3/11/11

- Finish 8.8

Prep for Monday:

- Finish Ch. 8 practice worksheet
- Finish homework

Wednesday

- Exam 3/Ch. 8
- 8.1–8.5, 8.7, 8.8
- HW due
  → 8.6 extra credit

Warm up:

① $\int \ln 2x \, dx$

$u = \ln 2x$

$du = \frac{2}{2x} dx$

$du = \frac{dx}{x}$

$dv = dx$

$v = x$

$= x\ln 2x - \int x \cdot \frac{dx}{\cancel{x}}$

$= \boxed{x\ln 2x - x + C}$

② $\int \frac{(\ln 2x)^3}{x} dx = \int \frac{u^3}{\cancel{x}} \cdot \cancel{x} du$

$u = \ln 2x$

$\frac{du}{dx} = \frac{2}{2x}$

$dx = x\,du$

$= \frac{u^4}{4} + C$

$= \boxed{\frac{(\ln 2x)^4}{4} + C}$

8.4 #38

$\int \frac{\sqrt{1-x}}{\sqrt{x}} dx = \int \frac{\cos\theta}{\cancel{\sqrt{x}}} \, 2\cancel{\sqrt{x}} \cos\theta \, d\theta = 2\int \cos^2\theta \, d\theta$

$\sqrt{1-(\sqrt{x})^2}$

$= \sqrt{1-\sin^2\theta}$

$= \cos\theta$

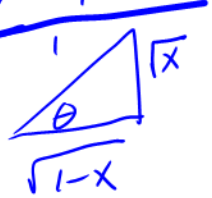
$= \cancel{2}\int \frac{1+\cos 2\theta}{\cancel{2}} d\theta$

$= \theta + \frac{\sin 2\theta}{2} + C$

$= \arcsin\sqrt{x} + (\sqrt{x})(\sqrt{1-x}) + C$

$= \boxed{\arcsin\sqrt{x} + \sqrt{x}\sqrt{1-x} + C}$

$\sqrt{x} = 1\sin\theta$



$\frac{d}{d\theta}\sqrt{x} = \frac{d}{d\theta}\sin\theta$

$\frac{dx/d\theta}{2\sqrt{x}} = \cos\theta$

$dx = 2\sqrt{x}\cos\theta \, d\theta$

$\theta = \arcsin\sqrt{x}$, $\frac{\sin 2\theta}{2} = \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}$

MATH 251/GRACEY NAME________________________

WORKSHEET/8.8

IMPROPER INTEGRALS

Definition of Improper Integrals with Infinite Integration Limits

1. If f is continuous on the interval $[a,\infty)$, then

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

2. If f is continuous on the interval $(-\infty,b]$, then

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

3. If f is continuous on the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx,\ c \text{ is any real number.}$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.

1. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

a. $\int_0^\infty (x-1)e^{-x}dx = \lim_{b\to\infty}\int_0^b (x-1)e^{-x}dx$

$\lim_{b\to\infty}\frac{-b}{e^b} = \lim_{b\to\infty}\frac{-1}{e^b} = 0$

So $\int_0^\infty (x-1)e^{-x}dx$ converges to 0

Consider the indefinite integral first

0.5: 8/8

$\int (x-1)e^{-x}dx = -e^{-x}(x-1) + \int e^{-x}dx = -e^{-x}(x-1) - e^{-x} = -e^{-x}(x-1+1)$

$= -xe^{-x}$

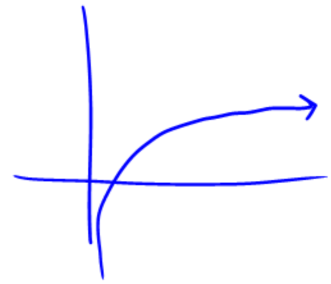
$u = x-1 \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

