2/7/11

- Warm up
  using 7.2 W.S
- Lecture 7.2

Wednesday

7.3

Friday

7.4

When you are done with your homework you should be able to…

- π Find the volume of a solid of revolution using the disk method
- π Find the volume of a solid of revolution using the washer method
- π Find the volume of a solid with known cross sections

Warm-up: Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

a. $f(x) = -x^2 + 4x + 1, \quad g(x) = x + 1$

① Find limits of int

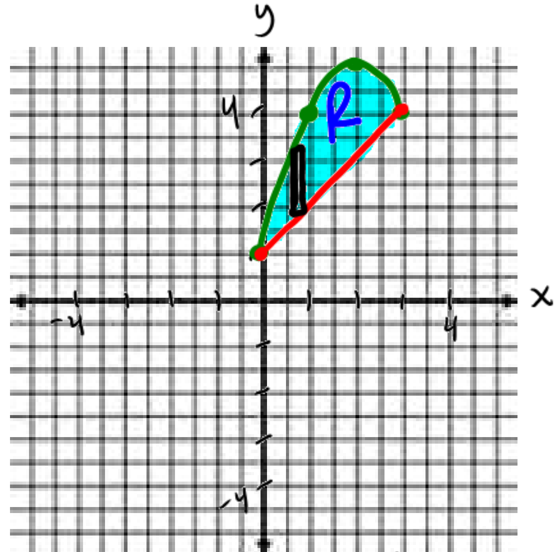
$-x^2 + 4x + 1 = x + 1$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0$ or $x - 3 = 0$

$x = 3$



② Sketch the graph

$f(0) = 1$

$f(3) = -9 + 12 + 1 = 4$

$f(1) = 4$

③ Find the area

$A = \int_0^3 [(-x^2 + 4x + 1) - (x + 1)]\,dx$

$A = \int_0^3 (-x^2 + 3x)\,dx$

$= \left(-\frac{x^3}{3} + \frac{3x^2}{2}\right)\Big|_0^3$

$= \left(-\frac{\overset{9}{\cancel{27}}}{\cancel{3}} + \frac{27}{2}\right) - (0 + 0)$

$= \frac{-18 + 27}{2}$

$= \boxed{\frac{9}{2} \text{ sq. units}}$

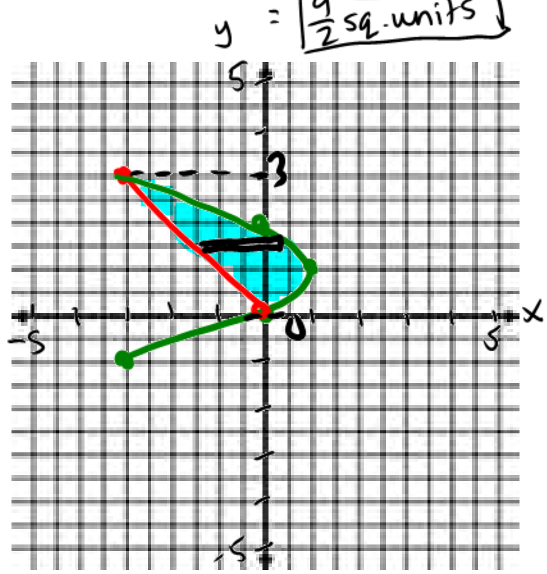
b. $f(y) = y(2 - y), \quad g(y) = \cancel{0}\,-y,$ ~~$y = -1, y = 2$~~

