2/25/11

- Lecture 8.1

Monday

8.2 : Integration by parts

MATH 251/GRACEY NAME ____________________________
WORKSHEET/8.1

BASIC INTEGRATION RULES ($a > 0$)

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C,\ n \neq -1$
5. $\int \dfrac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\dfrac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \arcsin\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\arctan\dfrac{u}{a} + C$
20. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\operatorname{arcsec}\dfrac{|u|}{a} + C$

PROCEDURES FOR FITTING INTEGRANDS TO BASIC RULES

TECHNIQUE	EXAMPLE
Expand (numerator)	$(x^2+8)^2 = x^4+16x^2+64$
Separate numerator	$\dfrac{x-3}{1+x^2} = \dfrac{x}{1+x^2} - \dfrac{3}{1+x^2}$
Complete the square	$\dfrac{1}{\sqrt{4x-x^2}} = \dfrac{1}{\sqrt{4-(x-2)^2}}$
Divide improper rational function	$\dfrac{x^2}{1+x^2} = 1 - \dfrac{1}{1+x^2}$
Add and subtract terms in numerator	$\dfrac{2x}{x^2+6x+9} = \dfrac{2x+6-6}{x^2+6x+9} = \dfrac{2x+6}{x^2+6x+9} - \dfrac{6}{(x+3)^2}$
Use trigonometric identities	$\cos^2 x = \dfrac{1+\cos 2x}{2}$ $\sin^2 x = \dfrac{1-\cos 2x}{2}$
Multiply and divide by Pythagorean conjugate	$\dfrac{1}{1+\cos x} = \dfrac{1}{1+\cos x}\left(\dfrac{1-\cos x}{1-\cos x}\right) = \dfrac{1-\cos x}{1-\cos^2 x} = \dfrac{1-\cos x}{\sin^2 x}$ $= \dfrac{1}{\sin^2 x} - \dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin x}$ $= \csc^2 x - \cot x \csc x$

1. $\displaystyle\int \frac{1-\sin 2\theta}{\cos 2\theta}\,d\theta = \int\left(\frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}\right)d\theta$

$= \int(\sec 2\theta - \tan 2\theta)\,d\theta$

$= \frac{1}{2}\left(\ln|\sec 2\theta + \tan 2\theta| + \ln|\cos 2\theta|\right) + C$

$= \frac{1}{2}\ln|\cos 2\theta(\sec 2\theta + \tan 2\theta)| + C$

$= \boxed{\ln\sqrt{|1+\sin 2\theta|} + C}$

2. $\int \frac{e^{1/t}}{t^2}dt = \int \frac{e^u}{\cancel{t^2}}(-\cancel{t^2}\,du)$

$u = \frac{1}{t}$

$\frac{du}{dt} = -\frac{1}{t^2}$

$dt = -t^2\,du$

$= -\int e^u\,du$

$= -e^u + C$

$= \boxed{-e^{1/t} + C}$

3. $\int \frac{2}{(t-9)^2}dt = 2\int u^{-2}\,du$

$u = t - 9$

$\frac{du}{dt} = 1$

$dt = du$

$= 2\left(\frac{u^{-1}}{-1}\right) + C$

$= \boxed{\frac{-2}{t-9} + C}$

4. $\int \tan x\left[\ln(\cos x)\right]dx = \int \cancel{\tan x} \cdot u\left(-\frac{du}{\cancel{\tan x}}\right)$

$u = \ln\cos x$

$\frac{du}{dx} = \frac{-\sin x}{\cos x}$

$\frac{du}{dx} = -\tan x$

$dx = -\frac{du}{\tan x}$

$= -\int u\,du$

$= -\frac{u^2}{2} + C$

$= \boxed{-\frac{(\ln\cos x)^2}{2} + C}$