3/21/11

- Finish lecture 10.1
- Lecture 10.2

Next Wednesday and Friday:
NO class
↳ out-of-class assignment due on Mon. 3/28

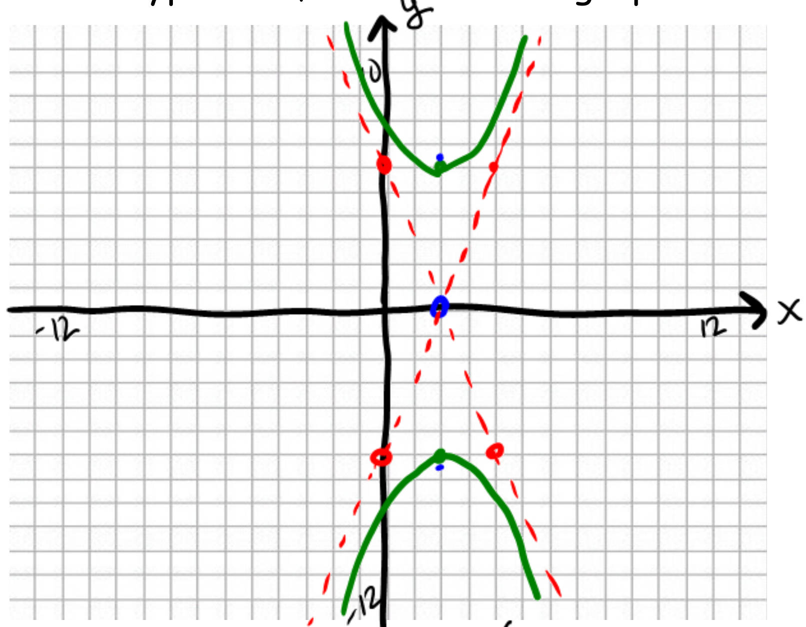
3. Find the center, foci, and vertices of the hyperbola, and sketch its graph using asymptotes as an aid.

$$y^2 - 9x^2 + 36x - 72 = 0$$

$y^2 - 9(x^2 - 4x + (-2)^2) = 72 - 36$

$y^2 - 9(x-2)^2 = 36$

$\frac{y^2}{6^2} - \frac{(x-2)^2}{2^2} = 1$



transverse axis vertical

center: $(2,0)$

$a=6$, $b=2$, $c=\sqrt{6^2+2^2} = \sqrt{40} = 2\sqrt{10} \approx 6.32$

Foci: $(2, 2\sqrt{10})$, $(2, -2\sqrt{10})$

Asymptotes: $y = 0 + \frac{6}{2}(x-2)$

$y = 3x - 6$

$y = 0 - \frac{6}{2}(x-2)$

$y = -3x + 6$

4. Find an equation of the conic section.

a. An ellipse with vertices $(0,2)$ and $(4,2)$ and eccentricity $\frac{1}{2}$.

$(h,k) = (2,2)$

$a=2$

$c^2 = a^2 - b^2$

$1 = 2^2 - b^2$

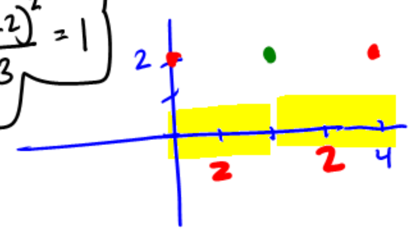
$-3 = -b^2 \rightarrow b^2 = 3$

$e = \frac{c}{a}$

$\frac{1}{2} = \frac{c}{2}$

$c = 1$

$\boxed{\frac{(x-2)^2}{4} + \frac{(y-2)^2}{3} = 1}$



b. A parabola with focus $(2,2)$ and directrix $x = -2$.

sideways parabola

$x = h - p \rightarrow -2 = h - p$

$F = h + p \rightarrow 2 = h + p$

$0 = 2h$ and $2 = 0 + p$

$0 = h$ $\quad 2 = p$

$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 4(2)(x-0)$

$\boxed{(y-2)^2 = 8x}$

c. A hyperbola with vertices $(0, \pm 3)$ and asymptotes: $y = \pm 3x$.

5. Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

a. $12x^2 - 2x + 12y^2 = 15$ circle

b. $x + y^2 - 5y = 1$ parabola

c. $-x^2 - 2x + 3y^2 = 5y^2 - 2 \rightarrow 2 = x^2 + 2x + 2y^2$ ellipse

d. $65x^2 = y^2 + 1$ hyperbola

e. $2x^2 - 8x + y^2 + 5y = 6$ ellipse

6. Find an equation for (a) the tangent lines and (b) normal lines to the hyperbola $\frac{d}{dx}\left(\frac{y^2}{4} - \frac{x^2}{2}\right) = \frac{d}{dx}1$ at $x = 4$.

