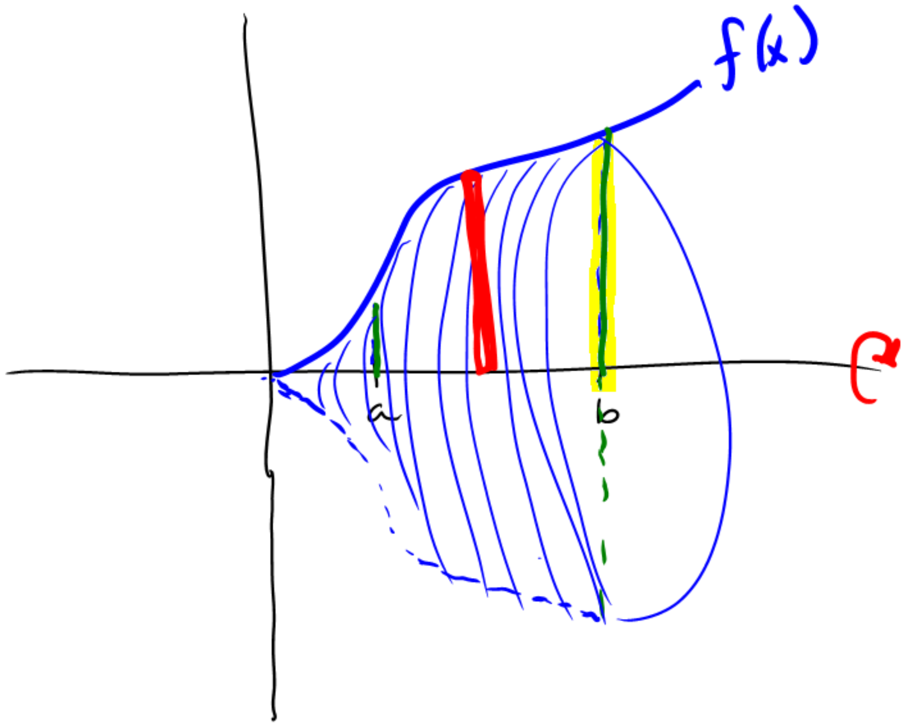
Review of Volumes of Solids of Revolution