① limits

$2y - y^2 = -y$

$0 = y^2 - 3y$

$0 = y(y - 3)$ → $y = 0$ or $y = 3$



② Sketch graph

$f(-1) = -3 \rightarrow (-3, -1)$

$f(2) = 0 \rightarrow (0, 2)$

$f(0) = 0 \rightarrow (0, 0)$

$f(1) = 1 \rightarrow (1, 1)$

$g(y) = -y \rightarrow x = -y$

$y = -x$

③ Find the area

$A = \int_0^3 [(2y - y^2) - (-y)]\,dy$

$A = \int_0^3 (3y - y^2)\,dy$

$A = \left(\frac{3y^2}{2} - \frac{y^3}{3}\right)\Big|_0^3$

$A = \left(\frac{3 \cdot 9}{2} - \frac{27}{3}\right) - (0 - 0)$

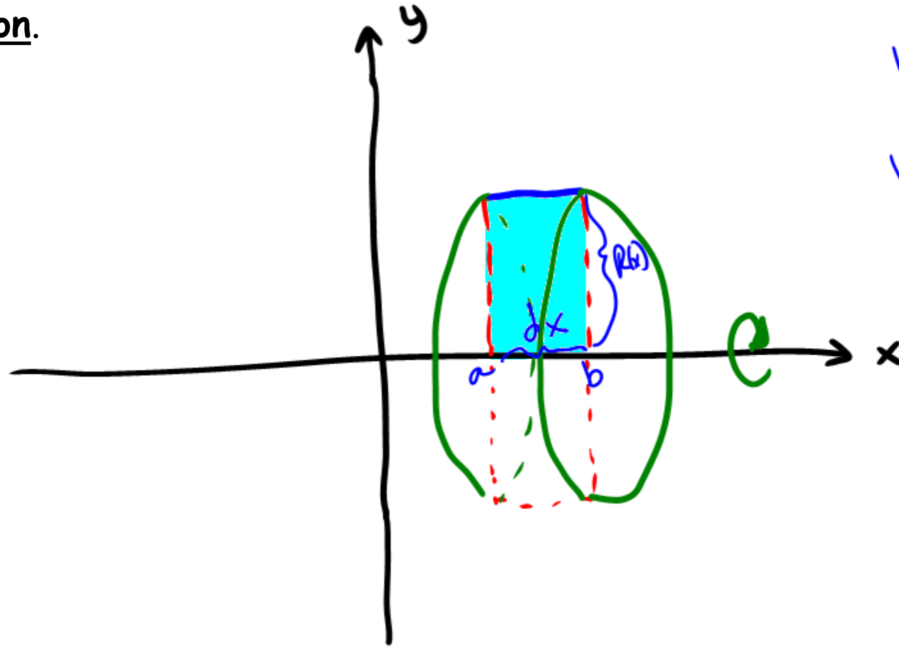
$A = \frac{9}{2}$ sq units

THE DISK METHOD

An important application of the ___definite___ integral is its use in finding the ___volume___ of a three-dimensional solid—one whose ___cross___ sections are ___similar___.

Solids of revolution are used commonly in engineering and manufacturing. Some examples are ___axles___, ___funnels___, ___pills___, ___bottles___, and ___pistons___.

If a ___region___ in the ___plane___ is ___revolved___ about a ___line___, the resulting ___solid___ is a **solid of revolution**, and the line is called the **axis of revolution**.



$V = \pi r^2 h$

$V = \pi \int_a^b [R(x)]^2 dx$

* dx is parallel to the axis of revolution
* R(x) is $\perp$ the axis of the rev.

THE DISK METHOD

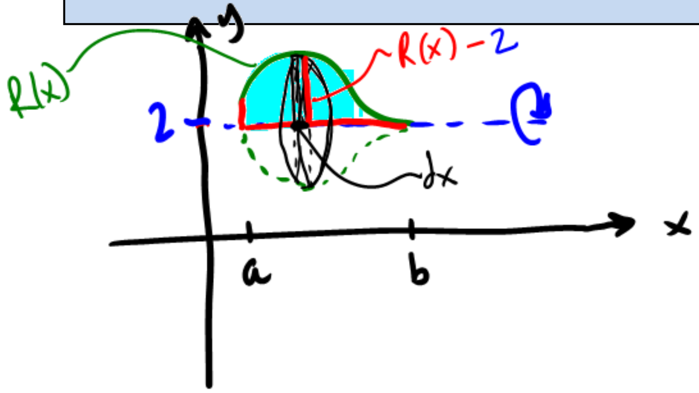
To find the volume of a solid of revolution with the **disk method** use one of the following:

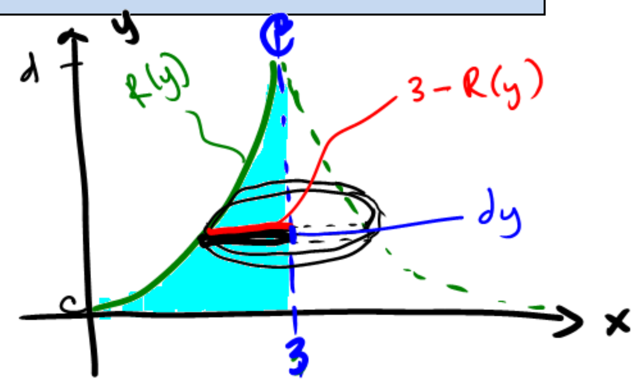
Horizontal Axis of Revolution

$$V=\pi\int_a^b\left[R(x)\right]^2dx$$

Vertical Axis of Revolution

$$V=\pi\int_c^d\left[R(y)\right]^2dy$$





Example 1: Find the volume of the solids generated by revolving the regions bounded by the graphs of the equations about the given line.

a) $y=2x^2,\ y=0,\ x=2,$ about the $x-$axis .

