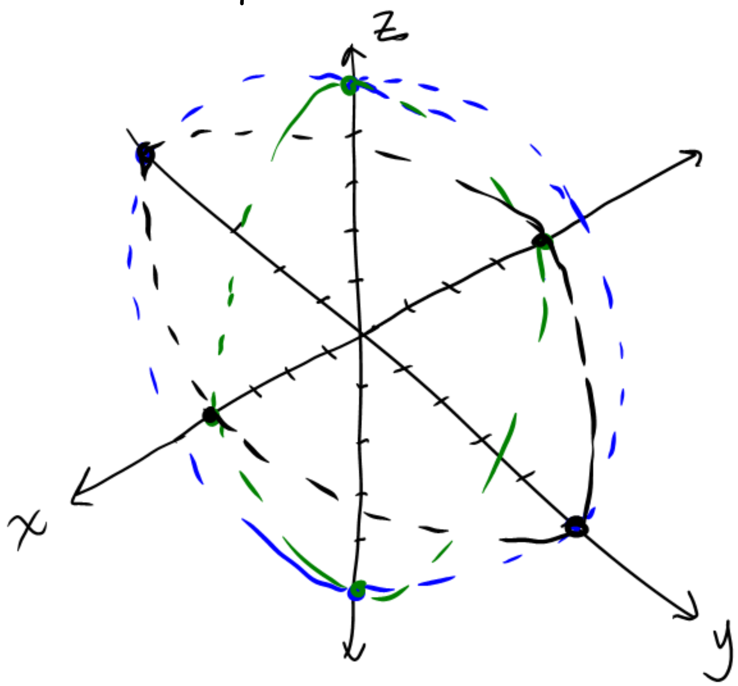
Example 2: Identify and sketch the quadric surface.

$$\frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{25}=1$$

yz-plane
$x=0$, $y^2+z^2=5^2$

xz-plane
$y=0$, $\frac{x^2}{4^2}+\frac{z^2}{5^2}=1$

xy-plane
$z=0$, $\frac{x^2}{4^2}+\frac{y^2}{5^2}=1$



ellipsoid

SURFACE OF REVOLUTION

If the graph of a radius function r is revolved about one of the coordinate axes, the equation of the resulting surface of revolution has one of the following forms:

1. Revolved about the x-axis: $y^2+z^2=\left[r(x)\right]^2$
2. Revolved about the y-axis: $x^2+z^2=\left[r(y)\right]^2$
3. Revolved about the z-axis: $x^2+y^2=\left[r(z)\right]^2$

Example 3: Find an equation for the surface of revolution generated by revolving the curve $z=3y$ in the yz-plane about the y-axis.

Need a function of y since we're revolving about the y-axis.
$z=3y \rightarrow r(y)=3y$

$x^2+z^2=(3y)^2$
$x^2+z^2=9y^2$