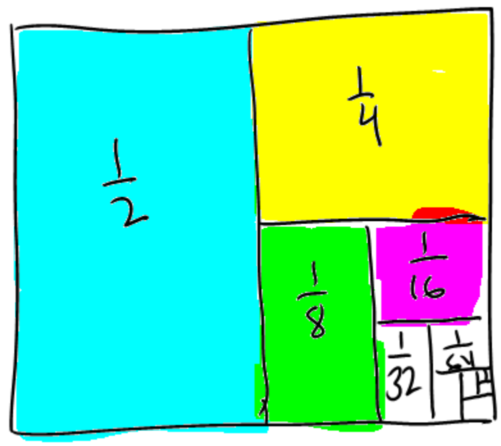
4/15/11

- Warmup by working #2 on 9.2 worksheet
- Finish 9.2
- Lecture 9.3

Next week

Spring break!

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$$



$S_1 = \frac{1}{2}$

$S_2 = \frac{3}{4}$

$S_3 = \frac{7}{8}$

$\vdots$

$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n}$

$S_n = \frac{2^n - 1}{2^n}$

$$\lim_{n \to \infty} \frac{2^n - 1}{2^n} = 1$$

So this series converges and its sum is 1.

<u>Theorem</u>: Convergence of a geometric series

A geometric series with ratio r diverges if $|r| \geq 1$. ($r \leq -1$ or $r \geq 1$)

If $0 < |r| < 1$, then the series converges to the sum

$$\sum_{n=0}^{\infty} a \cdot r^n = \frac{a}{1-r}, \quad 0 < |r| < 1.$$

1. Find the first 5 terms of the sequence of partial sums.

a. $\underbrace{1}_{a_1}+\underbrace{\frac{1}{2}}_{a_2}+\underbrace{\frac{1}{4}}_{a_3}+\underbrace{\frac{1}{8}}_{a_4}+\underbrace{\frac{1}{16}}_{a_5}+\cdots$

$S_1 = 1$

$S_2 = a_1 + a_2 = 1 + \frac{1}{2} = \frac{3}{2}$

$S_3 = a_1 + a_2 + a_3 = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$

$S_4 = a_1 + a_2 + a_3 + a_4 = \frac{7}{4} + \frac{1}{8} = \frac{15}{8}$

$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = \frac{15}{8} + \frac{1}{16} = \frac{31}{16}$

$\boxed{\{S_5\} = \{1, 3/2, 7/4, 15/8, 31/16\}}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$

$a_1 = 1$

$a_2 = -\frac{1}{2}$

$a_3 = \frac{1}{6}$

$a_4 = -\frac{1}{24}$

$a_5 = \frac{1}{120}$

$S_1 = 1$

$S_2 = 1 + \left(-\frac{1}{2}\right) = \frac{1}{2}$

$S_3 = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

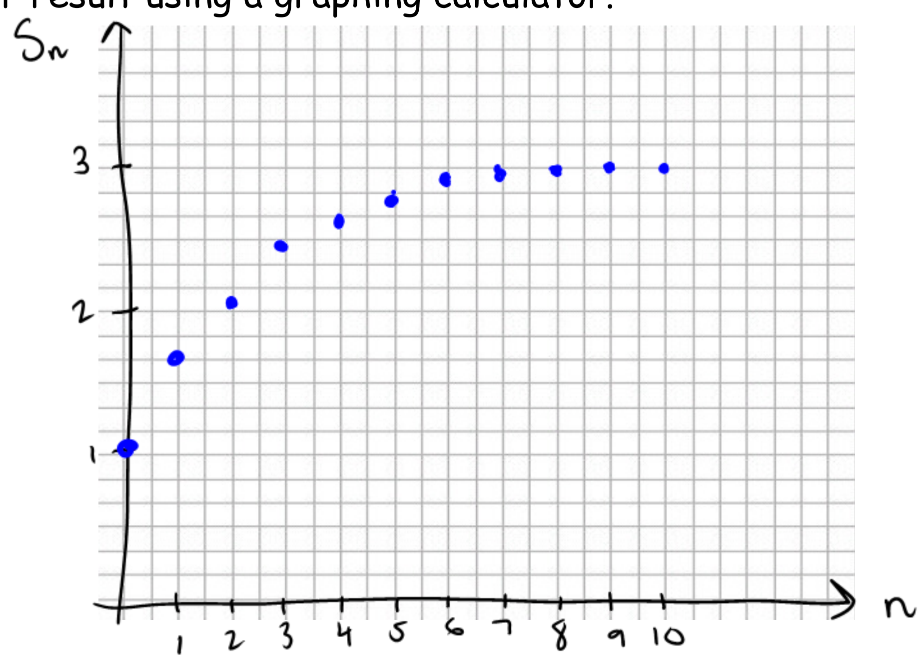
$S_4 = \frac{2}{3} + \left(-\frac{1}{24}\right) = \frac{15}{24}$

