MATH 251/GRACEY NAME Key

EXAM 1/CHAPTER 5.6-5.8

80 POINTS POSSIBLE DATE ______________________

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!

NO CALCULATOR!

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1. (12 POINTS, 3 POINTS EACH) Evaluate each expression without using a calculator. Fully simplify your answers to a single rational expression, when applicable.

a. $\cosh\left(\frac{1}{2}\right) = \frac{e^{1/2} + e^{-1/2}}{2}$

$= \frac{e^{-1/2}(e+1)}{2}$

$= \boxed{\frac{e+1}{2e^{1/2}}}$

b. $\arctan(\tan \pi)$

$\tan \pi = 0$

$\boxed{\arctan 0 = 0}$

c. $\operatorname{arc}\cot\left(-\sqrt{3}\right) = \boxed{\frac{5\pi}{6}}$

d. $\sinh^{-1} 0 = \ln\left(0 + \sqrt{0^2+1}\right)$

$= \ln 1$

$= \boxed{0}$

2. (24 POINTS, 6 POINTS EACH) Evaluate the following derivatives with respect to *x*. Simplify each result to a single rational expression with positive exponents, when applicable.

a. $g(x) = \arcsin x + \arccos x$

$g'(x) = \frac{1}{1-x^2} - \frac{1}{1-x^2}$

$\boxed{g'(x) = 0}$

b. $f(x) = x \arctan 2x - \frac{1}{4}\ln\left(1+4x^2\right)$

$f'(x) = \left[1 \arctan 2x + x\left(\frac{2}{1+4x^2}\right)\right] - \frac{1}{\cancel{4}}\left(\frac{\overset{2}{\cancel{8}}x}{1+4x^2}\right)$

$f'(x) = \arctan 2x + \frac{2x}{1+4x^2} - \frac{2x}{1+4x^2}$

$\boxed{f'(x) = \arctan 2x}$

c. $g(x) = \ln(\cosh x)$

$g'(x) = \frac{\sinh x}{\cosh x}$

$\boxed{g'(x) = \tanh x}$

d. $y = \operatorname{sech}^2 2x$

$\frac{dy}{dx} = 2(\operatorname{sech} 2x)^1(-\operatorname{sech} 2x \tanh 2x)(2)$

$\boxed{\frac{dy}{dx} = -4\operatorname{sech}^2 2x \tanh 2X}$

3. (8 POINTS) Verify the identity.

$$e^{2x} = \sinh 2x + \cosh 2x$$

$$e^{2x} = \frac{e^{2x} - e^{-2x}}{2} + \frac{e^{2x} + e^{-2x}}{2}$$

$$e^{2x} = \frac{e^{2x} - \cancel{e^{-2x}} + e^{2x} + \cancel{e^{-2x}}}{2}$$

$$e^{2x} = \frac{\cancel{2}e^{2x}}{\cancel{2}}$$

$$\Rightarrow e^{2x} = e^{2x} \quad //$$

4. (28 POINTS, 7 POINTS EACH) Evaluate the indefinite integral.

a. $\int x^2 \operatorname{sech}^2 x^3 \, dx$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$= \int \cancel{x^2} \operatorname{sech}^2 u \left(\frac{du}{3\cancel{x^2}}\right)$$

$$= \frac{1}{3}\int \operatorname{sech}^2 u \, du$$

$$= \frac{1}{3}\tanh u + C$$

$$= \boxed{\frac{\tanh x^3}{3} + C}$$

c. $\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\, du$

$$= \int \frac{1}{\cancel{\sqrt{x}}\sqrt{1+u^2}} (2\cancel{\sqrt{x}}\, du)$$

$$= 2\int \frac{du}{\sqrt{1+u^2}}$$

$$= 2\left(\ln\left(u + \sqrt{u^2+1}\right)\right) + C$$

$$= \boxed{2\ln\left(\sqrt{x} + \sqrt{x+1}\right) + C}$$

b. $\int \frac{\sin x}{7 + \cos^2 x} dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$a^2 = 7$

$a = \sqrt{7}$

$$= \int \frac{\cancel{\sin x}}{(\sqrt{7})^2 + u^2} \left(\frac{du}{-\cancel{\sin x}}\right)$$

$$= -\int \frac{du}{(\sqrt{7})^2 + u^2}$$

$$= -\frac{1}{\sqrt{7}} \arctan \frac{u}{\sqrt{7}} + C$$

$$= \boxed{-\frac{1}{\sqrt{7}} \arctan\left(\frac{\cos x}{\sqrt{7}}\right) + C}$$

d. $\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$

$u = \arcsin x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = \sqrt{1-x^2}\, du$

$$= \int \frac{u}{\cancel{\sqrt{1-x^2}}} \left(\cancel{\sqrt{1-x^2}} \cdot du\right)$$

$$= \int u \, du$$

$$= \frac{u^2}{2} + C$$

$$= \boxed{\frac{\arcsin^2 x}{2} + C}$$

5. (8 POINTS, 2 POINTS EACH) Find the following limits. (It is ok to write $\infty$ or $-\infty$).

a. $\lim\limits_{x\to\infty} \arctan x = \boxed{\frac{\pi}{2}}$

b. $\lim\limits_{x\to-\infty} \sinh x = \boxed{-\infty}$

c. $\lim\limits_{x\to-\infty} \operatorname{arc\,csc} x = \boxed{0}$

d. $\lim\limits_{x\to\infty} \left(\cosh 2x + \sinh 2x\right) = \infty + \infty$

$= \boxed{\infty}$