1/24/11

- Warm-up (11.5 worksheet)
- Lecture 11.5

Wednesday

11.6

Friday

11.7

Next Monday

Review Ch. 11

When you are done with your homework you should be able to…

- π Write a set of parametric equations for a line in space
- π Write a linear equation to represent a plane in space
- π Sketch the plane given by a linear equation
- π Find the distance between points, planes, and lines in space

Warm-up: Graph the following parametric curve, indicating the orientation.

$$x-3=\cos^2\theta, \text{ and } y=\sin^2\theta,\ 0\le\theta<2\pi$$

range: $[0,1]$

Hint: $\sin^2\theta+\cos^2\theta=1$

$x=\cos^2\theta+3 \rightarrow$ domain: $[3,4]$

$y+(x-3)=1$

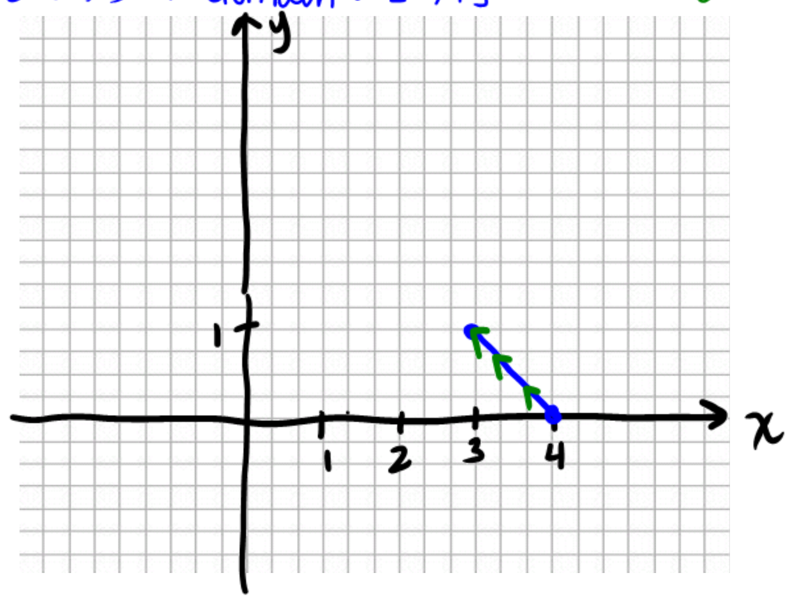
$y=-x+4,$

$D: [3,4]$

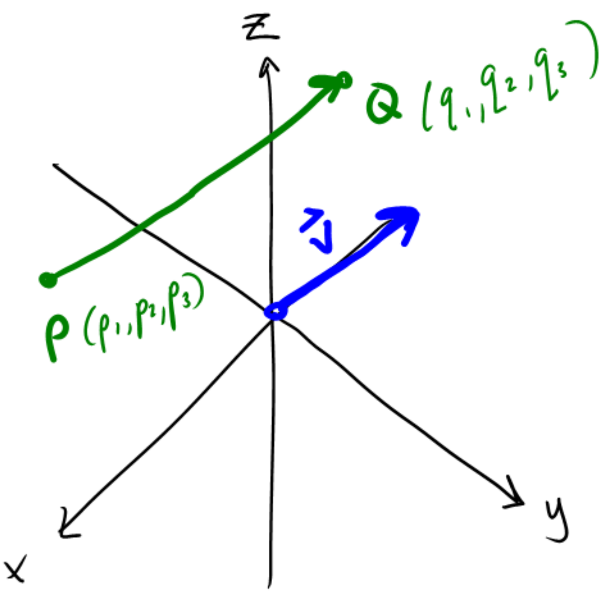
$R: [0,1]$

Let $\theta=0 \rightarrow x=1+3=4$

Let $\theta=\frac{\pi}{2} \rightarrow x=3$



In the plane **Slope** is used to determine an equation of a line. In space, it is convenient to use **vectors** to determine the equation of a line.