Disk & washer method uses right circular cylinders

Disk: no hole



$R(x) = f(x) - 0$

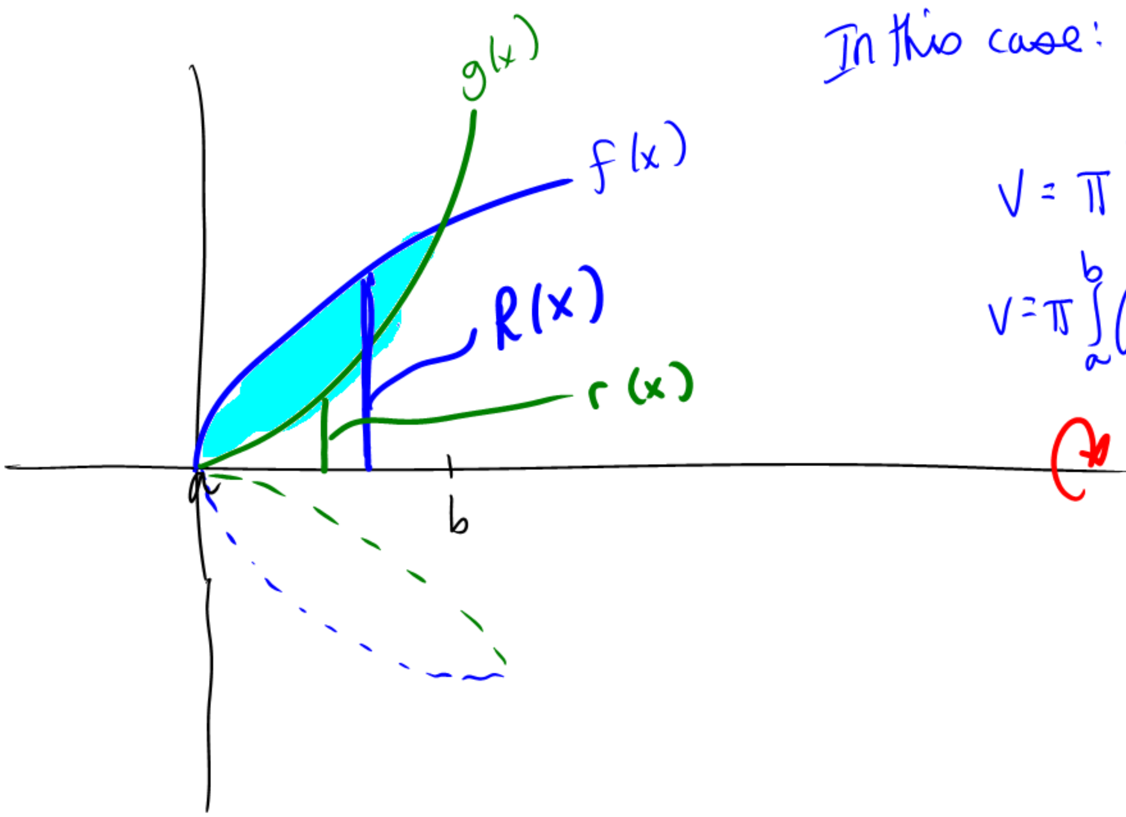
$R(x) = f(x)$ in this case

Each volume is $\pi r^2 h$

$\pi (R(x))^2 dx$

$V = \pi \int_a^b [R(x)]^2 dx$

Washer: there's a hole or hollow



In this case: $R(x) = f(x) - 0 = f(x)$

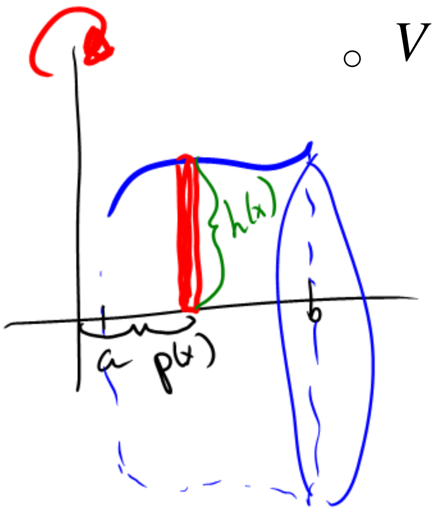
$r(x) = g(x) - 0 = g(x)$

$V = \pi \int_a^b [R(x)]^2 dx - \pi \int_a^b [r(x)]^2 dx$

$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$

MATH 251/GRACEY NAME________________________________
WS/7.3

Volume using the Shell Method



- The shell method uses cylindrical shells.
  - $V = lwh$
    - The length is the circumference of the cylinder, or $2\pi r$
      - $r = p(x)$ or $r = p(y)$
      - The radius is the distance between any given rectangle you draw and the axis of revolution
    - The width is the change in x or the change in y
    - The height is the height of any rectangle you draw
      - $h(x)$ or $h(y)$

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1. (15 POINTS) Find the volume of the solid bounded by the graph of $y = \cos x$, $x = 0$, $y = 0$, and $x = \frac{\pi}{2}$, which is then rotated about the line $y = 2$.

Washer method

$y = 2 \rightarrow R(x) = 2 - 0$
$y = 0 \quad R(x) = 2$

$V = \pi \int_0^{\pi/2} \left[(2)^2 - (2 - \cos x)^2\right] dx$

$V = \pi \int_0^{\pi/2} (4 - 4 + 4\cos x - \cos^2 x) dx$

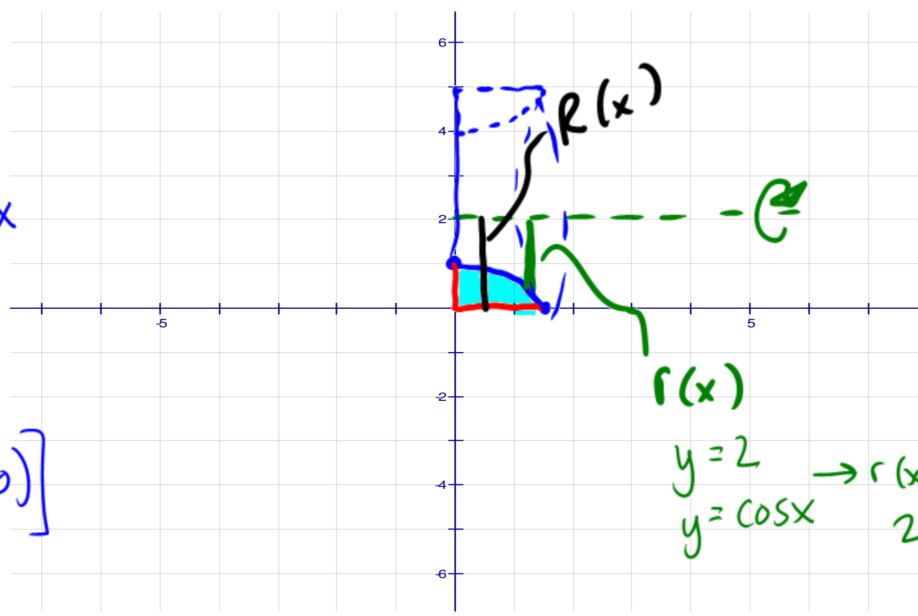
$V = \pi \int_0^{\pi/2} \left(4\cos x - \frac{1}{2}(1 + \cos 2x)\right) dx$

