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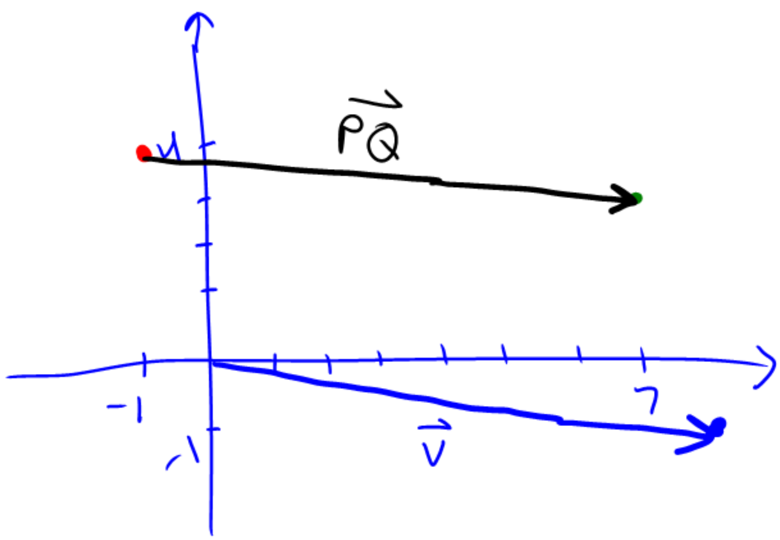
class website

P (-1,4) , Q (7,3)



When you are done with your homework you should be able to…

- π Write the component form of a vector
- π Perform vector operations and interpret the results geometrically
- π Write a vector as a linear combination of standard unit vectors
- π Use vectors to solve problems involving force or velocity

Warm-up: Find the distance between the points (2, 1) and (4, 7).

$d = \sqrt{(4-2)^2 + (7-1)^2} \rightarrow d = \sqrt{36+4} =$

$d = \sqrt{40}$

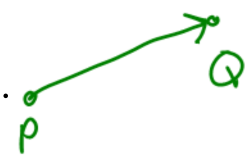
$d = \sqrt{10}\sqrt{4}$

$d = \boxed{2\sqrt{10}}$

What is a scalar quantity?

constant → no direction

Give examples of quantities which can be characterized by a scalar.



potential, distance, mass, volume

What is a vector?

A directed line segment

$\overrightarrow{PQ}$ → P is the initial point
Q is the terminal point

Give examples of quantities which are represented by vectors.

momentum, force, work

How do you find the length, aka magnitude, aka norm, of a vector?

Use the distance formula

What makes two vectors equivalent?

Same direction and magnitude

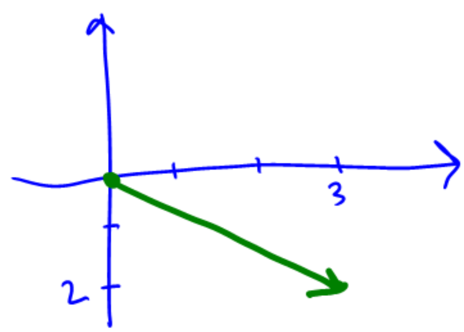
DEFINITION OF COMPONENT FORM OF A VECTOR IN THE PLANE

If $\mathbf{v}$ is a vector in the plane whose initial point is the origin and whose terminal point is (v_1, v_2), then the **component form** $\mathbf{v}$ is given by

$$\mathbf{v} = \langle v_1, v_2 \rangle.$$

The coordinates v_1 and v_2 are called the **components** of $\mathbf{v}$. If both the initial point and the terminal point lie at the origin, then $\mathbf{v}$ is called the **zero vector** and is denoted by $\mathbf{0} = \langle 0, 0 \rangle$.

Example 1: Sketch the vector whose initial point is the origin and whose terminal point is (3, -2).



DEFINITIONS OF VECTOR ADDITION AND SCALAR MULTIPLICATION

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let c be a scalar.

1. The **vector sum** of $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$.
2. The **scalar multiple** of c and $\mathbf{u}$ is the vector $c\mathbf{u} = \langle cu_1, cu_2 \rangle$.
3. The **negative** of $\mathbf{v}$ is the vector $-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle$.
4. The **difference** of $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$.

Example 2: Find the component form and length of the vector $\mathbf{v}$ that has initial point (-1, 4) and terminal point (7, 3). Find the norm of $\mathbf{v}$.

$P \rightarrow (-1,4)$, $Q \rightarrow (7,3)$

$\vec{v} = \langle 7-(-1), 3-4 \rangle$

$\boxed{\vec{v} = \langle 8,-1 \rangle}$

$\|\vec{v}\| = \sqrt{8^2 + (-1)^2}$

$\boxed{\|\vec{v}\| = \sqrt{65}}$

Example 3: Let $\mathbf{u} = \langle -1,-3 \rangle$ and $\mathbf{v} = \langle 2,-8 \rangle$ find the following vectors. Illustrate the vector operations geometrically.

