Integral Review

- Integration by parts
- trig substitution
- partial fraction decomp
- Integrals which yield logarithmic results
- $\int a^u du$

Shannon Gracey
www.swccd.edu/~sgracey
these notes will be posted on my calculus 3 page

Integration by parts

$$\int u\,dv = u\cdot v - \int v\,du$$

Example 1: Integrate

a) $\int \ln x\, dx$

$\frac{du}{dx} = \frac{d}{dx}\ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$\int dv = \int dx$

$v = x$

$$\int \ln x\,dx = x\ln x - \int \cancel{x}^{\,1}\,\frac{dx}{\cancel{x}_{\,1}}$$

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$

b) $\int e^{-x}\cos 2x\,dx$

$u_1 = \cos 2x$

$du_1 = -2\sin 2x\,dx$

$\int dv_1 = \int e^{-x}\,dx$

$v_1 = \frac{e^{-x}}{-1}$

$v_1 = -e^{-x}$

$u_2 = \sin 2x$

$du_2 = 2\cos 2x\,dx$

$\int dv_2 = \int e^{-x}\,dx$

$v_2 = -e^{-x}$

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x - \int(-e^{-x})(-2\sin 2x\,dx)$$

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,dx$$

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x - 2\left[-e^{-x}\sin 2x - \int(-e^{-x}\cdot 2\cos 2x\,dx)\right]$$

$$1\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx$$

$$+4\int e^{-x}\cos 2x\,dx \qquad\qquad +4\int e^{-x}\cos 2x\,dx$$

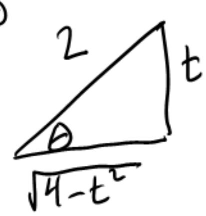
$$\frac{5\int e^{-x}\cos 2x\,dx}{5} = \frac{e^{-x}}{5}(2\sin 2x - \cos 2x) + C$$

$$\boxed{\int e^{-x}\cos 2x\,dx = \frac{e^{-x}}{5}(2\sin 2x - \cos 2x) + C}$$

Trig Substitution

$t = 2\sin\theta$

$\sin\theta = \frac{t}{2}$



Example 2: Integrate

a) $$\int \frac{t}{(4-t^2)^{3/2}}\,dt = \int \frac{t\,dt}{\left(\sqrt{(2)^2 - t^2}\right)^3} = \int \frac{(\cancel{2}\sin\theta)(\cancel{2}\cos\theta\,d\theta)}{\underset{2}{\cancel{8}}\cos^{\cancel{3}\,2}\theta}$$

Let $\frac{d}{d\theta}t = \frac{d}{d\theta}2\sin\theta$, $\rightarrow dt = 2\cos\theta\,d\theta$

$$\left(\sqrt{4-t^2}\right)^3 = \left(\sqrt{4-(2\sin\theta)^2}\right)^3$$
$$= \left(\sqrt{4-4\sin^2\theta}\right)^3$$
$$= \left(\sqrt{4(1-\sin^2\theta)}\right)^3$$
$$= \left(\sqrt{4\cos^2\theta}\right)^3$$
$$= (2\cos\theta)^3$$
$$= 8\cos^3\theta$$

$$= \frac{1}{2}\int \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$
$$= \frac{1}{2}\int \tan\theta\sec\theta\,d\theta$$
$$= \frac{1}{2}\sec\theta + C$$
$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{\sqrt{4-t^2}}\right) + C$$
$$= \boxed{\frac{1}{\sqrt{4-t^2}} + C}$$

Partial Fraction Decomposition

$$\int \frac{6x}{x^3-8}\,dx$$

Partial fraction decomp. first:

$$\frac{6x}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4} = \frac{1}{x-2} + \frac{-1x+2}{x^2+2x+4}$$

$$\frac{6x}{(x-2)(x^2+2x+4)} = \frac{A(x^2+2x+4)+(Bx+C)(x-2)}{(x-2)(x^2+2x+4)}$$

$$6x = Ax^2+2Ax+4A+Bx^2-2Bx+Cx-2C$$

$$0x^2+6x+0 = (A+B)x^2+(2A-2B+C)x+(4A-2C)$$

$$\left.\begin{aligned} A+B &= 0 \\ 2A-2B+C &= 6 \\ 4A \quad -2C &= 0 \end{aligned}\right\}$$

$B=-A$, $\quad -2C=-4A$

$C=2A$

$$2A-2(-A)+(2A)=6$$

$6A=6$

$A=1$

$B=-1$

$C=2$

$$x^2+2x+4 = (x^2+2x+1)+4-1$$

$$= 3+(x+1)^2$$

complete square

So we have

$u=x-2$, $du=dx$ $\qquad$ $u=x^2+2x+4$, $du=(2x+2)dx$

$$\int \frac{6x}{x^3-8}\,dx = \int \frac{dx}{x-2} - \int \frac{x-2}{x^2+2x+4}\,dx$$

$$= \int \frac{du}{u} - \frac{1}{2}\int \frac{2(x-2)}{x^2+2x+4}\,dx$$

$$= \ln|u| - \frac{1}{2}\int \frac{2x-4+6-6}{x^2+2x+4}\,dx$$

$$= \ln|x-2| - \frac{1}{2}\left[\int \frac{2x+2}{x^2+2x+4}\,dx - 6\int \frac{dx}{x^2+2x+4}\right]$$

$$= \ln|x-2| - \frac{1}{2}\int \frac{du}{u} + 3\int \frac{dx}{3+(x+1)^2}$$

$$= \ln|x-2| - \frac{1}{2}\ln|u| + \frac{3}{\sqrt{3}}\arctan\left(\frac{x+1}{\sqrt{3}}\right) + C$$

$$= \boxed{\ln|x-2| - \frac{1}{2}\ln|x^2+2x+4| + \sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}}\right) + C}$$