$\overrightarrow{PQ}=t\vec{v}$

$v=\langle a,b,c\rangle$

THEOREM: PARAMETRIC EQUATIONS OF A LINE IN SPACE

A line L parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P = (x_1, y_1, z_1)$ is represented by the **parametric equations**

$$x = x_1 + at,\ y = y_1 + bt,\ \text{and } z = z_1 + ct$$

If the direction numbers a, b, and c are all nonzero, you can eliminate the parameter t to obtain **symmetric equations** of the line.

$t = \frac{x - x_1}{a}$, $t = \frac{y - y_1}{b}$, $t = \frac{z - z_1}{c}$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Example 1: Find equations of the line which passes through the point $(-3, 0, 2)$ and is parallel to the vector $\mathbf{v} = 6\mathbf{j} + 3\mathbf{k}$ in

(x_1, y_1, z_1)

a) Parametric form

$\vec{v} = \langle 0, 6, 3 \rangle \rightarrow a = 0,\ b = 6,\ c = 3$

$x = x_1 + at$
$x = -3 + 0 \cdot t$
$x = -3$

$y = y_1 + bt$
$y = 0 + 6t$
$y = 6t$

$z = z_1 + ct$
$z = 2 + 3t$

$$\boxed{\begin{aligned} x &= -3 \\ y &= 6t \\ z &= 2 + 3t \end{aligned}}$$

b) Symmetric form

Since $a = 0$, we can't do the symmetric equations

Let's try a different problem:

parallel to $\vec{v} = -2\hat{i} + 8\hat{j} - 3\hat{k}$, passing through $(-3, 0, 2)$

parametric:

$x = -3 - 2t$
$y = 8t$
$z = 2 - 3t$

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

$$\boxed{-\frac{x + 3}{2} = \frac{y}{8} = -\frac{z - 2}{3}}$$

THEOREM: STANDARD EQUATION OF A PLANE IN SPACE

The plane containing the point (x_1, y_1, z_1) and having normal vector $\mathbf{n} = \langle a, b, c \rangle$ can be represented, in **standard form**, by the equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

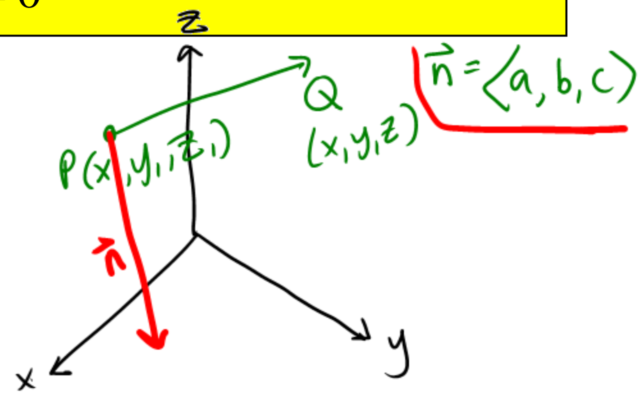
The **general form** is given by the equation

$$ax + by + cz + d = 0$$

$0 = \vec{n} \cdot \overrightarrow{PQ}$

$0 = \langle a, b, c \rangle \langle x - x_1, y - y_1, z - z_1 \rangle$

$0 = a(x - x_1) + b(y - y_1) + c(z - z_1)$



THEOREM: DISTANCE BETWEEN A POINT AND A PLANE

The distance between a plane and a point Q (not in the plane) is

$$D = \left\| \text{proj}_{\mathbf{n}} \overrightarrow{PQ} \right\| = \frac{\left| \overrightarrow{PQ} \cdot \mathbf{n} \right|}{\|\mathbf{n}\|}$$