$S_5 = \frac{15}{24} + \frac{1}{120} = \frac{76}{120} = \frac{19}{30}$

$\boxed{\{S_5\} = \left\{1, \frac{1}{2}, \frac{2}{3}, \frac{15}{24}, \frac{19}{30}\right\}}$

2. Graph the first 10 terms of the sequence of partial sums given by $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n$ by hand and then check your result using a graphing calculator.

n	S_n
0	1
1	1.67
2	2.11
3	2.41
4	2.61
5	2.74
6	2.83
7	2.89
8	2.93
9	2.97
10	3.00



Write $0.\overline{16}$ as the ratio of 2 integers

$$0.16161616\ldots = 0.16 + 0.0016 + .000016 + \ldots$$

$$= \frac{16}{100} + \frac{16}{10000} + \frac{16}{1000000}$$

$$= \frac{16}{10^2} + \frac{16}{10^4} + \frac{16}{10^6}$$

$$= \frac{16}{10^2}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots\right)$$

$$= \frac{16}{10^2}\sum_{n=0}^{\infty}\left(\frac{1}{10^2}\right)^n$$

$$= \frac{16}{100}\left(\frac{1}{1-\frac{1}{100}}\right)$$

$$= \frac{16}{100}\left(\frac{1}{\frac{99}{100}}\right)$$

$$= \frac{16}{\cancel{100}} \cdot \frac{\cancel{100}}{99}$$

$$= \boxed{\frac{16}{99}}$$

3. Verify that the infinite series converges.

a. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right)$

telescoping series

$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$

$1 = An + 2A + Bn$

$0n + 1 = (A+B)n + 2A$

$\left.\begin{aligned} A+B &= 0 \\ 2A &= 1 \end{aligned}\right\}$

$A = \frac{1}{2},\ B = -\frac{1}{2}$

$= \frac{1}{2}\left[\left(1 - \cancel{\tfrac{1}{3}}\right) + \left(\tfrac{1}{2} - \cancel{\tfrac{1}{4}}\right) + \left(\cancel{\tfrac{1}{3}} - \tfrac{1}{5}\right) + \left(\cancel{\tfrac{1}{4}} - \tfrac{1}{6}\right) + \ldots\right]$

As we approach infinity, all terms will zero out except for the sum $1 + \frac{1}{2}$. So $S = \frac{1}{2}\left(\frac{3}{2}\right) = \boxed{\frac{3}{4}}$

Telescoping Series

$(b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + \cdots$

$S = b_1 - \lim_{n\to\infty} b_{n+1}$

b. $\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n = -2 + \sum_{n=0}^{\infty} 2\left(-\frac{1}{2}\right)^n$

$a_n = 2\left(-\frac{1}{2}\right)^n$

$a_0 = 2 \cdot 1 = 2$

$\sum_{n=0}^{\infty} 2\left(-\frac{1}{2}\right)^n$

$r = -\frac{1}{2},\ |r| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

so $0 < |r| = \frac{1}{2} < 1$ so

this is a convergent geometric series

$a = 2$

$r = -\frac{1}{2}$

$= -2 + \frac{2}{1 - \left(-\frac{1}{2}\right)}$

$= -2 + \frac{2}{1 + \frac{1}{2}}$

$= -2 + 2 \cdot \frac{2}{3}$

$= -2 + \frac{4}{3}$

$= -\frac{6}{3} + \frac{4}{3}$

$= \boxed{-\frac{2}{3}}$

4. Find the sum of the convergent series.

a. $8+6+\frac{9}{2}+\frac{27}{8}+\cdots = \sum_{n=0}^{\infty} 8\left(\frac{3}{4}\right)^n = \frac{8}{1-\frac{3}{4}} = \frac{8}{\frac{1}{4}} = \boxed{32}$

to see if there's a common ratio:

