MATH 251/GRACEY NAME Key

EXAM 2/CHAPTER 7.1-7.4

70 POINTS POSSIBLE

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!

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1. (10 POINTS) Sketch the region bounded by the graphs of $f(y) = y^2$, $g(y) = y+2$. and find the area

left most right most

$y^2 = y+2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$

limits of integration

$A = \int_{-1}^{2} \left[(y+2) - (y^2)\right] dy$

$A = \left(\frac{y^2}{2} + 2y - \frac{y^3}{3}\right)\Big|_{-1}^{2}$

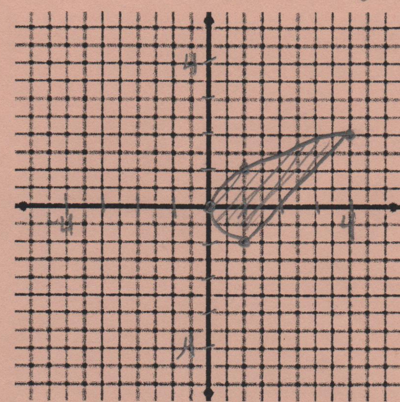
$A = \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$A = 6 - \frac{9}{3} - \frac{1}{2} + 2$

$A = 8 - 3 - \frac{1}{2}$

$A = 5 - \frac{1}{2}$

$\boxed{A = \frac{9}{2} \text{ sq. units}}$



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2. (10 POINTS) Sketch the region bounded by the equations $y = \sqrt{x}$, $y = 2$, $x = 0$ and find the volume of the solid generated by revolving the plane region bounded by the equations about the line $x = -1$.

$x = y^2$

Washer

$R(y) = y^2 - (-1) = y^2 + 1$

$r(y) = 0 - (-1) = 1$

$V = \pi \int_0^2 \left[(y^2+1)^2 - 1^2\right] dy$

$V = \pi \int_0^2 (y^4 + 2y^2) dy$

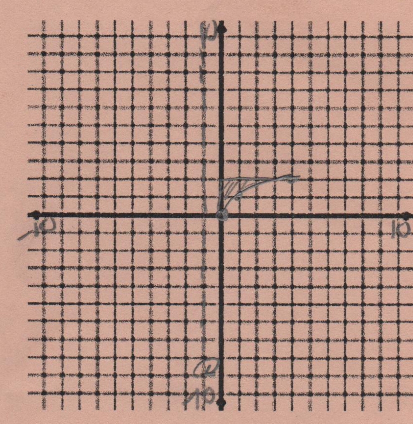
$V = \pi \left(\frac{y^5}{5} + \frac{2}{3}y^3\right)\Big|_0^2$

$V = \pi\left[\left(\frac{32}{5} + \frac{16}{3}\right) - (0)\right]$

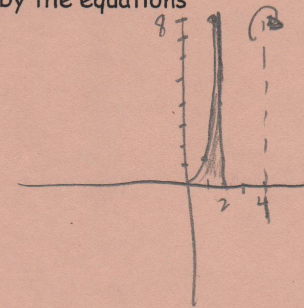
$V = \pi\left(\frac{96+80}{15}\right)$

$V = \frac{\pi}{15}(176)$

$\boxed{V = \frac{176\pi}{15} \text{ cubic units}}$



3. (14 POINTS) SET UP the definite integrals which would find the volume of the solid generated by revolving the plane region bounded by the equations $y = x^3$, $y = 0$, $x = 2$ about the line $x = 4$.



a. Disk and washer method

$R(y) = 4 - \sqrt[3]{y}$, $r(y) = 4 - 2 = 2$

$$\boxed{V = \pi \int_0^8 \left[(4 - \sqrt[3]{y})^2 - (2)^2 \right] dy}$$

b. Shell method

$p(x) = 4 - x$, $h(x) = x^3 - 0 = x^3$

$$\boxed{V = 2\pi \int_0^2 (4-x)(x^3)\, dx}$$

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4. (12 POINTS) Find the arc length of the function $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ over the interval $[1, 2]$.