$R(x)=2x^2-0$

$R(x)=2x^2$

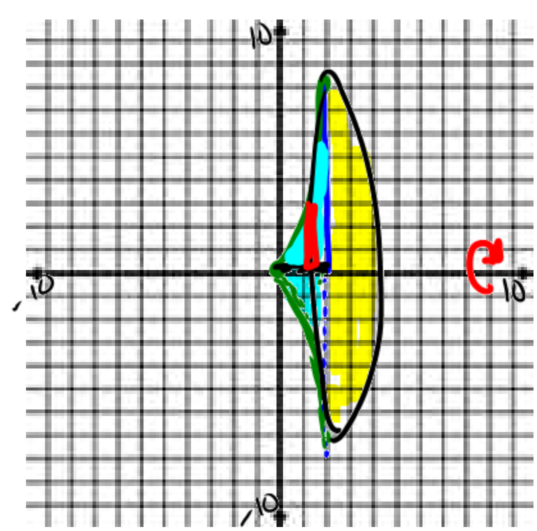
$V=\pi\int_0^2(2x^2)^2dx$

$V=\pi\int_0^2 4x^4dx$

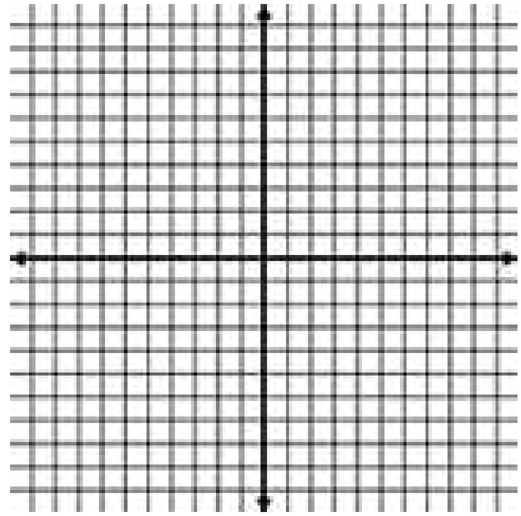
$V=4\pi\left(\frac{x^5}{5}\right)\Big|_0^2$

$V=\frac{4\pi}{5}(32-0)$

$\boxed{V=\frac{128}{5}\pi \text{ cu. units}}$



b) $y = 2x^2, \; y = 0, \; x = 2,$ about the $y-$axis .



THE WASHER METHOD

The disk method can be extended to cover solids of revolution with holes

by replacing the representative disk with a representative

washer.

THE WASHER METHOD

To find the volume of a solid of revolution with the **washer method** use one of the following:

Horizontal Axis of Revolution

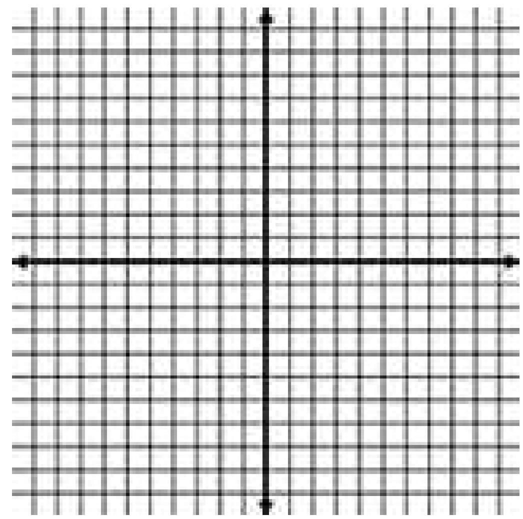
$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

Vertical Axis of Revolution

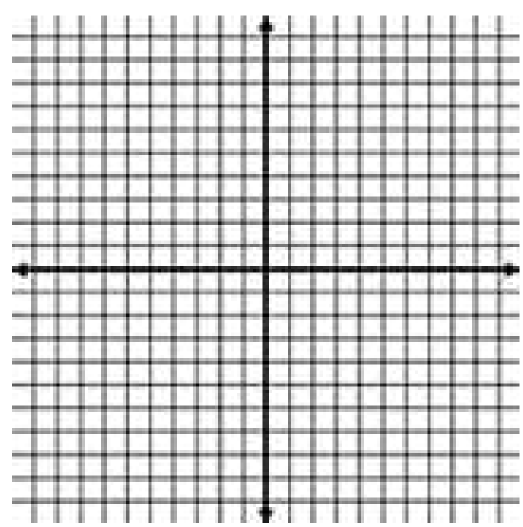
$$V = \pi \int_c^d \left([R(y)]^2 - [r(y)]^2 \right) dy$$

Example 2: Find the volume of the solids generated by revolving the regions bounded by the graphs of the equations about the given line.

a) $y = 2x^2,\ y = 0,\ x = 2,$ about the line $x = 6$.



b) $y = \cos x,\ y = 1,\ x = 0\ , x = \dfrac{\pi}{2}$ about the line $y = 2$.



SOLIDS WITH KNOWN CROSS SECTIONS

With the disk method, you can find the ______________________________ of a solid

having a ___________________________ cross section whose area is _______________. This method can be generalized to solids of any shape, as long as you know a formula for the area of an arbitrary cross section. Some common cross sections

are _________________________, ________________________________,

____________________________, ________________________________, and

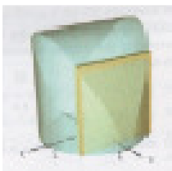
____.

VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the *x*-axis,
$$V = \int_a^b A(x)dx$$
2. For cross sections of area $A(y)$ taken perpendicular to the *y*-axis,
$$V = \int_c^d A(y)dy$$

Example 3: Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the *x*-axis.

a) Squares



b) Semicircles