1/14/11

- Warm-up (11.2 worksheet)
- Lecture 11.2

Monday

HOLIDAY

Wednesday

11.3

When you are done with your homework you should be able to…

- π Understand the three-dimensional rectangular coordinate system
- π Analyze vectors in space
- π Use three-dimensional vectors to solve real-life problems

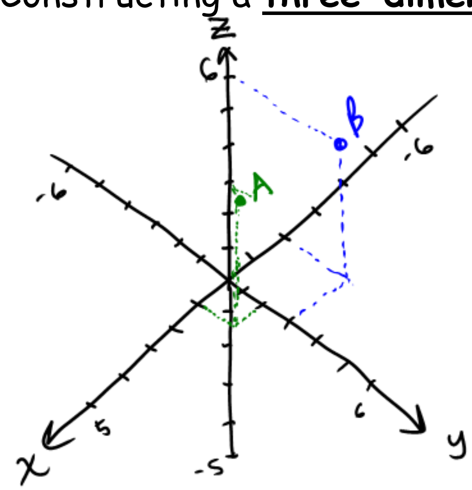
Warm-up: Find the vector $\mathbf{v}$ with magnitude 4 and the same direction as $\mathbf{u} = \langle -1, 1 \rangle$.

① need to make $\vec{u}$ unit vector

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle -1, 1 \rangle}{\sqrt{(-1)^2 + (1)^2}} = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$$

② Find $\vec{v}$: $\vec{v} = 4\left(\frac{1}{\sqrt{2}} \langle -1, 1 \rangle\right) = \boxed{2\sqrt{2} \langle -1, 1 \rangle}$

Constructing a **three-dimensional coordinate system**:



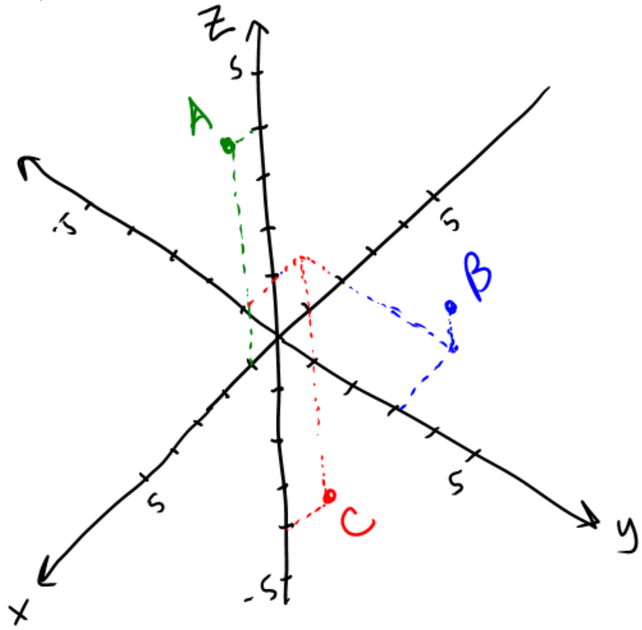
A (1, 2, 3)

B (-2, 3, 6)

- Taken as pairs, the axes determine three **coordinate planes**: the *xy*-plane, the *xz*-plane, and the *yz*-plane
  - These planes separate the three-space into ___8___ octants
- In this three-dimensional system, a point *P* in space is determined by an ordered ___triple___, denoted ___(x, y, z)___
  - x = directed distance from *yz*-plane to *P*
  - y = directed distance from *xz*-plane to *P*
  - z = directed distance from *xy*-plane to *P*

- A three-dimensional coordinate system can either have a **left-handed** or **right-handed** orientation
    - The right-handed system has the right hand pointing along the *x*-axis
        - Our text uses the right-handed system

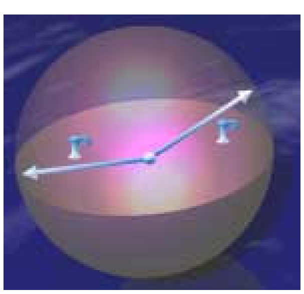
Example 1: Draw a three-dimensional coordinate system and plot the following points: A(1, 0, 4), B(-2, 3, 1) and C(-2, -1, -4)



THE DISTANCE BETWEEN TWO POINTS IN SPACE

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example 2: Find the standard equation of the sphere that has the points (0, 1, 3) and (-2, 4, 2) as endpoints of a diameter.



(x_0, y_0, z_0) is the center of the sphere

Standard equation of a sphere

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

$$d = \sqrt{(-2-0)^2 + (4-1)^2 + (2-3)^2}$$

$$d = \sqrt{14} \rightarrow r = \frac{\sqrt{14}}{2}$$

midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

center of sphere: $\left(\frac{0+(-2)}{2}, \frac{1+4}{2}, \frac{3+2}{2}\right) = \left(-1, \frac{5}{2}, \frac{5}{2}\right)$

$$(x-(-1))^2 + \left(y-\frac{5}{2}\right)^2 + \left(z-\frac{5}{2}\right)^2 = \left(\frac{\sqrt{14}}{2}\right)^2$$

$$\boxed{(x+1)^2 + \left(y-\frac{5}{2}\right)^2 + \left(z-\frac{5}{2}\right)^2 = \frac{7}{2}}$$

DEFINITIONS OF VECTOR ADDITION AND SCALAR MULTIPLICATION

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors in space and let c be a scalar.

1. *Equality of Vectors.* $\mathbf{u} = \mathbf{v}$ if and only if $u_1 = v_1,\ u_2 = v_2,$ and $u_3 = v_3$.
2. *Component Form.* If $\mathbf{v}$ is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, then $\mathbf{v} = \langle v_1, v_2, v_3 \rangle = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$
3. *Length.* $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ ~~is the vector~~ $\cancel{-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle}$.
4. *Unit Vector in the Direction of* $\mathbf{v}$. $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|}\langle v_1, v_2, v_3 \rangle,\ \mathbf{v} \neq \mathbf{0}$
5. *Vector Addition.* $\mathbf{v} + \mathbf{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$
6. *Scalar Multiplication.* $c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$

Example 3: Find the component form of the vector $\mathbf{v}$ that has initial point (-1, 6, 4) and terminal point (0, -5, 3). Find a unit vector in the direction of $\mathbf{v}$.

$\vec{v} = \langle 0-(-1), -5-6, 3-4 \rangle$

$\vec{v} = \boxed{\langle 1, -11, -1 \rangle}$

$\frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 1, -11, -1 \rangle}{\sqrt{(1)^2 + (-11)^2 + (-1)^2}} = \boxed{\frac{1}{\sqrt{123}}\langle 1, -11, -1 \rangle}$

DEFINITION: PARALLEL VECTORS

Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are **<u>parallel</u>** if there is some scalar c such that

$$\mathbf{u} = c\mathbf{v}.$$

Example 4: Vector $\mathbf{z}$ has initial point (5, 4, 1) and terminal point (-2, -4, 4). Determine which of the vectors is parallel to $\mathbf{z}$.

$\vec{z} = \langle -2-5, -4-4, 4-1 \rangle$

$\vec{z} = \langle -7, -8, 3 \rangle$

a) $\langle 7, 6, 2 \rangle$

not parallel

b) $\langle 14, 16, -6 \rangle = -2\langle -7, -8, 3 \rangle$

parallel

Example 5: Find the component form of the unit vector $\mathbf{v}$ in the direction of the diagonal of the cube shown in the figure.