$V = \pi \left(4\sin x - \frac{x}{2} - \frac{\sin 2x}{4}\right)\Big|_0^{\pi/2}$

$V = \pi\left[\left(4(1) - \frac{\pi/2}{2} - 0\right) - (4(0) - 0 - 0)\right]$

$V = \pi\left(4 - \frac{\pi}{4}\right)$

$\boxed{V = \frac{\pi}{4}(16 - \pi) \text{ units cubed}}$



$y = 2$
$y = \cos x \rightarrow r(x) = 2 - \cos x$

2. (15 POINTS) Find the volume of the solid bounded by the graph of $y=\sqrt{x}$, $y=0$ ~~and~~ , $x=0$ $x=4$ which is then rotated about the line $x=7$.

Shell

distance between rectangle and axis of rev. at any given time

$p(x) = 7 - x$

$h(x) = \sqrt{x} - 0 = \sqrt{x}$

$$V = 2\pi \int_0^4 (7-x)x^{1/2}\,dx$$

$$V = 2\pi \int_0^4 \left(7x^{1/2} - x^{3/2}\right)dx$$

$$V = 2\pi \left(\frac{14}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right)\Big|_0^4$$

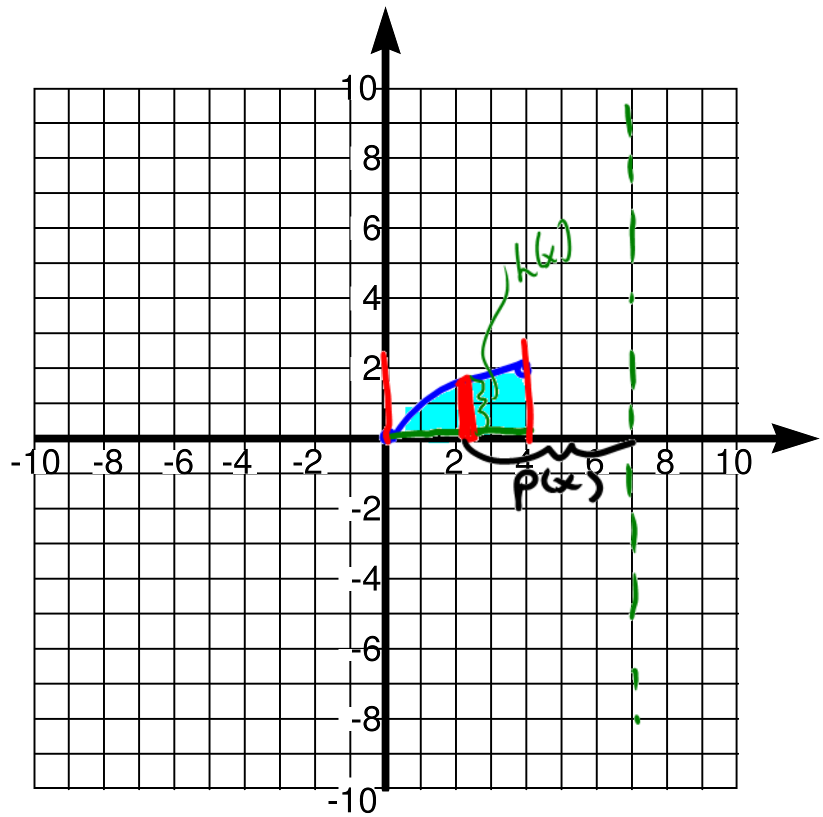
$$V = 2\pi\left[\left(\frac{14}{3}\cdot 8 - \frac{2}{5}\cdot 32\right) - (0-0)\right]$$