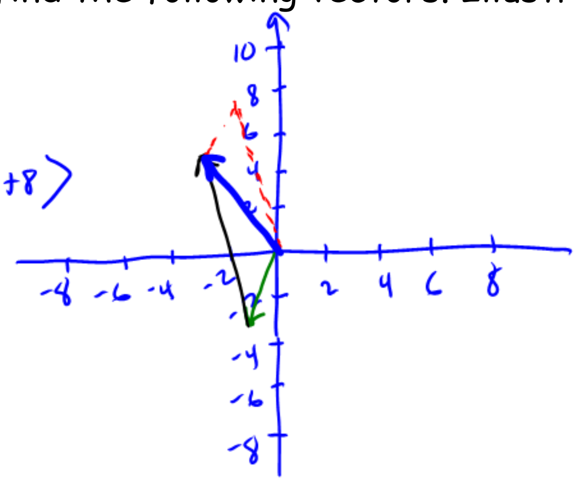
a) $\mathbf{u} - \mathbf{v}$

$\vec{u} + -\vec{v}$

$-\vec{v} = \langle -2, 8 \rangle$

$\vec{u} + -\vec{v} = \langle -1+-2, -3+8 \rangle$

$= \boxed{\langle -3,5 \rangle}$



b) $-2\mathbf{v}$

THEOREM: PROPERTIES OF VECTOR OPERATIONS

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in the plane, and let c and d be scalars.

1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ commutative
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$ associative
3. $\mathbf{u}+\mathbf{0}=\mathbf{u}$ add. identity
4. $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$ add. inverse
5. $c(d\mathbf{u})=(cd)\mathbf{u}$
6. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$
7. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$
8. $1(\mathbf{u})=\mathbf{u}$, and $0(\mathbf{u})=\mathbf{0}$

THEOREM: LENGTH OF A SCALAR MULTIPLE

Let $\mathbf{v}$ be a vector, and let c be a scalar. Then

$$\|c\mathbf{v}\| = |c|\|\mathbf{v}\|.$$

THEOREM: UNIT VECTOR IN THE DIRECTION OF $\mathbf{v}$

If $\mathbf{v}$ is a nonzero vector in the plane, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Has length 1 and the same direction as **v**.

Example 4: Find a unit vector in the direction of $\mathbf{v} = \langle 7, -5 \rangle$. Verify that it has length 1.

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\vec{u} = \frac{\langle 7, -5 \rangle}{\sqrt{(7)^2 + (-5)^2}}$$

$$\vec{u} = \frac{\langle 7, -5 \rangle}{\sqrt{74}}$$

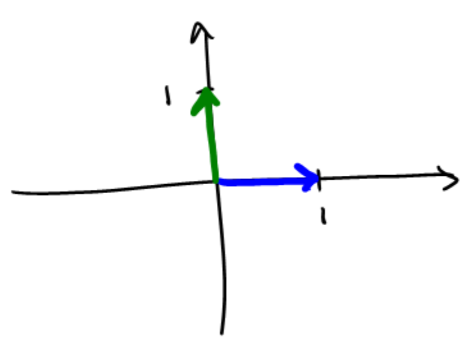
$$\vec{u} = \frac{1}{\sqrt{74}} \langle 7, -5 \rangle$$

OR

$$\vec{u} = \left\langle \frac{7}{\sqrt{74}}, -\frac{5}{\sqrt{74}} \right\rangle$$

Standard Unit Vectors

$\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$



Example 5: Let $\mathbf{u}$ be the vector with initial point (-4, 1) and terminal point (3, -1) and let $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$. Write each vector as a linear combination of **i** and **j**.

a) $\mathbf{u} = \langle 3-(-4), -1-1 \rangle$

$\vec{u} = \langle 7, -2 \rangle$

$\vec{u} = 7\langle 1, 0 \rangle - 2\langle 0, 1 \rangle$

$\boxed{\vec{u} = 7\hat{i} - 2\hat{j}}$

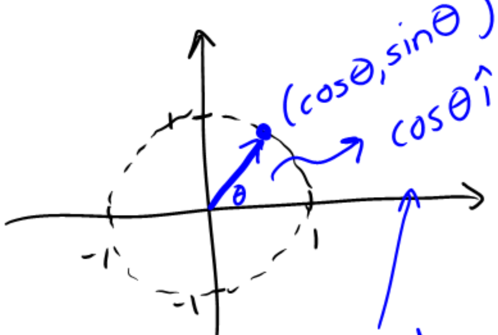
b) $\mathbf{w} = 4\mathbf{u} - 2\mathbf{v}$

$\vec{w} = 4(7\hat{i} - 2\hat{j}) - 2(5\hat{i} + 2\hat{j})$

$\vec{w} = 28\hat{i} - 8\hat{j} - 10\hat{i} - 4\hat{j}$

$\boxed{\vec{w} = 18\hat{i} - 12\hat{j}}$

Example 6: The vector **v** has a magnitude of 2 and makes an angle of $\frac{\pi}{3}$ with the positive *x*-axis. Write **v** as a linear combination of the unit vectors **i** and **j**.



$(\cos\theta, \sin\theta)$

$\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

magnitude is $\sqrt{\cos^2\theta + \sin^2\theta}$

$= \sqrt{1}$

$= 1$

$\frac{\vec{v}}{\|\vec{v}\|}$ is a unit vector

$\frac{v}{\|v\|} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\vec{v} = \|\vec{v}\|\cos\theta\,\hat{i} + \|v\|\sin\theta\,\hat{j}$

$\vec{v} = 2\cos\frac{\pi}{3}\,\hat{i} + 2\sin\frac{\pi}{3}\,\hat{j}$

$\vec{v} = 2\left(\frac{1}{2}\right)\hat{i} + 2\left(\frac{\sqrt{3}}{2}\right)\hat{j}$

$\boxed{\vec{v} = \hat{i} + \sqrt{3}\,\hat{j}}$