So $-xe^{-x}\Big|_0^b = -be^{-b} - 0$

$= -be^{-b} = \frac{-b}{e^b}$

b. $\displaystyle\int_1^{\infty}\frac{5}{x}dx = \lim_{b\to\infty}\int_1^b \frac{5}{x}dx$



$$= \lim_{b\to\infty} 5\ln|x| \Big|_1^b$$

$$= 5\left[\lim_{b\to\infty}\left[\ln b - \overset{0}{\ln 1}\right]\right]$$

$$= 5\cdot\infty$$

$$= \infty$$

$\boxed{\text{So } \int_1^{\infty}\frac{5}{x}dx \text{ diverges}}$

Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval $[a,b)$ and has an infinite discontinuity at *b*, then $\displaystyle\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$
2. If f is continuous on the interval $(a,b]$ and has an infinite discontinuity at *a*, then $\displaystyle\int_c^b f(x)\,dx = \lim_{c\to a^+}\int_a^c f(x)\,dx$
3. If f is continuous on the interval $[a,b]$, except for some *c* in (a,b) at which *f* has an infinite discontinuity, then
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise the improper integral **diverges**. In the third case, the improper integral on the left diverges if **either** of the improper integrals on the right diverges.

2. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

V.A. at $x=0$

a. $\displaystyle\int_{-1}^{2}\frac{dx}{x^3} = \int_{-1}^{0}\frac{dx}{x^3} + \int_{0}^{2}\frac{dx}{x^3}$

$$= \lim_{c\to 0^-}\int_{-1}^{c}\frac{dx}{x^3} + \lim_{c\to 0^+}\int_{c}^{2}\frac{dx}{x^3}$$

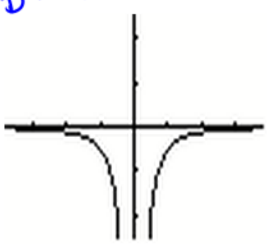
$$= \lim_{c\to 0^-}\left(-\frac{1}{2x^2}\Big|_{-1}^{c}\right) + \cancel{\lim_{c\to 0^+}\left(-\frac{1}{2x^2}\Big|_{c}^{2}\right)}$$

don't need to worry about this one since the first diverges

$$= -\infty$$

$$-\frac{1}{2\left(\frac{-1}{100}\right)^2} = -\frac{10000}{2}$$

BIG



So $\displaystyle\int_{-1}^{2}\frac{dx}{x^3}$ diverges

V.A. at $x=6$

b. $\displaystyle\int_{0}^{6}\frac{4}{\sqrt{6-x}}\,dx = \lim_{c\to 6^-} -\int_{0}^{c} -4(6-x)^{-1/2}\,dx$

$$= -4\lim_{c\to 6^-} 2(6-x)^{1/2}\Big|_{0}^{c}$$

$$= -8\left[\lim_{c\to 6^-}(6-c)^{1/2} - \lim_{c\to 6^-}(6-0)^{1/2}\right]$$

$$= -8\left[(6-6)^{1/2} - 6^{1/2}\right]$$

$$= 8\sqrt{6}$$

So $\displaystyle\int_{0}^{6}\frac{4}{\sqrt{6-x}}\,dx$ converges to $8\sqrt{6}$

Theorem: A Special Type of Improper Integral

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \dfrac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \le 1 \end{cases}$$

3. Evaluate the definite integral.

a. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}dx = \int_1^\infty \frac{1}{x^{1/2}}\,dx$

$$\boxed{p = \tfrac{1}{2} \text{ so } \int_1^\infty \frac{1}{\sqrt{x}}\,dx \text{ diverges } [p \le 1]}$$

b. $\displaystyle\int_1^\infty \frac{5}{\sqrt[5]{x^6}}dx = 5\int_1^\infty \frac{1}{x^{6/5}}\,dx$

$p = \frac{6}{5} > 1$, so $\displaystyle\int_1^\infty \frac{5}{\sqrt[5]{x^6}}\,dx$ converges to $= 5\left(\frac{1}{\frac{6}{5}-1}\right)$

$= \boxed{25}$

c. $\displaystyle\int_0^2 (2-t)\sqrt{t}\,dt$

$= \displaystyle\int_0^2 \left(2t^{1/2} - t^{3/2}\right)dt$

$= \left[2\left(\frac{2}{3}t^{3/2}\right) - \frac{2}{5}t^{5/2}\right|_0^2$

$= \left(\frac{4}{3}2^{3/2} - \frac{2}{5}\cdot 2^{5/2}\right) - (0)$

$= \frac{4}{3}\left(2\sqrt{2}\right) - \frac{2}{5}\left(2^2\sqrt{2}\right)$

$= \frac{8}{3}\sqrt{2} - \frac{8}{5}\sqrt{2}$

$= \dfrac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \boxed{\frac{16}{15}\sqrt{2}}$