$$V = 2\pi\left(\frac{112}{3} - \frac{64}{5}\right)$$

$$V = 2\pi\left(\frac{560-192}{15}\right)$$

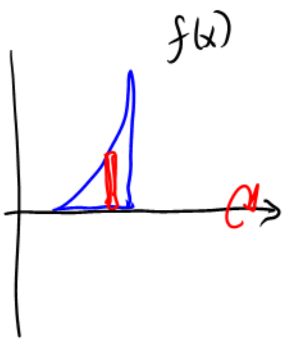
$$V = 2\pi\left(\frac{368}{15}\right)$$

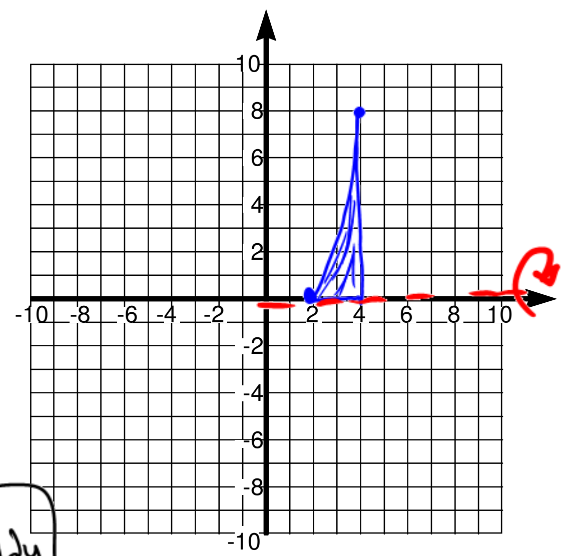
$$\boxed{V = \frac{736\pi}{15} \text{ cubic units}}$$



3. Sketch the region bounded by the graphs of $y=(x-2)^3$, $y=0$ and $x=4$, then **SET UP** the integrals which will find the volume of the solid created by rotated the region about

$R(x) = (x-2)^3 - 0$



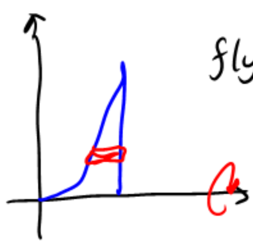


a. the line $y=0$

i. using the disc/washer method

$$\boxed{V = \pi \int_2^4 \left[(x-2)^3\right]^2 dx}$$

ii. using the shell method.



$f(y) \nearrow \sqrt[3]{y} = \sqrt[3]{(x-2)^3}$

$\sqrt[3]{y} + 2 = x$

$f(y) = y^{1/3} + 2$

$p(y) = y$

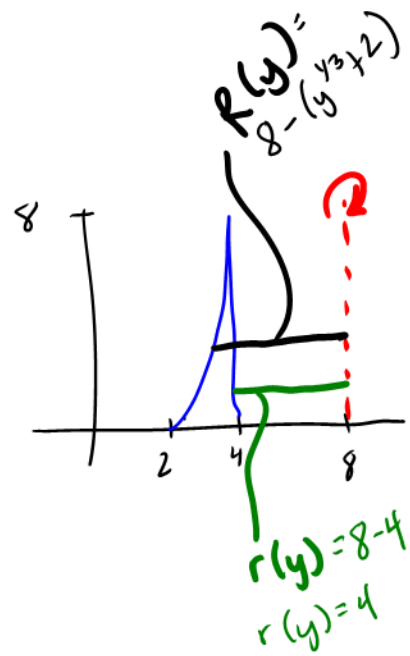
$h(y) = 4 - (y^{1/3}+2)$

$$\boxed{V = 2\pi \int_0^8 y\left[4 - (y^{1/3}+2)\right] dy}$$

b. the line $x=0$

i. using the disc/washer method

ii. using the shell method.



c. the line $x=8$

i. using the disc/washer method

$$\boxed{V = \pi \int_0^8 \left[(8-(y^{1/2}+2))^2 - (4)^2\right] dy}$$

ii. using the shell method.

$$\boxed{V = 2\pi \int_2^4 (8-x)(x-2)^3 dx}$$