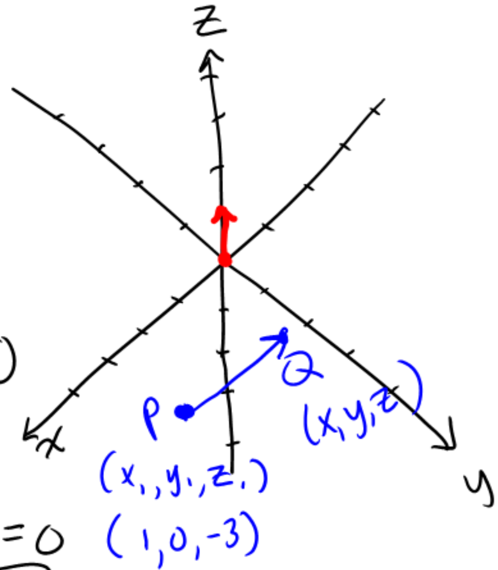
where P is a point in the plane and $\mathbf{n}$ is normal to the plane. Other forms of this distance from a point $Q(x_0, y_0, z_0)$ and the plane given by $ax + by + cz + d = 0$ are as follows:

$$D = \frac{\left| a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1) \right|}{\sqrt{a^2 + b^2 + c^2}} \quad \text{or} \quad D = \frac{\left| ax_0 + by_0 + cz_0 \right|}{\sqrt{a^2 + b^2 + c^2}}$$

Example 2: Find an equation of the plane passing through the point $(1,0,-3)$ perpendicular to the vector $\mathbf{n}=\mathbf{k}$.

$\hat{k} = \langle 0, 0, 1 \rangle$

a b c



$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$0(x-1)+0(y-0)+1(z-(-3))=0$

$\boxed{z+3=0}$

THEOREM: DISTANCE BETWEEN A POINT AND A LINE IN SPACE

The distance between a point Q and a line in space is given by

$$D=\frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\left\|\mathbf{u}\right\|}$$

where $\mathbf{u}$ is a direction vector for the line and P is a point on the line.

Example 3: Find the distance between the point $\overset{Q}{(3,2,1)}$ and the plane $x-y+2z=4$.

$x-y+2z-4=0$

$a=1$
$b=-1$
$c=2$

$\vec{n}=\langle 1,-1,2\rangle$ is normal to the given plane

need another point on the plane, so let $x=0$, $y=0 \rightarrow 2z=4 \rightarrow z=2$, $P(0,0,2)$

$\overrightarrow{PQ}=\langle 3-0, 2-0, 1-2\rangle$

$\overrightarrow{PQ}=\langle 3,2,-1\rangle$

$D=\dfrac{|\overrightarrow{PQ}\cdot\vec{n}|}{\|\vec{n}\|}$

$D=\dfrac{|3(1)+(2)(-1)+(-1)(2)|}{\sqrt{1^2+(-1)^2+2^2}}$

$\boxed{D=\dfrac{1}{\sqrt{6}}\text{ units}}$

(105) Find the distance between the point and the line given by the set of parametric equations.

Q

$(1,-2,4)$; $x=2t$, $y=t-3$, $z=2t+2$

Step 1: Find direction vector for the line

$a=2$, $b=1$, $c=2$ So $\vec{u}=\langle 2,1,2\rangle$

Step 2: Find a point on the line, and $\overrightarrow{PQ}$

let $t=0$, $P(0,-3,2)$

$\overrightarrow{PQ}=\langle 1-0, -2-(-3), 4-2\rangle$

$\overrightarrow{PQ}=\langle 1,1,2\rangle$

Step 3: Find distance

$$D=\frac{\|\overrightarrow{PQ}\times\vec{u}\|}{\|\vec{u}\|}$$

$$\overrightarrow{PQ}\times\vec{u}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 2\\ 2 & 1 & 2\end{vmatrix}$$

$$=(2-2)\hat{i}-(2-4)\hat{j}+(1-2)\hat{k}$$

$$=2\hat{j}-\hat{k}$$

$$=\langle 0,2,-1\rangle$$

$\|\vec{u}\|=\sqrt{(2)^2+(1)^2+(2)^2}$

$\|\vec{u}\|=3$

$\|\overrightarrow{PQ}\times\vec{u}\|=\sqrt{(0)^2+(2)^2+(-1)^2}=\sqrt{5}$

$$D=\frac{\|\overrightarrow{PQ}\times\vec{u}\|}{\|\vec{u}\|}=\boxed{\frac{\sqrt{5}}{3}\text{ units}}$$