$\frac{a_{n+1}}{a_n} \rightarrow \frac{6}{8} = \frac{3}{4}$

$\frac{\frac{9}{2}}{6} = \frac{\cancel{9}^{3}}{2}\cdot\frac{1}{\cancel{6}_{2}} = \frac{3}{4}$

common ratio: $r=\frac{3}{4}$ and $0<\frac{3}{4}<1$ so this is a convergent geometric series

b. $\sum_{n=1}^{\infty}\left[(0.7)^n+(0.9)^n\right]$

$\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$

$= -2 + \left[\sum_{n=0}^{\infty}(0.7)^n + \sum_{n=0}^{\infty}(0.9)^n\right]$

$= -2 + \left[\frac{1}{1-0.7} + \frac{1}{1-.9}\right]$

$= -2 + \frac{10}{3} + 10$

$= \boxed{34/3}$

$r_1 = 0.7$, $r_2 = 0.9$

$0<.7<1$ $\quad 0<.9<1$

both convergent geometric series

5. Write the repeating decimal $0.\overline{9}$ as a geometric series and write its sum as the ratio of two integers.

$0.9999\ldots = 0.9 + 0.09 + 0.009 + 0.0009 + \cdots$

$= \frac{9}{10^1} + \frac{9}{10^2} + \frac{9}{10^3} + \frac{9}{10^4} + \cdots$

$= \frac{9}{10^1}\left(1 + \frac{1}{10^1} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots\right)$

$= \frac{9}{10}\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n$

$= \frac{9}{10}\left(\frac{1}{1-\frac{1}{10}}\right)$

$= \frac{9}{10}\cdot\frac{1}{\frac{9}{10}}$

$= \frac{9}{10}\cdot\frac{10}{9}$

$= 1$

WTF?! LOL

Properties of Infinite series

A, B, and c are real numbers. $\sum a_n$ and $\sum b_n$ are convergent series.

① $\displaystyle\sum_{n=1}^{\infty} c a_n = cA$

② $\displaystyle\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n = A + B$

③ $\displaystyle\sum_{n=1}^{\infty} (a_n - b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n = A - B$

Thm: Limit of the nth term of a convergent series

If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$.

Thm: nth term test for *DIVERGENCE*

If $\displaystyle\lim_{n\to\infty} a_n \neq 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ DIVERGES

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6. Determine the convergence or divergence of the series.

a. $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

$\lim_{n \to \infty} \frac{n+1}{2n-1} = \frac{1}{2} \neq 0$

So... $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ diverges by the nth term test for divergence

b. $\sum_{n=1}^{\infty} \ln \frac{1}{n}$

$\sum_{n=1}^{\infty} \ln \frac{1}{n}$ diverges by the nth term test for divergence

$\ln \frac{1}{n} = \ln 1 - \ln n$
$= -\ln n$

So... $\lim_{n \to \infty} -\ln n = -\infty \neq 0$

c. $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

Diverges by the nth term test for divergence

$\lim_{n \to \infty} \frac{3^n}{n^3} = \lim_{n \to \infty} \frac{\ln 3 \cdot 3^n}{3n^2}$
$= \lim_{n \to \infty} \frac{(\ln 3)^2 \cdot 3^n}{6n}$
$= \lim_{n \to \infty} \frac{(\ln 3)^3 \cdot 3^n}{6}$
$= \infty$
$\neq 0$

d. $a_n = \left(-\frac{2}{3}\right)^n$ not a series!

Whoops!!

e. $a_n = ne^{-n/2}$

7. Use the Bounded Monotonic Sequences theorem to show that the sequence with the given *n*th term converges and use a graphing calculator to graph the first 10 terms of the sequence and find its limit.

$$a_n = 4 + \frac{1}{2^n}$$