$y' = \dfrac{3x^2}{6} - \dfrac{1}{2x^2} = \dfrac{x^2}{2} - \dfrac{1}{2x^2}$

$(y')^2 = \dfrac{x^4}{4} - \cancel{2}\left(\dfrac{\cancel{x^2}}{\cancel{2}}\right)\left(\dfrac{1}{\cancel{2x^2}}\right) + \dfrac{1}{4x^4} = \dfrac{x^4}{4} - \dfrac{1}{2} + \dfrac{1}{4x^4}$

$\sqrt{(y')^2 + 1} = \sqrt{\dfrac{x^4}{4} + \dfrac{1}{2} + \dfrac{1}{4x^4}} = \sqrt{\left(\dfrac{x^2}{2} + \dfrac{1}{2x^2}\right)^2} = \dfrac{x^2}{2} + \dfrac{1}{2x^2}$

$s = \dfrac{1}{2} \int_1^2 (x^2 + x^{-2})\, dx$

$s = \dfrac{1}{2} \left(\dfrac{x^3}{3} - \dfrac{1}{x} \right) \Big|_1^2$

$s = \dfrac{1}{2} \left[\left(\dfrac{8}{3} - \dfrac{1}{2} \right) - \left(\dfrac{1}{3} - 1 \right) \right]$

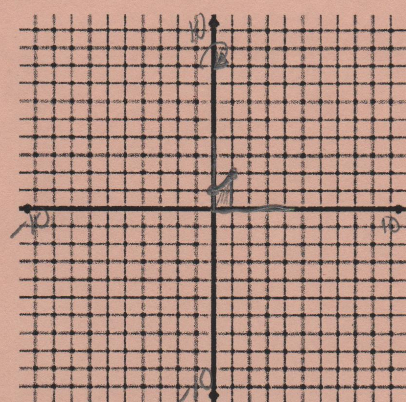
$s = \dfrac{1}{2} \left(\dfrac{7}{3} + \dfrac{1}{2} \right)$

$s = \dfrac{1}{2} \left(\dfrac{14 + 3}{6} \right)$

$\boxed{s = \dfrac{17}{12} \text{ units}}$

5. (12 POINTS) Sketch the region bounded by the equations $y = x^2 + 1,\ y = 0,\ x = 0,$ and $x = 1$ and find the volume of the solid generated by revolving the plane region bounded by the equations about the *y*-axis.

Shell $\quad p(x) = x,\ h(x) = (x^2+1) - 0 = x^2+1$



$$V = 2\pi \int_0^1 x(x^2+1)\,dx$$

$$V = 2\pi \int_0^1 (x^3+x)\,dx$$

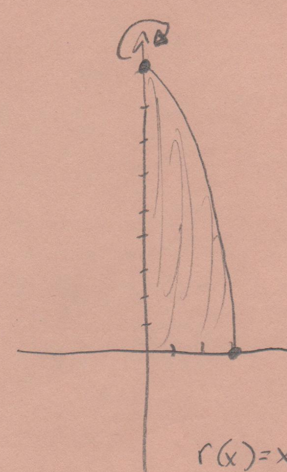
$$V = 2\pi \left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Big|_0^1$$

$$V = 2\pi\left[\left(\frac{1}{4}+\frac{1}{2}\right) - 0\right]$$

$$V = 2\pi\left(\frac{3}{4}\right)$$

$$\boxed{V = \frac{3\pi}{2} \text{ cubic units}}$$

6. (12 POINTS) Find the area of the surface generated by revolving the curve $y = 9 - x^2,\ 0 \le x \le 3$ about the y-axis.



$r(x) = x$

$$y' = -2x$$

$$\sqrt{1+(y')^2} = \sqrt{1+4x^2}$$

$$S = \frac{2\pi}{8} \int_0^3 8x\sqrt{1+4x^2}\,dx$$

$$S = \frac{\pi}{4}\left(\frac{2}{3}(1+4x^2)^{3/2}\right)\Big|_0^3$$

$$S = \frac{\pi}{6}\left[(1+36)^{3/2} - (1+0)^{3/2}\right]$$

$$\boxed{S = \frac{\pi}{6}\left(37^{3/2} - 1\right) \text{ sq. units}}$$