Step1: Find y when x = 4

$\frac{y^2}{4} - \frac{(4)^2}{2} = 1$

$\frac{y^2}{4} = 9$

$y^2 = 36$

$y = \pm 6$

Step2: Find slope at (4,6) and (4,−6)

$\frac{2y}{4}\frac{dy}{dx} - \frac{2x}{2} = 0$

$\frac{y}{2}\frac{dy}{dx} = x$

$\frac{dy}{dx} = \frac{2x}{y}$

$\left.\frac{dy}{dx}\right|_{(4,6)} = \frac{2(4)}{(6)} = \frac{4}{3}$

$\left.\frac{dy}{dx}\right|_{(4,-6)} = \frac{2(4)}{(-6)} = -\frac{4}{3}$

normal line slopes

at $(4,6) \rightarrow -\frac{3}{4}$

at $(4,-6) \rightarrow \frac{3}{4}$

Step3: Equations

Tangent lines:

at $(4,6) \rightarrow y - 6 = \frac{4}{3}(x-4)$

at $(4,-6) \rightarrow y + 6 = -\frac{4}{3}(x-4)$

Normal lines

at $(4,6) \rightarrow y - 6 = -\frac{3}{4}(x-4)$

at $(4,-6) \rightarrow y + 6 = \frac{3}{4}(x-4)$

7. Find the point on the graph of $x^2 = 8y$ that is closest to the focus of the parabola.

$y = \frac{x^2}{8}$

Step 1: Analysis

Let (x, y) be a point on the parabola

vertex is $(0,0)$ and the focus is $(0, 0+p) = (0, 2)$

$4p = 8$

$p = 2$

Let S be the square of the distance

Step 2: Primary Equation

$S = (x-0)^2 + (y-2)^2$

$S = x^2 + (y-2)^2$

Step 3: Reduce Primary

$S(x) = x^2 + \left(\frac{x^2}{8} - 2\right)^2$

Step 4: Optimize

$S'(x) = 2x + 2\left(\frac{x^2}{8} - 2\right)' \cdot \frac{x}{4}$

$S'(x) = 2x + \frac{x^3}{16} - x$

$S'(x) = x + \frac{x^3}{16}$

$S'(x) = \frac{16x(1+x^2)}{16}$

$0 = 16x(1+x^2)$

$x = 0$

$y = \frac{0^2}{8} = 0$

Step 5: Conclusion

$\sqrt{S} = \sqrt{0^2 + (0-2)^2} = \sqrt{4} = 2$

The point closest to the focus is $(0,0)$ and the min. distance is 2

8. Find the arc length of the parabola $4x - y^2 = 0$ over the interval $0 \le y \le 4$.

10.2: Plane curves and parametric equations

Definition of a Plane Curve

If f and g are continuous functions of t on an interval I, then the equations

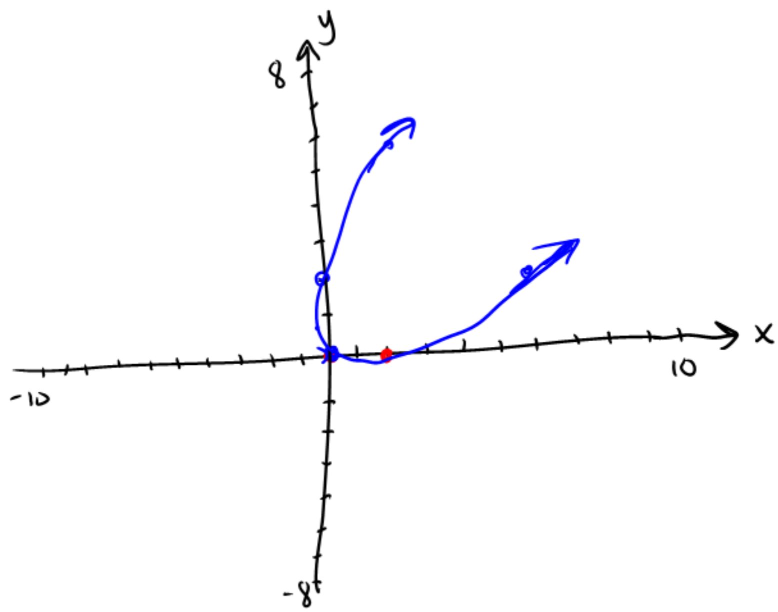
$$x = f(t) \quad \text{and} \quad y = g(t)$$

are called **parametric equations** and t is called the **parameter**. The set of points (x, y) obtained as t varies over the interval I is called the graph of the parametric equations. Taken together, the parametric equations and the graph are called a **plane curve**, denoted by C.

Example:

$x = t^2 + t$, $y = t^2 - t$

t	x	y	(x, y)
0	0	0	(0, -1)
1	2	0	(2, 0)
2	6	2	(6, 3)
-1	0	2	(0, 2)
-2	+2	6	(2, 3)
$-1/2$	$-1/4$	$+3/4$	$(-1/4, -3/4)$



eliminate the parameter

$x = t^2 + t$, $y = t^2 - t$

$$\begin{array}{r} t^2 + t = x \\ -(t^2 - t) = -y \\ \hline 2t = x - y \end{array}$$

$t = \dfrac{x - y}{2}$

$y = \dfrac{(x-y)^2}{4} - \dfrac{x - y}{2}$