3/7/11

- Warm up using prob. e from WS 8.4
- lecture 8.5
  - ↳ integrals requiring partial fraction decomp.

Wednesday

Lecture 8.7
↳ L'Hôpital's Rule

Friday

8.8
↳ Improper Integrals

Wed., 3/16/11

- Exam 3/Ch. 8 (no table formulas permitted)
- HW 8.1-8.5, 8.7, 8.8 is due
- 8.6 HW is extra credit

$$\frac{5}{2} = \frac{2}{2} + \frac{3}{2}$$

or $$= \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$$

MATH 251/GRACEY NAME______________________
WORKSHEET/8.5

PARTIAL FRACTION DECOMPOSITION

Decomposition of $N(x)/D(x)$ into Partial Fractions

1. **Divide if improper:** If $N(x)/D(x)$ is an improper fraction (the degree of the numerator is greater than the degree of the denominator), divide the denominator into the numerator to obtain
$$\frac{N(x)}{D(x)} = (\text{a polynomial}) + \frac{N_1(x)}{D(x)}$$
Where the degree of $N_1(x)$ is less than the degree of $D(x)$. Then

apply steps 2, 3, and 4 to the proper rational expression $\frac{N_1(x)}{D(x)}$.

2. **Factor the denominator:** Completely factor the denominator into factors of the form
$$(px+q)^m \quad \text{and} \quad (ax^2+bx+c)^n$$
Where ax^2+bx+c is irreducible.

3. **Linear factors:** For each factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of *m* fractions.
$$\frac{A_1}{(px+q)^1} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

4. **Quadratic factors:** For each factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of *n* fractions.
$$\frac{B_1x+C_1}{(ax^2+bx+c)^1} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

1. Write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

a. $\dfrac{x}{\left(x^2+3\right)^3} = \dfrac{A_1x+B_1}{(x^2+3)^1} + \dfrac{A_2x+B_2}{(x^2+3)^2} + \dfrac{A_3x+B_3}{(x^2+3)^3}$

b. $\dfrac{3x^2-2}{x^4-4x^2+4} = \dfrac{A_1x+B_1}{(x^2-2)^1} + \dfrac{A_2x+B_2}{(x^2-2)^2}$

$(x^2-2)^2$

<u>Guidelines for solving the basic equation</u>

1. Expand the basic equation.
2. Collect terms according to powers of *x*.
3. Equate the coefficients of like powers to obtain a system of linear equations involving *A*, B, C, and so on.
4. Solve the resulting system of equations.

2. Rewrite the given rational expression as a sum of partial fractions.

$$\frac{4x^2+2x-1}{x^3+x^2} = \boxed{\frac{3}{x} - \frac{1}{x^2} + \frac{1}{x+1}}$$

$$\frac{4x^2+2x-1}{(x)^2(x+1)} = \frac{A_1}{x^1} + \frac{A_2}{x^2} + \frac{A_3}{x+1}$$

$$\frac{4x^2+2x-1}{x^2(x+1)} = \frac{A_1(x(x+1)) + A_2(x+1) + A_3(x^2)}{x^2(x+1)}$$

$$4x^2+2x-1 = A_1x^2 + A_1x + A_2x + A_2 + A_3x^2$$

$$4x^2+2x-1 = (A_1+A_3)x^2 + (A_1+A_2)x + A_2$$

$$\begin{cases} A_1 + A_3 = 4 \\ A_1 + A_2 = 2 \\ A_2 = -1 \end{cases}$$

$A_1 + (-1) = 2$

$A_1 = 3$

$(3) + A_3 = 4$

$A_3 = 1$

3. Find the integral.

a. $\int \frac{x+2}{x^2-4x}dx = \int\left(-\frac{1}{2x}+\frac{3}{2(x-4)}\right)dx = -\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x-4| + C$

$= \ln|x^{-1/2}| + \ln|(x-4)^{3/2}| + C$

$= \ln\left[x^{-1/2}(x-4)^{3/2}\right] + C$

$= \boxed{\ln\sqrt{\frac{(x-4)^3}{x}} + C}$

PFD

$\frac{x+2}{x(x-4)} = \frac{A_1}{x} + \frac{A_2}{x-4} = -\frac{1}{2x} + \frac{3}{2(x-4)}$

$\frac{x+2}{x(x-4)} = \frac{A_1(x-4)+A_2(x)}{x(x-4)}$

$1x+2 = A_1x - A_14 + A_2x$

$1x+2 = (A_1+A_2)x - 4A_1$

$\begin{cases} A_1 + A_2 = 1 \\ -4A_1 = 2 \end{cases}$

$-4A_1 = 2$, $A_1 = -\frac{1}{2}$ | $-\frac{1}{2} + A_2 = 1$, $A_2 = \frac{3}{2}$

b. $\int \frac{x^3-x+3}{x^2+x-2}dx = \int(x-1)dx + \int\frac{2x+1}{x^2+x-2}dx$

$= \boxed{\frac{x^2}{2} - x + \ln|x^2+x-2| + C}$

$u = x^2+x-2$

$\frac{du}{dx} = 2x+1$

Divide

$$\begin{array}{r} x-1+\frac{2x+1}{x^2+x-2} \\ (x^2+x-2)\overline{)x^3+0x^2-x+3} \\ -(x^3+x^2-2x) \\ \hline -x^2+x+3 \\ -(-x^2-x+2) \\ \hline 2x+1 \end{array}$$

c. $\int \frac{\sin x \cancel{+2}}{\cos x + \cos^2 x} dx = \int \frac{\cancel{\sin x}}{u+u^2}\left(\frac{du}{-\cancel{\sin x}}\right)$

$u = \cos x$

$\frac{du}{dx} = -\sin x dx$

$dx = \frac{du}{-\sin x}$

PFD

$\frac{1}{u(u+1)} = \frac{A_1}{u} + \frac{A_2}{u+1} = \frac{1}{u} - \frac{1}{u+1}$

$\frac{1}{u(u+1)} = \frac{A_1(u+1) + A_2(u)}{u(u+1)}$

$1 = A_1 u + A_1 + A_2 u$

$0u + 1 = (A_1 + A_2)u + A_1$

$\begin{cases} A_1 + A_2 = 0 \\ A_1 = 1 \end{cases}$

$1 + A_2 = 0$

$A_2 = -1$

$= -\int \frac{1}{u(u+1)} du$

$= -\int \left(\frac{1}{u} - \frac{1}{u+1}\right) du$

$= -\left(\ln|u| - \ln|u+1|\right) + C$

$= \ln|1+\cos x| - \ln|\cos x| + C$

$= \ln\left|\frac{1+\cos x}{\cos x}\right| + C$

$= \boxed{\ln|1+\sec x| + C}$